$$1\,N_p = 20\cdot\lg(e)\,dB \approx 8.7\,dB$$

$\frac{rad}{m}$ $\frac{Np}{m}$

$$\Gamma(l) = \Gamma_{load}\cdot e^{-2j\beta l}\cdot e^{-2\alpha l}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L - 1}{Z_L + 1}$$

$$Z_L = Z_0\cdot\frac{1+\Gamma}{1-\Gamma}$$

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = \frac{SWR-1}{SWR+1}$$

$$SWR = \frac{1+\sqrt{\frac{P_{ref}}{P_{fwd}}}}{1-\sqrt{\frac{P_{ref}}{P_{fwd}}}}$$

$$V = V_0^+ e^{\alpha l} e^{j\beta l}\left(1+\Gamma e^{-2\alpha l} e^{-j2\beta l}\right)$$

$$= V_0^+ e^{\alpha l} e^{j\beta l}\left[1+\Gamma(l)\right]$$

$$V = V_0^+ e^{l\gamma} + V_0^- e^{-l\gamma}$$

$$I = I_0^+ e^{\gamma l} + I_0^- e^{-l\gamma}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\beta\cdot l = 2\pi\cdot\frac{l}{\lambda}$$ el. length

$\frac{1}{2}$ runde: $\frac{1}{4}\lambda \Rightarrow \beta l = \pi/2$

1 runde: $\frac{1}{2}\lambda \Rightarrow \beta l = \pi$

$$v_p = \frac{c}{\sqrt{\epsilon_r}} \qquad \lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$

Lengde på kabelen i radianer. På Smith-diagrammet blir det dobbelt så langt

$$Z_{in} = \frac{Z_L + jZ_0\cdot\tan\beta l}{Z_0 + jZ_L\cdot\tan\beta l}\cdot Z_0$$

$$Z_{in} = \frac{Z_L + Z_0\cdot\tanh(\gamma l)}{Z_0 + Z_L\cdot\tanh(\gamma l)}\cdot Z_0$$

$$\boxed{\gamma = \alpha + j\beta}$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x}-1}{e^{2x}+1} = -j\cdot\tan(j\cdot x)$$

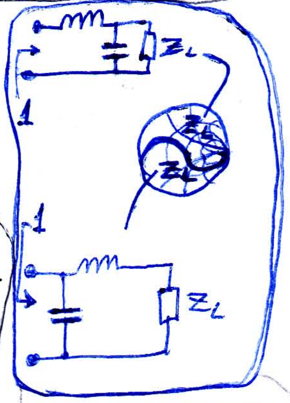
el. lengde $0 \sim 90°$ $\Big\{$ C og L : $l < \frac{\lambda}{4}$

$$Z_{in} = jZ_0\tan(\beta l) = j\omega L$$

$$L = \frac{Z_0\tan(\beta l)}{\omega}$$

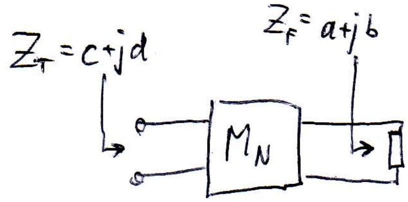
$$Z_{in} = -j\cdot Z_0/\tan(\beta l) = \frac{-j}{\omega C}$$

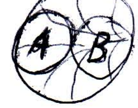
$$C = \frac{\tan(\beta l)}{\omega\cdot Z_0}$$



$$Im1 = \sqrt{Re - Re^2}$$

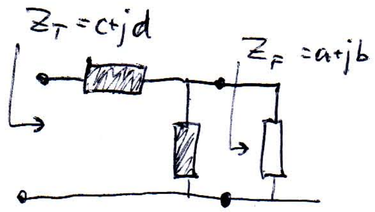
$$Im2 = \frac{Im1}{Re}$$





$a > c$

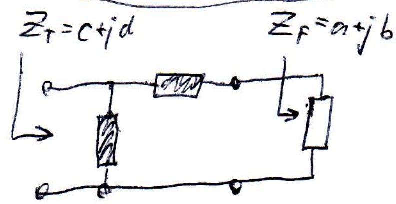
$b^2 > (c-a)\cdot a$



Normalize to $\underline{c = Re\{Z_T\}\ \Omega}$

- → Z_F is outside A
- → Y_F outside B
- → add parallel susceptance (circle A)
- → Change $Y \to Z$
- → add series reactance
- → done.

$ac < c^2+d^2 \Leftrightarrow ac < |Z_T|^2$



Normalize to

$Re\{Y_T\} = Re\{\frac{1}{Z_T}\}$ ℧ adm

or $\frac{1}{Re\{\frac{1}{Z_T}\}} = \underline{c + \frac{d^2}{c}\ \Omega}$

- → Z_F is outside B (real part < 1)
- → Add series reactance (circle A)
- → Change $Z \to Y$
- → Add parallel susceptance to reach $Y_T = \frac{1}{Z_T}$.
- → done

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64 $L = \frac{Z_0 \cdot \tan(\beta l)}{\omega} \approx \frac{Z_0 \cdot \beta l}{\omega}$

$C = \frac{\tan(\beta l)}{\omega Z_0} \approx \frac{\beta l}{\omega Z_0}$

$\beta l < \frac{\pi}{6}$

$l < \frac{\lambda}{12}$

$\beta = \frac{2\pi}{\lambda}$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

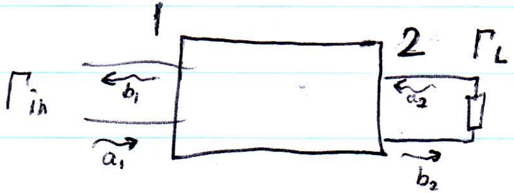
$$V_n = \sqrt{Z} \cdot (a_n + b_n)$$

ref. $[\sqrt{W}] = \left[\frac{V}{\sqrt{\Omega}}\right]$ $=$ $\sqrt{\frac{P_1}{P_2}}$ (no dimension) $\cdot$ Incident $[\sqrt{W}] = \left[\frac{V}{\sqrt{\Omega}}\right]$

$S_{nn} = \Gamma_{inn}$ for den n'te porten

$1 + S_{nn} = T$ og forteller hvor stor andel av spenningen som går videre, uten hensyn til impedans.

a, b og S må beregnes også ut ifra porten sin impedans!



$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{21} \cdot \Gamma_L \cdot S_{12}}{1 - S_{22} \cdot \Gamma_L}$$

$$S = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

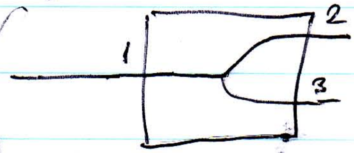
Tapsfritt, lineært, resiprokt $\longrightarrow$ $|d|^2 + |e|^2 + |f|^2 = 1$ (alle kolonner)

$ga^* + hb^* + ic^* = 0$ (mult. av to kolonner)

$$a_1 = \frac{V_{01}^+}{\sqrt{Z_{01}}} \qquad b_1 = \frac{V_{01}^-}{\sqrt{Z_{01}}} \qquad \left[\frac{V}{\sqrt{\Omega}}\right] = [\sqrt{W}]$$

Couplers

3-port



"lossy"

There are no matched, lossless 3-port couplers!

$|S_{11}|^2$ = reflection loss

$|S_{21}|^2$ = Insertion loss

$|S_{31}|^2$ = coupled power

T-junction

$$\begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \end{bmatrix}$$

lossless

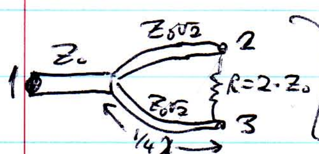
Res. T

$$\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

50% av eff → narme

matched

Wilkinson



$$S = \frac{-j}{\sqrt{2}} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

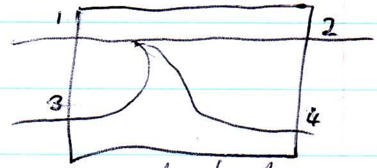
Combiner:

$b_1 = \frac{-j}{\sqrt{2}} \cdot (a_2 + a_3)$

$P_{ut} = \frac{1}{2} \cdot (\sqrt{P_2} + \sqrt{P_3})^2$

(Unequal power div: pg. 74)

4-port



"lossless"

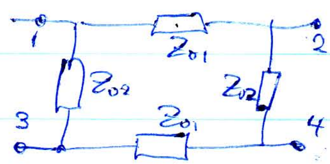
$|S_{11}|^2$ = Refl. loss (0)

$|S_{21}|^2$ = Insertion loss (0.5, 0.9, 0.99)

$|S_{31}|^2$ = Isolation $(0 = -\infty\, dB)$

$|S_{41}|^2$ = Coupled power $\underset{3dB}{(0.5)}\ \underset{10dB}{(0.1)}\ \underset{20dB}{(0.01)}$

Branch-line (90°) (Applications: pg. 80-81)



$Z_{01} = Z_0/\sqrt{2}$

$Z_{02} = Z_0$

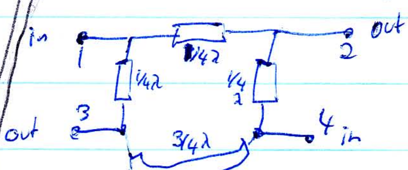
length $= \lambda/4$

$$S = \frac{-1}{\sqrt{2}} \cdot \begin{bmatrix} 0 & j & 0 & 1 \\ j & 0 & 1 & 0 \\ 0 & 1 & 0 & j \\ 1 & 0 & j & 0 \end{bmatrix}$$

2: −90°
4: −180°
over
tvers

Unequal Power division: Lecture 10 pg 78~79

Rat-race (0-180°)



3dB:
$Z = Z_0\sqrt{2}$

inp a_1: in-phase

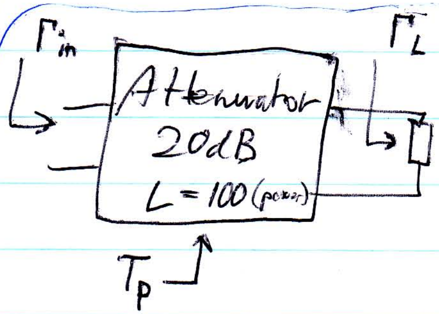
inp a_4: out of phase

(Not 3dB: page 79, Lecture 10)

$$S = \frac{-j}{\sqrt{2}} \cdot \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

Noise

Attenuator



$T_0 = 290K$

$$\Gamma_{in} = \frac{1}{L} \cdot \Gamma_L$$

$$NF_{att} = L$$

$$T_{eq} = (NF - 1) \cdot T_p = (L-1) \cdot T_p$$

$$NF_{att} = 1 + \frac{T_{eq}}{T_0} = 1 + (L-1)\frac{T_p}{T_0} \quad \left(= L \text{ når } T_p = T_0\right)$$

$$\text{støy ut} = \frac{1}{L} \cdot \left(\begin{array}{c}\text{støy vi sender}\\ \text{inn på inngangen}\end{array}\right) + \frac{L-1}{L} \cdot \left(\begin{array}{c}\text{attenuatoren}\\ \text{sin støy}\end{array}\right)$$

$$NF = \frac{T_0 + T_{eq}}{T_0} = 1 + \frac{T_{eq}}{T_0} = \frac{SNR_i}{SNR_o}$$

$$T_{eq} = (NF - 1) \cdot T_0$$

Cascade:

$$T_{eq} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 \cdot G_2} + \ldots$$

$$NF_{eq} = 1 + \frac{T_{eq}}{T_0} = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} + \ldots$$

$$P = kTB \quad (\cdot G \text{ for } P_{noise\ out} \text{ of ampl.})$$

$k = 1.38 \cdot 10^{-23}\ J/K$

$h = 6.626 \cdot 10^{-34}\ Js$

$$V_n^2 = 4kTBR$$

Elektromotorisk RMS-verdi?

$$V_n = 2\sqrt{kTR} \leftarrow \frac{V}{\sqrt{Hz}}$$

Romtemperatur:

$4 \cdot 10^{-21}\ \frac{W}{Hz}$

$1k\Omega$ mots.: $E_{rms} = 4\ \frac{nV}{\sqrt{Hz}}$

$$G_T = \frac{P_L}{P_{AVS}} = |S_{21}|^2 \cdot \frac{(1-|\Gamma_s|^2)\cdot(1-|\Gamma_L|^2)}{|1-\Gamma_s\Gamma_{in}|^2 \cdot |1-S_{22}\Gamma_L|^2}$$ Transducer Gain

$P_L = \frac{1}{2}\cdot\frac{|V_2^-|^2}{Z_0}\cdot(1-|\Gamma_L|^2)$ til M_L $\qquad P_{AVS} = \frac{\left(\frac{V_g}{\sqrt{2}}\cdot\frac{1}{2}\right)^2}{Z_0} = \frac{|V_g|^2}{8Z_0}$

$P_{in} = \frac{1}{2}\cdot\frac{|V_1^+|^2}{Z_0}\cdot(1-|\Gamma_{in}|^2)$ til transistor

$$G = \frac{P_L}{P_{in}}$$ "gain" (useless)

$$G_A = \frac{P_{AVL}}{P_{AVS}} = \frac{P_L|_{\Gamma_L=\Gamma_{out}^*}}{P_{AVS}} = |S_{21}|^2\cdot\frac{(1-|\Gamma_s|^2)}{|1-S_{11}\cdot\Gamma_s|^2\cdot(1-|\Gamma_{out}|^2)}$$ Available gain

Vanligvis: $G_A > G > G_T$

eller: $G > G_A > G_T$

$$G_T = \underbrace{\frac{1-|\Gamma_s|^2}{|1-\Gamma_s\Gamma_{in}|^2}}_{G_S \text{ due to source matching}} \cdot \underbrace{|S_{21}|^2}_{\text{transistor gain @ } 50\Omega \text{ inp/outp.}} \cdot \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}}_{\text{Gain due to load matching}}$$

<u>Unilateral</u>: $S_{12}=0 \Rightarrow G_T = G_{TU} = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s\cdot S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$

For maximum unilateral gain, we need $\Gamma_s = S_{11}^*$ og $\Gamma_L = S_{22}^* \Rightarrow$

$$G_{TU(max)} = \underbrace{\frac{1}{1-|S_{11}|^2}}_{G_{S(max)}}\cdot|S_{21}|^2\cdot\underbrace{\frac{1}{1-|S_{22}|^2}}_{G_{L(max)}}$$

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pg. 61

Bilateral

max conjugate gain

$$\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21} \qquad K = \overbrace{\frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12}| \cdot |S_{21}|}}^{\text{stability factor}}$$

$$G_{T(max)} = MAG = \frac{|S_{21}|}{|S_{12}|} \cdot \left(k \pm \sqrt{k^2 - 1}\right)$$

$+ \rightarrow |\Delta| > 1$, $- \rightarrow |\Delta| < 1$

$\leftarrow$ {Har løsning bare når $k > 1$

$$\boxed{\frac{1}{(1+u)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-u)^2}}$$

min — max

unilateral figure of merit $\boxed{u = \frac{|S_{12}||S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}}$

$U = 0$ $(S_{12} = 0)$ $G_T = G_{TU}$ ok

$U = 0.1 \sim$ ok

$U = 0.2$ not acceptable

Stability:

For hver frekvens må Γ_s (input stability) og Γ_L (output stab.) være utenfor sirklene som er "unstable regions".

Unconditionally stable:

For alle Γ_s og Γ_L $\Rightarrow$ Sirklene må aldri krysse Smith diagrammet.

$$\Rightarrow k > 1 \text{ og } |\Delta| < 1$$

Når $k < 1$ $\Rightarrow$ potentially unstable

teoretisk grense: $MSG = MAG|_{k=1} = \frac{|S_{21}|}{|S_{12}|}$

} I praksis prøver vi å få til $G = MSG - 3dB$ for bra stabilitet.