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Ref. Khan Academy: "Showing that an eigenbasis makes for good coordinate systems."

Hodgkiss	Khan
Data autocorrelation matrix R	Transformation matrix A
Q	Change of basis matrix C
Λ	Transformation matrix w.r.t. the new basis D

- Multiply by Q^H to go from the standard basis to eigenbasis (p linearly indep. eigenvectors of R are the basis vectors).

- In this new basis, the correlation matrix is diagonal with eigenvalues of R as the elements: Λ

$$
\begin{tikzcd}
\underline{x} \arrow[r, "R"] \arrow[d, shift right, "Q^H"'] & T(\underline{x}) = R\underline{x} \arrow[d, shift left, "Q^H"] \\
{[\underline{x}]_B = \underline{x}'} \arrow[r, "\Lambda"] \arrow[u, shift right, "Q"'] & {[T(\underline{x})]_B = \Lambda \underline{x}' = Q^H R \underline{x}} \arrow[u, shift left, "Q"]
\end{tikzcd}
$$

(Going right is a transformation. Going up/down is a change of basis.)

- We still have all the information that was contained in the original vector, but in a different basis. If we want to revert to the original basis, we can multiply by the change of basis matrix Q.

- Khan's point is that the transformation $T(\underline{x})$ is computationally easier in the other basis since that transformation matrix is diagonal.

- Hodgkiss' point: In doing the coordinate transformation from $\underline{x}$ to $\underline{x}'$, we achieve a diagonal correlation matrix $\Lambda \Rightarrow$

$\boxed{\text{Elements of } x' \text{ are uncorrelated to each other!}}$