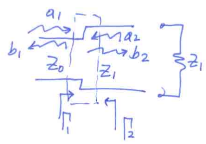
Impedance Step:



Terminate port 2 with Z_1

$$\Gamma_1 = \frac{b_1}{a_1} = S_{11} = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

Terminate port 1 with Z_0

$$\Gamma_2 = \frac{b_2}{a_2} = S_{22} = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1$$

S-Matrix

$$\begin{bmatrix} \Gamma_1 & \sqrt{1-|\Gamma_1|^2} \\ \sqrt{1-|\Gamma_1|^2} & -\Gamma_1 \end{bmatrix} \Leftarrow$$

Since it is lossless

$$|S_{11}|^2 + |S_{21}|^2 = 1 \Rightarrow S_{21} = \sqrt{1-|\Gamma_1|^2} = S_{12} \text{ (Reciprocity)}$$

If we expand $\sqrt{1-|\Gamma_1|^2}$, we find

$$S_{21} = S_{12} = \frac{2\sqrt{Z_0 Z_1}}{(Z_0 + Z_1)}$$

Let us look at it differently: Terminate Port 2 with Z_1 & calculate $V_2 \left(\equiv b_2\sqrt{Z_1}\right)$

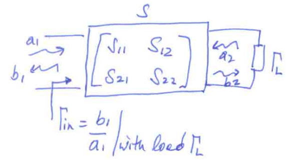
$$V_2 = (1+\Gamma_1)V_1^+ = \frac{2Z_1}{Z_0+Z_1} V_1^+ \longrightarrow \text{Note: } \frac{V_2}{V_1^+} \neq S_{21}$$

because we need to normalize to two different impedances Z_0 & Z_1 to get b_2 & a_1.

and $b_2\sqrt{Z_1} = \dfrac{2Z_1}{Z_0+Z_1} a_1\sqrt{Z_0}$

$$\Rightarrow S_{21} = \frac{b_2}{a_1} = \frac{2\sqrt{Z_1 Z_0}}{Z_0 + Z_1}$$

General S-parameter 2-Port:



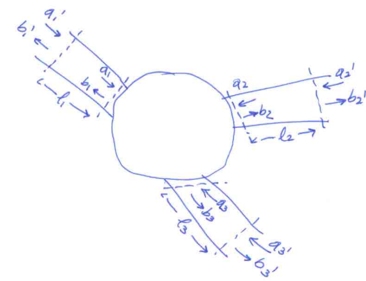
$\Gamma_{in} = \dfrac{b_1}{a_1}\Big|$ with load Γ_L

$$\left\{ \begin{array}{l} \dfrac{a_2}{b_2} = \Gamma_L \\ + \\ b_1 = S_{11}a_1 + S_{12}a_2 \\ b_2 = S_{21}a_1 + S_{22}a_2 \end{array} \right.$$

$$\Rightarrow \boxed{\frac{b_1}{a_1} = S_{11} + \frac{S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L}}$$

← See explanation in class "Follow the waves"

Shift in Terminal Planes:



$$a_1' = a_1 e^{+j\beta_1 l_1}$$

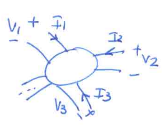
$$b_1' = b_1 e^{-j\beta_1 l_1}$$

$$S_{11}' = S_{11} e^{-2j\beta_1 l_1}$$

& $\boxed{S_{nn}' = S_{nn} e^{-2j\beta_n l_n}}$

Also $\boxed{S_{mn}' = S_{mn} e^{-j(\beta_m l_m + \beta_n l_n)}}$

Impedance & Admittance Matrices:



$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \dots + Z_{1N}I_N$$

$$\vdots$$

$$V_N = Z_{N1}I_1 + Z_{N2}I_2 + \dots Z_{NN}I_N$$

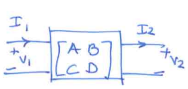
$$[V] = [Z][I] \quad \& \quad [Z] = [Y]^{-1} \quad \left(\text{or } [Y] = [Z]^{-1}\right)$$

also $[I] = [Y][V]$

To calculate $Z_{ij} = \dfrac{V_i}{I_j}\Big|_{I_k = 0,\ k \neq j}$

- Reciprocal Network: $Z_{nm} = Z_{mn}$
- Lossless Network: $Re\{Z_{mn}\} = 0$ $m \neq n$, $m = n$.

Transmission Matrix:



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

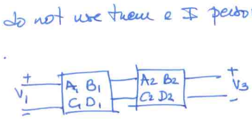
$A = \dfrac{V_1}{V_2}\Big|_{I_2 = 0}$ (open circuit port 2) Put voltage source at port 2 & measure O.C. voltage V_1 −

$B = \dfrac{V_1}{I_2}\Big|_{V_2 = 0}$ (short circuit port 2) ⇝ ...

$C = \dfrac{I_1}{V_2}\Big|_{I_2 = 0}$ (open circuit port 2) ⇝ ...

$D = \dfrac{I_1}{I_2}\Big|_{V_2 = 0}$ (short circuit port 2)

The good thing about transmission matrices is that you can cascade them. The bad thing is that they can be unstable in very lossy lines. Most people do not use them & I personally do not like them (& never use them).



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

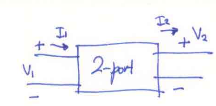
EECS 411: Conversion between Two-Port Network Parameters

TABLE 5.2 Conversions Between Two-Port Network Parameters

	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\frac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0-Y_{11})(Y_0+Y_{11})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\frac{2Z_{12}Z_0}{\Delta Z}$	$\frac{-2Y_{12}Y_0}{\Delta Y}$	$\frac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\frac{2Z_{21}Z_0}{\Delta Z}$	$\frac{-2Y_{21}Y_0}{\Delta Y}$	$\frac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\frac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\frac{Y_{22}}{\|Y\|}$	$\frac{A}{C}$
Z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\frac{-Y_{12}}{\|Y\|}$	$\frac{AD-BC}{C}$
Z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\frac{-Y_{21}}{\|Y\|}$	$\frac{1}{C}$
Z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\frac{Y_{11}}{\|Y\|}$	$\frac{D}{C}$
Y_{11}	$Y_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{22}}{\|Z\|}$	Y_{11}	$\frac{D}{B}$
Y_{12}	$Y_0\frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{12}}{\|Z\|}$	Y_{12}	$\frac{BC-AD}{B}$
Y_{21}	$Y_0\frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{21}}{\|Z\|}$	Y_{21}	$\frac{-1}{B}$
Y_{22}	$Y_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{11}}{\|Z\|}$	Y_{22}	$\frac{A}{B}$
A	$\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{\|Z\|}{Z_{21}}$	$\frac{-1}{Y_{21}}$	B
C	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{-\|Y\|}{Y_{21}}$	C
D	$\frac{(1-S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	D

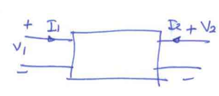
$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$; $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$; $\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$

ABCD Matrix



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Z & Y matrices:



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$