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ECE 264A

Jan 08, 2008 E.G.

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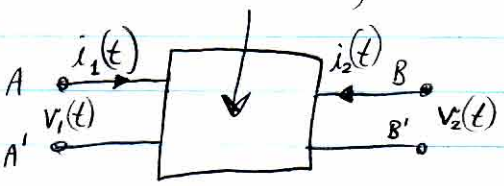
Poles Zeros & Freq. Response

SSM (small signal model) of circ. containing trans. and RLC networks



①

A, A' ≡ any two nodes of the circuit
B, B' ≡ " ——— " ——— "

Def. Let $x(t)$ be any real signal then $\underbrace{\mathcal{L}\{x(t)\}}_{X(s)} = \int_{-\infty}^{\infty} x(t)e^{-st}dt$ ≡ Bilateral Laplace transform

lowercase → $X(s)$

Let $G(s) = \frac{x(s)}{y(s)}$ where $x(t) = v_1(t), i_1(t), v_2(t), i_2(t)$
$y(t) =$ " ——— "

e.g. $G(s) = \frac{v_2(s)}{v_1 s}$ = Laplace transf. of gain from A,A' to B,B'

or $G(s) = \frac{v_1(s)}{i_1(s)}$ = " ——— " impedance at A,A' terminals.

Fact G(s) is always rational with real coefficients.

i.e. $G(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}$ ② ← rational ‖ ratio of two polynomial functions

where $a_k, b_k \in \mathbb{R}$

Q Do ∃ practical circ. for which $G(s) \neq$ rational

A Yes! Ex: Delay element: $G(s) = e^{-sT}$ ≡ T second delay

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Can write ② as

$$G(s) = G' \frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_n)} \quad ③ \quad \text{where } G', z_k, p_k \in \mathbb{C}$$

$$z_k = \text{zeros of } G(s)$$
$$p_k = \text{poles of } G(s)$$

Fact

$$\left.\begin{array}{l} G(s),\ s \in \mathbb{C} \\ G(j\omega),\ \omega \in \mathbb{R} \quad (\text{if it exists}) \\ g(t) = \mathcal{L}^{-1}\{G(s)\} \end{array}\right\} \text{contain equiv. info. about ①}$$

"impulse response"

freq. response → Also: Magnitude response & phase response

transfer fcn.

Goal: Find simple way to deduce "shape" of $G(j\omega)$ from poles & zeros of $G(s)$

Claim 1 real coeff $\Leftrightarrow$ If $s_0 = \alpha + j\beta$ $(\alpha, \beta \in \mathbb{R})$ is a pole (or zero) of $G(s)$
Then $s_0^* = \alpha - j\beta$ is also a pole (or zero) of $G(s)$

Proof: exercise

Ex $$(s-s_0)(s-s_0^*) = s^2 - s\overbrace{(s_0+s_0^*)}^{2\cdot Re\{s_0\}} + \overbrace{s_0 s_0^*}^{|s_0|^2} = s^2 + s(2\zeta\cdot\omega_0) + \omega_0^2$$

where $\omega_0 \equiv |s_0| \equiv$ resonant requency

$\zeta = -Re\{s_0\}/\omega_0 \equiv$ damping ratio (Claim 1 $\Rightarrow \zeta, \omega_0 \in \mathbb{R}$)

$\therefore$ Can group conjugate poles & zeros to get

$$G(s) = G' \left[\prod_{\substack{\text{real}\\\text{zeros}}} (s-z_k)\right] \cdot \left[\prod_{\substack{\text{complex}\\\text{zero pairs}}} \left(s^2 + s(2\zeta_i\omega_{0i}) + \omega_{0i}^2\right)\right] \cdot \left[\prod_{\substack{\text{real}\\\text{poles}}} \left(\frac{1}{s-p_k}\right)\right] \cdot \left[\prod_{\substack{\text{complex}\\\text{pole pairs}}} \left(\frac{1}{s^2 + s(2\cdot\zeta_i\cdot\omega_{0i}) + \omega_0^2}\right)\right] \quad ④$$

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Let ω_{zi}, $i = 1, 2, \ldots N \equiv$ the set of (non-zero) real zeros
ω_{pi} $i = 1, 2, \ldots N \equiv$ " ———————— " poles

$$\therefore \; G(s) = G(0)\, s^{n_0} \left[\prod_{\substack{\text{real}\\\text{zeros}}} \left(1 - s/\omega_{zk}\right)\right] \left[\prod_{\substack{\text{complex}\\\text{zero pairs}}} \left(\frac{s^2}{\omega_{0zi}^2} + \frac{2\,\zeta_{zi}}{\omega_{0zi}} s + 1\right)\right]$$

$$\cdot \left[\prod_{\substack{\text{real}\\\text{pole}}} \left(\frac{1}{1 - s/\omega_{pk}}\right)\right] \left[\prod_{\substack{\text{complex}\\\text{pole pair}}} \left(\frac{1}{\frac{s^2}{\omega_{opi}^2} + \frac{2\,\zeta_{pi}}{\omega_{opi}} s + 1}\right)\right]$$

Where $G(0) = \frac{a_0}{b_0}$, n_0 = # of DC - zeros ÷ # of DC - poles

Usually interested in $\underbrace{10 \cdot lg\left(\left|G(j\omega)\right|^2\right)}_{\text{gain in dB}}$ — dB : viser større range, på en meningsfull måte

and $\angle G(j\omega) = \tan^{-1}\left(\frac{Im\{G(j\omega)\}}{Re\{G(j\omega)\}}\right)$

Gain (assume $n_0 = 0$ for now)

$$10 \cdot lg\, |G(j\omega)|^2 = 10 \cdot lg\, |G(0)|^2 \quad ⑥$$

$$+ \sum_{\substack{\text{real}\\\text{zeros}}} 10 \cdot lg \left(1 + \omega^2/\omega_{zk}^2\right) \quad ⑦$$

$$- \sum_{\substack{\text{real}\\\text{poles}}} 10 \cdot lg \left(1 + \omega^2/\omega_{pk}^2\right) \quad ⑧$$

$$+ \sum_{\substack{\text{complex}\\\text{zero pairs}}} 10 \cdot lg \left(\left(1 - \frac{\omega^2}{\omega_{0zi}^2}\right)^2 + 4\, \frac{\zeta_{zi}^2}{\omega_{0zi}^2} \cdot \omega^2\right) \quad ⑨$$

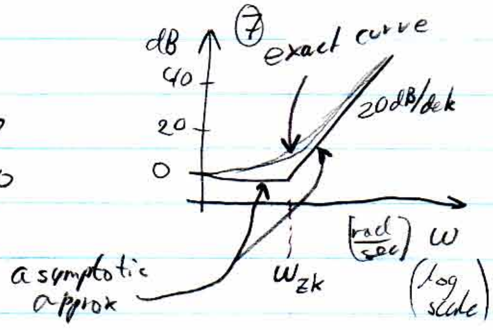
$$- \sum_{\substack{\text{complex}\\\text{pole pairs}}} \text{(and so on ..)} \quad ⑩$$

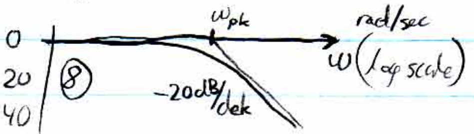
⇒ only 2 "types" of curves summed together

⑦, ⑧ $\rightarrow 0$ as $\omega \rightarrow 0$

⑦ $\rightarrow 20 \cdot lg(\omega) + \text{const}$ as $\omega \rightarrow \infty$

⑧ $\rightarrow -20\, lg\, (\omega) - \text{const}$ as $\omega \rightarrow \infty$





New room: Tuesday 1/15 (website)
Homework on Tuesday. Gjør tutorial for Cadence

cont.:
recall ⑩ $\ldots - \sum_{\substack{\text{complex}\\\text{pole}\\\text{pairs}}} 10 \cdot \lg\left[\left(1-\frac{\omega^2}{\omega_{opi}^2}\right)^2 + 4\frac{\zeta_{pi}^2}{\omega_{opi}^2}\omega^2\right]$

Claim 2 The max departure of the exact curve for real pole and zero factors from the asymptotic approx is a factor of 0.707 in both ⑦ and ⑧.

Proof exercise

$\boxed{Q = \frac{1}{2 \cdot \zeta}}$

⑦, ⑩ $\to 0\,dB$ as $\omega \to 0$

⑨ $\to 40 \cdot \lg \omega - \text{const}$ as $\omega \to \infty$
⑩ $\to -40 \cdot \lg \omega - \text{const}$ as $\omega \to \infty$

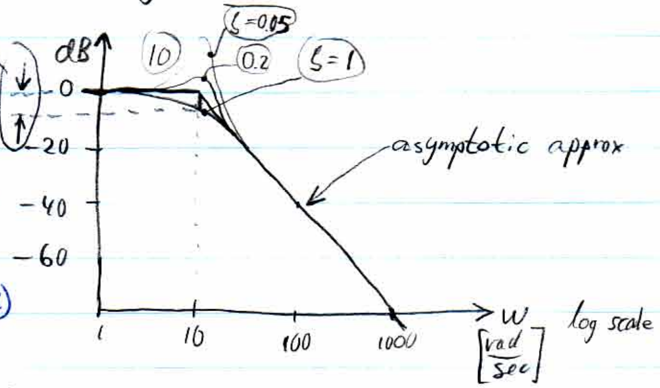
Peaking for $0 \leq \zeta < \frac{1}{\sqrt{2}}$ — $Q > \frac{1}{\sqrt{2}}$

Peak @ DC for $\zeta \geq \frac{1}{\sqrt{2}}$

ζ	Mag. at $\omega = \omega_{opi}$	
0	∞	∞
1/4	+6 dB	2
1/2	0 dB	1
$1/\sqrt{2}$	−3 dB	$1/\sqrt{2}$ (Butterworth)
1	−6 dB	$\frac{1}{2}$ (critically damped)
		Q =

Voltage amplitude at $\omega = \omega_{opi}$

e.g. (picture for $\omega_{opi} = 10\,\frac{rad}{sec}$)

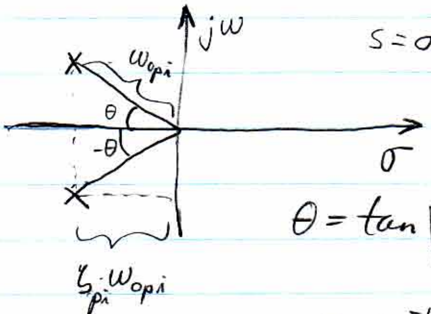


$\therefore$ asymptotic approx $\neq$ good approx near $\omega = \omega_{opi}$ for $\zeta \lesssim 0.2$

Voltage amplitude at the peak freq.:
$\boxed{\frac{1}{2\zeta \cdot \sqrt{1-\zeta^2}}}$ or $\boxed{\frac{Q^2}{\sqrt{Q^2 - 1/4}}}$ or $\boxed{\frac{1}{\sin(2\cdot\theta)}}$

poles accosiated with ⑩ (verify)

$s = \sigma + j\omega$



$\theta = \tan\sqrt{\frac{1}{\zeta_{pi}^2} - 1}$

$= \cos^{-1}(\zeta_{pi}) \quad (\zeta_{pi} < 1)$

RLC:	Parallel	Series	
damping:	$\frac{L}{4RC}$	R	} Ω
critical damping:	$\frac{1}{2}\sqrt{\frac{L}{C}}$	$2 \cdot \sqrt{\frac{L}{C}}$	
damping ratio ζ:	$\frac{1}{2R} \cdot \sqrt{\frac{L}{C}}$	$\frac{R}{2}\sqrt{\frac{C}{L}}$	
Q-factor	$R\sqrt{\frac{C}{L}}$	$\frac{1}{R}\sqrt{\frac{L}{C}}$	

$\omega_r = \omega_0 \cdot \sqrt{1 - 2 \cdot \zeta^2}$
$\omega_d = \omega_0 \cdot \sqrt{1 - \zeta^2}$

("Start of "New" lecture) ECE264A Jan 10 E.G.

$$G(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_1 s + a_0}{b_n s^n + \cdots \quad + b_1 s + b_0} \quad ①$$

where $a_k, b_k \in \mathbb{R}$ and $s \in \mathbb{C}$

$$\left.\begin{array}{l}
\text{Let } \{\omega_{zk} : k = 1, 2, \ldots, N\} = \text{non-zero real zeros of ① (if any)} \\
\{\omega_{pk} : k = 1, 2, \ldots, N'\} = \text{— " — poles — " —} \\
\{s_{zk}, s_{zk}^* : k = 1, \ldots N''\} = \text{non-real conj. zeros of ①} \\
\{s_{pk}, s_{pk}^* : k = 1, \ldots N'''\} = \text{— " — poles — " —} \\
\omega_{0xk} = |s_{xk}| \quad (x = z \text{ or } p) \quad (\equiv \text{natural freq. when } x = p) \\
\zeta_{xk} = -\frac{\mathrm{Re}\{s_{xk}\}}{\omega_{0xk}} \quad (\equiv \text{damping ratio when } x = p)
\end{array}\right\} ②$$

Using ②, ① becomes

$$\left.\begin{array}{l}
G(s) = G(0) \cdot s^{n_0} \left[\prod_{k=1}^{N} \left(1 - s/\omega_{zk}\right)\right] \left[\prod_{k=1}^{N''} \frac{1}{\omega_{0zk}^2} \left(s^2 + 2\zeta_{zk}\omega_{0zk} s + \omega_{0zk}^2\right)\right] \\
\qquad \cdot \left[\prod_{k=1}^{N'} \left(1 - s/\omega_{pk}\right)^{-1}\right] \left[\prod_{k=1}^{N'''} \omega_{0pk}^2 \left(s^2 + 2\zeta_{pk}\omega_{0pk} s + \omega_{0pk}^2\right)^{-1}\right] \\
\text{where } G(0) = \frac{a_0}{b_0}, \quad n_0 = \left(\begin{array}{l} \#\text{ zero-freq zeros} \\ -\#\text{ — " — poles} \end{array}\right)
\end{array}\right\} ③$$

$$G(j\omega) = \underbrace{|G(j\omega)|}_{\text{magnitude}} e^{j\underbrace{\measuredangle G(j\omega)}_{\text{phase}}}$$

Phase response

Exercise: verify $\measuredangle G(j\omega) = \sum \measuredangle(\text{each factor in ③})\Big|_{s=j\omega}$

where

$$(\text{factor in ③}) \longrightarrow \measuredangle(\text{factor in ③})\Big|_{s=j\omega}$$

$$G(0) = \frac{a_0}{b_0} \longrightarrow \begin{cases} 0 & \text{if } G(0) > 0 \\ \pi & \text{if } G(0) < 0 \end{cases}$$

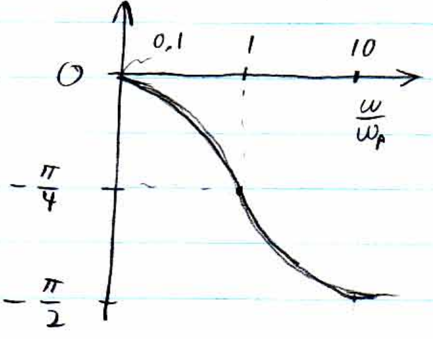
$$s^{n_0} \longrightarrow \frac{\pi}{2} \cdot n_0$$

$$\left(1 - s/\omega_{xk}\right)^{\pm1} \longrightarrow \mp \tan^{-1}\left(\frac{\omega}{\omega_{xk}}\right) \quad (x = p \text{ or } z)$$

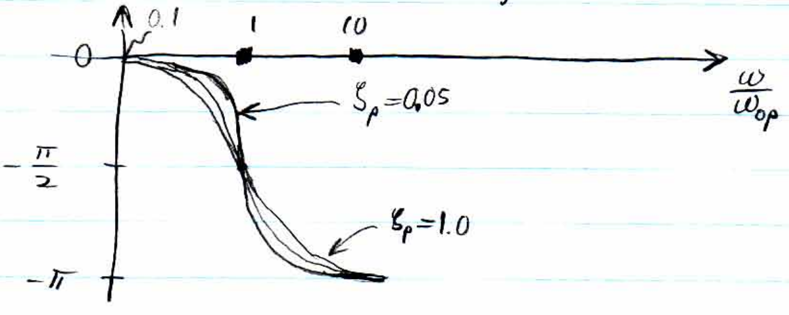
$$\left[\frac{1}{\omega_{0xk}^2}\left(s^2 + 2\zeta_{xk}\omega_{0xk}s + \omega_{0xk}^2\right)\right]^{\pm1} \rightarrow \pm\tan^{-1}\left(\frac{2\zeta_{xk}\omega_{0xk}\omega}{\omega_{0xk}^2 - \omega^2}\right) \quad x = p \text{ or } z$$

Picture (for LHP poles; only the sign changes for LHP zeros. same for RHP zeros)

$\measuredangle\left(\frac{1}{1 - j\omega/\omega_p}\right)$, $\omega_p < 0$



$\measuredangle\left(\frac{\omega_{op}^2}{s^2 + 2\omega_{op}\zeta_p s + \omega_{op}^2}\right)\Big|_{s=j\omega}$ $Re\{s_p\} < 0$



Poles & zeros – time response

$$\text{Let } \left.\begin{array}{l} p(s) = a_m s^m + a_{m-1} s^{m-1} + \cdots a_0 \\ q(s) = b_n s^n + b_{n-1} s^{n-1} + \cdots b_0 \end{array}\right\} \Rightarrow G(s) = \frac{P(s)}{q(s)}$$

$$= \frac{P(s)}{b_1 (s-p_1)(s-p_2)\cdots(s-p_n)}$$

Recall "Partial Fraction Expansion" (PFE)

Ex 1 $\quad m < n \quad \underbrace{p_i \neq p_j \text{ for } i \neq j}$

$\Leftrightarrow$ roots of $q(s)$ are all "first order"

$$\text{Then } G(s) = \sum_{k=1}^{n} \frac{A}{s-p_k}, \; A_k \in \mathbb{R} \quad \underbrace{\text{where } A_k = \lim_{s\to p_k}\left[(s-p_k)\,G(s)\right]}_{④}$$

Ex 2 $\quad m < n \quad p_1, p_2, p_3, \ldots \underset{7}{p_n} = 1^{\text{st}}$ order roots of $q(s)$

where $n' \leq n-2$
$4 \leq 7-2$

and $\underbrace{p_{n'+1} = p_{n'+2} = \cdots = p_{n^7}}$

r "multiple roots" of $q(s)$ where $\overset{3\,=\,7-4}{r = n-n'}$

} minst 2 like rotter
(example no.s)

$$\text{Then } G(s) = \sum_{k=1}^{4\to n'} \left(\frac{A_k}{s-p_k}\right) + \sum_{k=1}^{3\to r} \frac{B_k}{(s-p_n)^k}$$

where $A_k =$ ④

$$B_k = \lim_{s\to p_n} \frac{1}{(r-k)!} \frac{d^{r-k}}{ds^{r-k}}\left[(s-p_n)^r G(s)\right] \Big\} \; ⑤$$

Similar results hold for any comb. of 1^{st} order and multiple roots of $q(s)$.

Can extend for $m \geq n$

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Cont. from last time ↓   (Mid-term calc.: **Maybe**)

let $g(t) = \mathcal{L}^{-1}\{G(s)\}$

PFE ⇒ $g(t)$ = weighted sum of terms of types

$$\begin{cases} \mathcal{L}^{-1}\left\{\frac{1}{s-p_k}\right\} = e^{p_k t}\cdot u(t) & \textcircled{6} \\ \mathcal{L}^{-1}\left\{\frac{1}{(s-p_k)^r}\right\} = \frac{t^{r-1}}{(r-1)!}\cdot e^{p_k t}\cdot u(t) & \textcircled{7} \end{cases}$$

$$\left(\text{Note}: \textcircled{7} = \underbrace{\textcircled{6} * \textcircled{6} * \textcircled{6} * \cdots * \textcircled{6}}_{r \text{ times}}\right)$$

In circuits, rarely have multiple roots of $q(s)$ (i.e., rarely have multiple equal poles)
⇒ only concerned with ⑥ usually

For real non-zero poles, $\textcircled{6} \propto \mathcal{L}^{-1}\left\{\frac{1}{1-s/\omega_{pk}}\right\}$

For non-real poles, we can group conj. pairs for which ⑥ results in terms of form

$$\mathcal{L}^{-1}\left\{\frac{\omega_{opk}^2}{s^2+s(2\zeta_{pk}\omega_{opk})+\omega_{opk}^2}\right\}$$

ω_{opk} is an absolute value (Jan 10, pg. 2)

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Step Response

$$\mathcal{L}\{u(t)\} = \frac{1}{s}$$

Response of "output" with the input $= u(t) \equiv$ "step response"

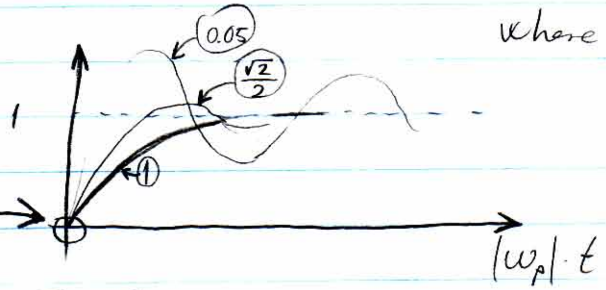
$$\therefore\ g_{step} = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot G(s)\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\ \frac{1}{1-s/\omega_p}\right\} = u(t)\cdot\left(1-e^{\omega_p t}\right)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\cdot\frac{\omega_{op}^2}{s^2+s(2\zeta_p\omega_{op})+\omega_{op}^2}\right\} = u(t)\cdot\left[1-\frac{1}{\sqrt{1-\zeta_p}}e^{-\zeta_p\omega_{op}t}\cdot\sin\left(\omega_{op}\sqrt{1-\zeta_p^2}\cdot t+\theta\right)\right\}$$

where $\theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta_p}\right)$

Den deriverte i starten av step-responsen er 0 dersom vi ikke har zeros i t.f.



For $\zeta_p = 0$, poles on img. axis $\Rightarrow$ oscillation.

For $\zeta_p < 0$ poles in RHP $\Rightarrow$ oscillation with exp. increasing amplitude

For $0 < \zeta_p < 1$ "overshoots" but settles to 1.

$$= u(t)\cdot\left[1-\frac{1}{\sqrt{1-\zeta_p^2}}\cdot e^{-\zeta_p\cdot\omega_{op}\cdot t}\cdot\cos\left[\omega_{op}\cdot\sqrt{1-\zeta^2}\cdot t-\tan^{-1}\left(\frac{\zeta_p}{\sqrt{1-\zeta_p^2}}\right)\right]\right]$$

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Dominant pole approx.

Provided have no DC poles or zeros

$$G(s) = G(0) \cdot \frac{(1 - s/z_1)(1 - s/z_2) \dots (1 - s/z_m)}{(1 - s/p_1)(1 - s/p_2) \dots (1 - s/p_n)}$$

Suppose $|p_1| << |p_k|, \; 2 \le k \le n$

$|p_1| << |z_k|, \; 1 \le k \le m$ $\quad (\Rightarrow p_1 \in \mathbb{R}$ why?)

Then $|G(j\omega)| \cong \dfrac{G(0)}{\sqrt{1 + \frac{\omega^2}{|p_1|^2}}}$ $\quad \therefore |G(j\omega_{-3dB})| = \dfrac{G(0)}{\sqrt{2}}$

$\Rightarrow \omega_{-3dB} = |p_1|$ (3dB Bandwidth)

(New numbering)

Comon source amplifiers freq. response

Aside Course convention

$$I_d = I_D + i_d$$
$$V_{gs} = V_{GS} + v_{gs}, \text{ etc...}$$

v_{gs}: Variations about a constant "bias" value

V_{GS}: a constant "bias" level

V_{gs}: "total" or "absolute" value

Lære boken bruker i_D og v_{GS} for dette. ⟷ page 7

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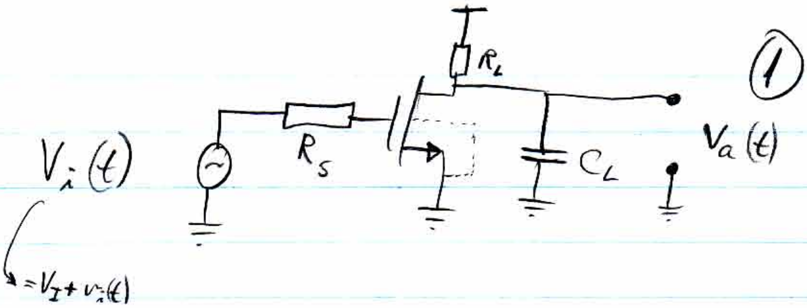
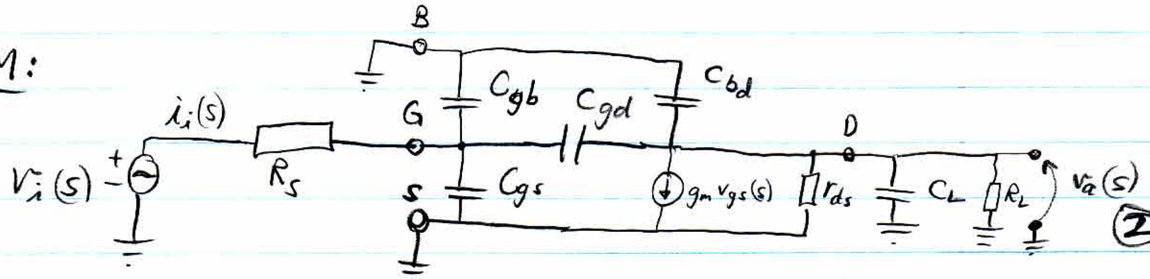


①

(R_L & R_s often from active circuits, not actual resistors)

SSM:



②

Let $C_{gs}' = C_{gs} + C_{gb}$ ($\approx C_{gs}$ usually)

$C_d = C_{bd} + C_L$

$R_L' = R_L \| r_{ds}$

eg. for $I_D = 200\mu A$ $\frac{W}{L} = 50$, $C_{gs}' = 90fF$, $C_{gd} = 20fF$, $C_d = 80fF$, $g_m = 1.7 \cdot 10^{-3}\,\Omega^{-1}$, $r_{ds} = 500k\Omega$ ③

Note: ② also SSM of diff pair DM ½ circuit

KCL at G: $\frac{v_i(s) - v_{gs}(s)}{R_s} - v_{gs}(s) \cdot s \cdot C_{gs}' - [v_{gs}(s) - v_a(s)] \cdot s \cdot C_{gd} = 0$

$$\therefore\ v_{gs}(s) = \left[\frac{1}{1/R_s + s(C_{gs}' + C_{gd})}\right]\left(v_a(s) \cdot s \cdot C_{gd} + v_i(s) \cdot \frac{1}{R_s}\right) \quad ④$$

KCL at D: $[v_{gs} - v_a(s)] \cdot s \cdot C_{gd} - g_m v_{gs}(s) - v_a(s)\left[\frac{1}{R_L'} + sC_d\right] = 0$

$$\therefore\ v_a(s) = \left[\frac{1}{\frac{1}{R_L'} + s(C_d + C_{gd})}\right](s\,C_{gd} - g_m) \cdot v_{gs}(s) \quad ⑤$$

$$\left.\begin{aligned} &\text{Let } G(s) = \frac{1}{R_s}\left[\frac{1}{1/R_s + s(C_{gs}' + C_{gd})}\right] \\ &A(s) = \frac{s\,C_{gd} - g_m}{1/R_L' + s(C_d + C_{gd})},\quad F(s) = -s \cdot C_{gd}\left[\frac{1}{1/R_s + s(C_{gs}' + C_{gd})}\right] \end{aligned}\right\} ⑥$$

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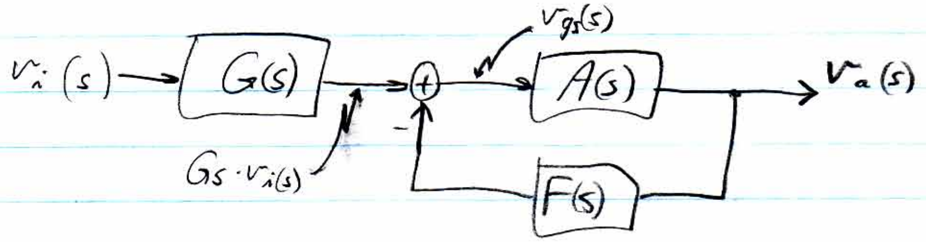
$$\left.\begin{aligned} &④, ⑥ \Rightarrow v_{gs}(s) = -F(s)\,v_a(s) + G(s)\,v_i(s) \\ &⑤, ⑥ \Rightarrow v_a(s) = A(s)\cdot v_{gs}(s) \end{aligned}\right\} ⑦$$

⑦ ⇒ "Block Diagram" of ②:



} ⑧

⑧ using Masons gain formula

OR

⑦ using algebra

$$\Rightarrow A_v(s) = G(s)\frac{A(s)}{1+A(s)F(s)} \quad ⑨$$

$$= \left(\frac{v_a(s)}{v_i(s)}\right)$$

Block Diagrams

SSM ⟺ system of equations in s ⟺ block diagram
(eg. ②) (eg. ④ & ⑤) (e.g. ⑧)

- SSM is symbolic ⇒ good for insight
- but components are bidirectional ⇒ bad for insight
- can't go directly from SSM to gain & impedances (need syst. of eqs. first)
- Block diagrams provide insight because components are uni-directional ⇒ Feedback paths are obvious
- Can go directly from block diagrams to gain & impedances (using Mason's gain formula)

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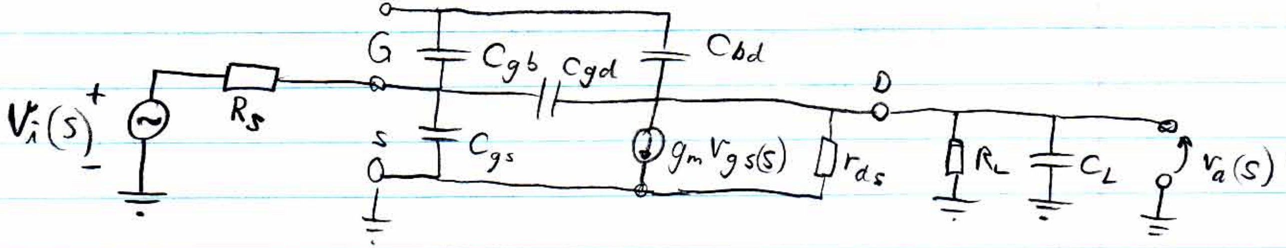
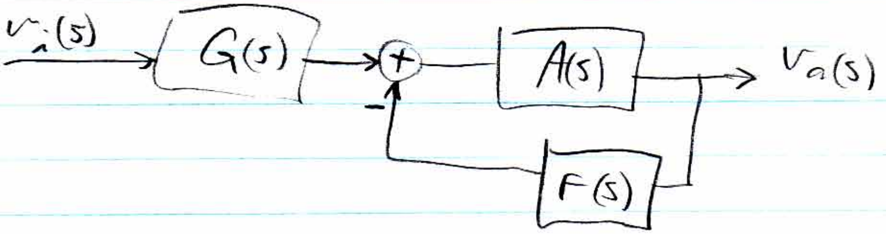
SSM of CS Amp. (recall from last time)



$$\left.\begin{aligned} v_{gs}(s) &= -F(s)\cdot v_a(s) + G(s)\, v_i(s) \\ v_a(s) &= A(s)\cdot v_{gs}(s) \end{aligned}\right\} ⑦$$



⑧

$$A_v(s) = G(s)\cdot\frac{A(s)}{1+A(s)F(s)} \quad ⑨$$

Recall Stability ⟺ no poles in RHP (incl. imag axis?)$_{E.G.}$

⑨ ⇒ — " — ⟺ $A(s_0)F(s_0) \neq -1$ for any s_0 with $\mathrm{Re}\{s_0\} \geq 0$
(assumes $F(s) \neq 0$ and no pole of $A(s)$ is a zero of $F(s)$)

First consider $A_{v_0} = A_v(j\omega)\big|_{\omega=0}$ "DC-gain"

$$A_{v_0} = G(j\omega)\,\frac{A(j\omega)}{1+A(j\omega)F(j\omega)}\Bigg|_{\omega=0}$$

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$F(j\omega)\big|_{\omega=0} = 0$ (makes sence because $F(s)$ represents feedback through C_{gd})

$\therefore A_{v_0} = -g_m R_L'$ ⑩

Now consider poles & zeros: Let $H(j\omega) = \frac{1}{A_{v_0}} \cdot A_v(j\omega)$

$\therefore A_v(j\omega) = A_{v_0} \cdot H(j\omega)$

⑥, ⑨ $\Rightarrow H(s) = \frac{1 - s/z_1}{1 + a_1 s + a_2 \cdot s^2}$

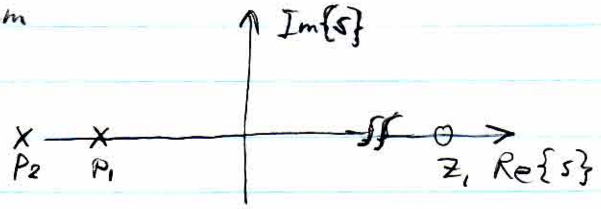
$$\text{where} \begin{cases} z_1 = g_m / C_{gd} \\ a_1 = R_S\left[C_{gs}' + C_{gd}(1 + g_m R_L')\right] + R_L'(C_{gd} + C_d) \\ a_2 = R_S R_L'\left[C_d C_{gs}' + C_d C_{gd} + C_{gs} C_{gd}\right] \end{cases}$$

$$\therefore A_v(s) = A_{v_0}\left[\frac{1 - s/z_1}{(1 - s/p_1)(1 - s/p_2)}\right] \text{ where } \left.\begin{aligned} p_1 &= -\frac{1}{2a_2}\left(a_1 + \sqrt{a_1^2 - 4a_2}\right) \\ p_2 &= -\frac{1}{2a_2}\left(a_1 - \sqrt{\text{---}''\text{---}}\right) \end{aligned}\right\} \text{⑪}$$

Usually, device parameters $\Rightarrow a_1^2 > 4a_2$ for CS amp.

$\Rightarrow p_1, p_2$ are real

pole-zero diagram



Hand-analysis:
10~20%
accuracy

Ex using ③ with $R_L = 10k\Omega$, $R_S = 5k\Omega$ gives

$A_{v_0} = -15.5$, $f_{z_1} = \frac{z_1}{2\pi} = 13.5\,GHz$, $f_{p_1} = \left|\frac{p_1}{2\pi}\right| = 56\,MHz$

$f_{p_2} = \left|\frac{p_2}{2\pi}\right| = 860\,MHz$

$$\left.\begin{aligned} |p_1| &\ll |p_2| \\ |p_1| &\ll |z_1| \end{aligned}\right\} \Rightarrow p_1 = \text{dominant pole}$$

$\Rightarrow |A_v(j\omega)| \approx \left|A_{v_0}\left(\frac{1}{1 + j\omega/(2\pi \cdot 56MHz)}\right)\right|$ for $\omega \lesssim f_{p_2} \cdot 2\pi$

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$\Rightarrow$ 3dB BW = 56 MHz

Now consider ① with capacitor C_c connected between D and G.

$\Rightarrow C_c$ in parallel with C_{gd} in ②

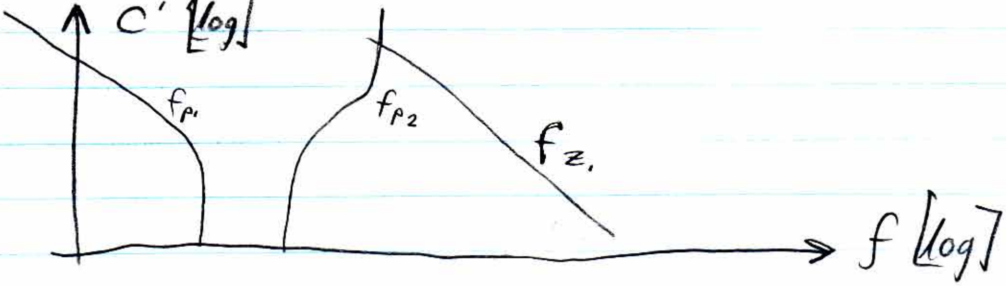
$\Rightarrow$ All eq. so far hold if C_{gd} is replaced by $C' = C_{gd} + C_c$ } (12)

Can show using (11) & (12) (exercise)

$$p_1 \approx -\frac{1}{(C_d + C')R_L' + (C_{gs}' C')R_s + g_m \cdot R_S \cdot R_L' C'} \quad (13)$$

$$\approx \frac{-1}{g_m R_s R_L' C'} \quad \text{(for large } R_s \,\&\, R_L\text{)}$$

$$p_2 \approx \frac{-g_m \cdot C'}{C_d C_{gs}' + C'(C_d + C_{gs}')} \qquad Z_1 = \frac{g_m}{C'}$$



$\therefore$ Increasing C' splits poles but moves zero closer to origin

Reset Numbering system!

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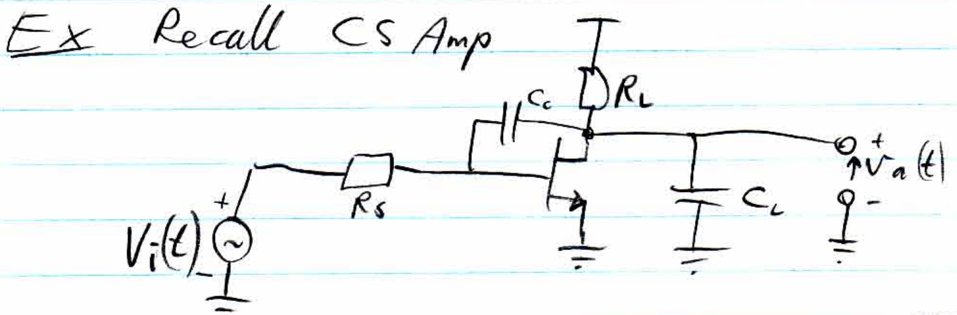
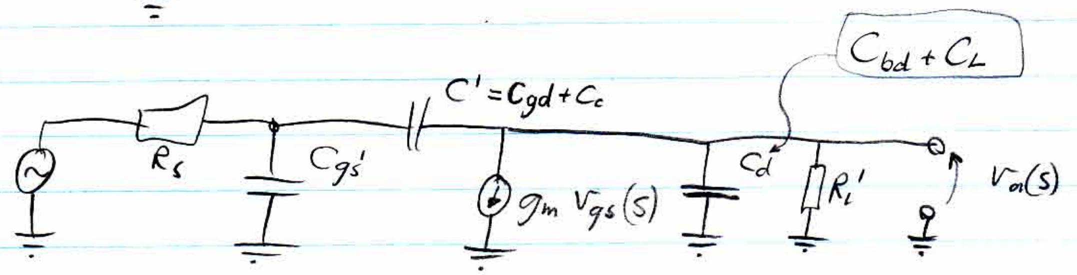
Zero-Value Time Constant Analysis

Ex Recall CS Amp





②

Last time: hard work gave us $p_1 \cong \dfrac{-1}{(C_d+C')R_L' + (C_{gs}'+C')R_S + g_m R_L' R_S C'}$ ③

p_1 = dom. pole so 3dB BW $= \left|\dfrac{p_1}{2\pi}\right|$

Q If only interested in dom. pole ($\Rightarrow$ 3dB BW) is there an easier way?

A Yes – ZVTC Analysis

ZVTC Theorem

β_1 = negative of Σ of all poles

Consider circuit containing only sources, resistors and capacitors and poles $p_1, p_2, \dots p_n$ where $p_k \neq 0$

Let $\beta_1 = -\sum_{k=1}^{n} +1/p_k$

$-\Sigma$(invers av pol) $= \Sigma$ tidskonst.

Then $\beta_1 = \sum_{k=1}^{k} R_k C_k$ ④

where C_k = value of k^{th} cap in circuit
and R_k = resistance between nodes of circuit to which C_k is connected (ie. impedance with all caps removed

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Ex (cont...)

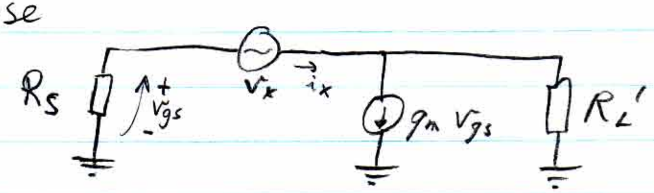
$$|p_1| << |p_2| \Rightarrow \beta_1 \approx |1/p_1| \quad \therefore p_1 \approx -\frac{1}{\beta_1}$$

Let $C_{gs}' \equiv C_1 \quad C_d \equiv C_2, \quad C' \equiv C_3$ ⑤

Inspection of ② ⇒ $R_1 = R_s$

$R_2 = R_L'$

To find R_3 use



$$\left.\begin{aligned} \Rightarrow v_{gs} &= -i_x R_s \\ v_x &= (i_x - g_m \cdot v_{gs}) R_L' - v_{gs} \end{aligned}\right\} R_3 = R_L' + R_s + g_m R_L' R_s$$

$$④ \Rightarrow p_1 \approx \frac{-1}{\underbrace{R_s C_{gs}'}_{\tau_1} + \underbrace{R_L' C_d}_{\tau_2} + \underbrace{(R_L' + R_s + g_m R_L' R_s) C'}_{\tau_3}} = ③ \quad \checkmark$$

Z.V.T.C. Analysis

Vi vet ikke sikkert vilket RC-ledd som er størst...
Men vi gjør et estimat:

$$\omega_{3dB} \approx |p_{dom.}| \approx \frac{1}{\sum_k C_k \cdot R_k}$$

$$\boxed{\approx \sum \left|\frac{1}{p_k}\right|}$$

"Zero Value" refererer muligens til at alle "de andre" kondensatorne er nullstilte. På norsk heter det "åpen krets tidskonstant metoden".

(Nedre grensefrekvens: "Kortslutnings-tidskonstant-metoden", basert på $\sum_k \frac{1}{C_k \cdot R_k}$)

LEC #5

JAN 2 2018

ZVTC Theorem (contd.) (same # system)

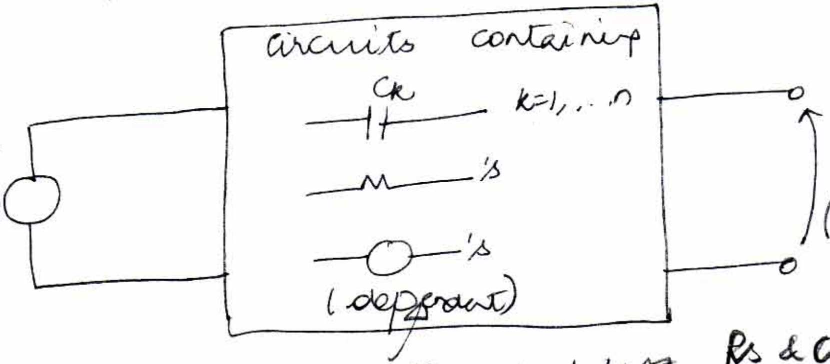
PROOF:-

Have:

Impulse:

$v_i(t) = \delta(t)$

$(v_i(s) = 1)$

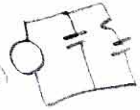


Impulse Response

$v_a(t) = g(t)$

$(v_a(s) = G(s))$

②



∴ For calculating Rs & Cs we need to turn off independent sources

where $G(s) = A_0 \dfrac{p(s)}{(1-s/p_1)(1-s/p_2)\cdots(1-s/p_n)}$ ← some polynomial

Note: In ②, $v_i(t) = \delta(t)$ sets initial condition of each capacitor $C_k, k=1,\dots n$. After $t=0$, $v_i(t)=0$

(Atleast one cap will get charged)

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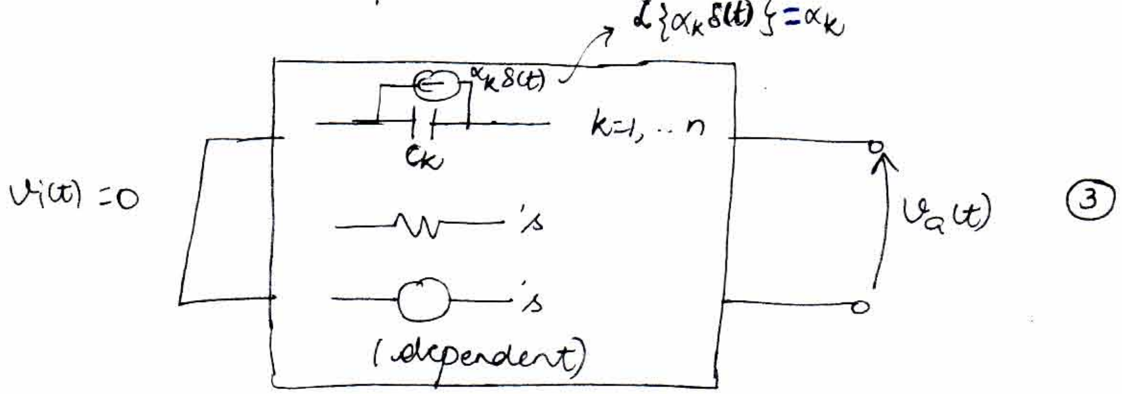
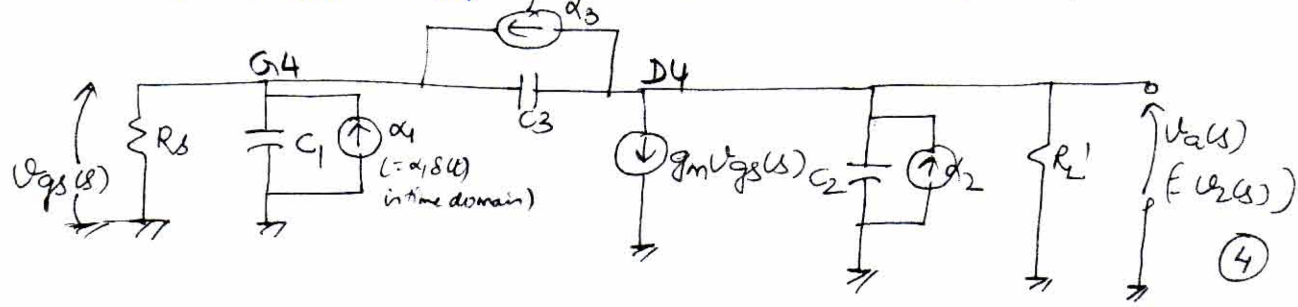
⇒ $g(t)$ = transient of ② if $v_i(t) = 0$ but with appropriate initial conditions on $C_1, C_2, \ldots, C_n$.

∴ ② is equivalent to



e.g. S.S.M. of C.S. amplifier (last time)

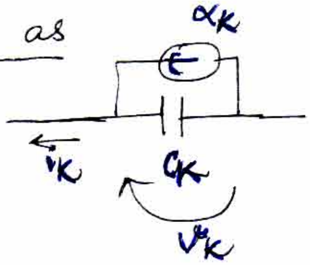
if $v_i(t) = \delta(t)$ produces same $v_a(t)$ as



NOTE: To be specific will use ④ in place of ③ for today

But all results we find generalize to ③.

Define v_k and i_k as



∴ $i_K(s) = \alpha_K - v_K(s) \cdot sC_K$ — ⑤

KCL @ any node ⇒ equation of the form:

$$d_1 i_1(s) + d_2 i_2(s) + d_3 i_3(s) + g_1 v_1(s) + g_2 v_2(s) + g_3 v_3(s) = 0$$

where $d_k = 1, 0,$ or -1 (corresponding to current entering, [illegible] leaving a node)

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and g_k = some conductance

e.g. KCL @ gate of ④:

$$-i_1(s) - i_3(s) + \frac{1}{R_S} v_i(s) = 0$$

Can solve all the equations of this form to get:

$$\left.\begin{aligned} i_1(s) &= a_{11} v_1(s) + a_{12} v_2(s) + a_{13} v_3(s) \\ i_2(s) &= a_{21} v_1(s) + a_{22} v_2(s) + a_{23} v_3(s) \\ i_3(s) &= a_{31} v_1(s) + a_{32} v_2(s) + a_{33} v_3(s) \end{aligned}\right\} \text{⑥ where } a_{jk} = \text{some conductance}$$

Note: To **force** ⑥ to be linearly independent, may need to add tiny resistors in series w/ some of the caps (add conceptually, not physically)

("tiny" ⇔ small enough to have negligible effect on ckt.)

In matrix **notation**, ⑥ is $\underline{i}(s) = \underline{A} \cdot \underline{v}(s)$

$$\underline{i}(s) = \underset{3\times 1}{\begin{bmatrix} i_1(s) \\ i_2(s) \\ i_3(s) \end{bmatrix}}, \quad \underline{A} = \underset{3\times 3}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}} \quad \& \quad \underline{v}(s) = \underset{3\times 1}{\begin{bmatrix} v_1(s) \\ v_2(s) \\ v_3(s) \end{bmatrix}}$$

Ladning på kondensatorne i start øyeblikket?

(Real life: CS Amp: 3 caps, 2 poles)

Using ⑤,:

$$\underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}}_{\text{Call } \underline{\alpha}} = \underbrace{\begin{bmatrix} (a_{11} + sC_1) & a_{12} & a_{13} \\ a_{21} & (a_{22} + sC_2) & a_{23} \\ a_{31} & a_{32} & (a_{33} + sC_3) \end{bmatrix}}_{\text{Call } \underline{B}} \underline{v}(s) \qquad ⑦$$

Recall "Cramer's Rule"

$\Rightarrow v_k(s) = \frac{\Delta_k(s)}{\Delta(s)}$ where $\Delta(s) = \det(\underline{B})$ (= polynomial in s)

$\Delta_k(s) = \det(\underline{B}$ with kth column replaced by $\underline{\alpha})$

	unit
$\underline{i}(s)$	A·sec
$\underline{v}(s)$	V·sec
$\underline{A}, \underline{B}$	$\mho = \frac{A}{V} =$ Siemens
α	$C = A\cdot sec$

⑤ & ⑥ $\Rightarrow$

$$\begin{aligned}\alpha_1 &= i_1(s) + v_1(s)\cdot s\cdot C_1\\ &= a_{11}\cdot v_1(s) + a_{12}\cdot v_2(s) + a_{13}\cdot v_3(s) + v_1(s)\cdot s\cdot C_1\\ &= (a_{11} + sC_1)\cdot v_1(s) + a_{12}\cdot v_2(s) + a_{13}\cdot v_3(s)\end{aligned}$$

$\Rightarrow$ ⑦

Note: $v_a(s) = v_2(s)$ in ④, so $G(s) = \underbrace{\frac{\Delta_2(s)}{\Delta(s)}}_{⑧}$

(i.e. we can calculate impulse response from cramer's rule)

We know denominator of $G(s)$ has form: $b_0 + b_1 s + b_2 s^2$

$= b_0(1 + \beta_1 s + \beta_2 s^2)$

(Because of pole-zero cancellation, we have 2nd order polynomial in real life rather than 3rd order polynomial)

$= b_0(1 - s/p_1)(1 - s/p_2)$

Algebra $\Rightarrow \underbrace{\beta_1 = -\sum_{k=1}^{n} 1/p_k, \; n=2}_{\text{true for any } n \geq 1}$

⑦, ⑧, defn. of det(B) $\Rightarrow$

$$\left.\begin{aligned} b_0 &= \Delta(s)\Big|_{c_1=c_2=c_3=0} \\ &= \Delta(0) \\ \& \quad b_0\beta_1 s \equiv b_1 s &= h_1 s c_1 + h_2 s c_2 + h_3 s c_3 \end{aligned}\right\} ⑨$$

where h_1, h_2, h_3 are real #s, i.e. $\in \mathbb{R}$

Let $\Delta_{ij} = (-1)^{i+j} \det(\underline{B}_{ij})$ where $\underline{B}_{ij} = \underline{B}$ with ith row & jth column deleted.

Can expand $\det(\underline{B})$ as

$$\Delta(s) = \underbrace{(a_{11} + sc_1)\Delta_{11} + a_{21}\Delta_{21} + a_{31}\Delta_{31}}$$

inspection of ⑦ $\Rightarrow$ c_1 only occurs in 1st term

$\therefore \; h_1 = \Delta_{11}\Big|_{c_2=c_3=0} = \Delta_{11}'(0)$

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Similarly, $h_2 = \Delta_{22}(0)$, $h_3 = \Delta_{33}(0)$

$$\therefore \; ⑨ \Rightarrow \beta_1 = \frac{\Delta_{11}(0)}{\Delta(0)} \cdot c_1 + \frac{\Delta_{22}(0)}{\Delta(0)} \cdot c_2 + \frac{\Delta_{33}(0)}{\Delta(0)} c_3$$

Now set $c_1 = c_2 = c_3 = 0$

& $i_2 = i_3 = 0$ to get

$$\begin{bmatrix} i_1 \\ 0 \\ 0 \end{bmatrix} = \underline{A} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Cramer's Rule $\Rightarrow$

$$v_1 = \frac{i_1 \det \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}}{\det(\underline{A})}$$

$$= i_1 \frac{\Delta_{11}(0)}{\Delta(0)} \quad \text{by defn.}$$

$$\therefore \left. \frac{v_1}{i_1} \right|_{\substack{c_1 = c_2 = c_3 = 0 \\ i_2 = i_3 = 0}} = R_1 = \frac{\Delta_{11}(0)}{\Delta(0)}$$

Similarly for R_2 and R_3

$$\therefore \beta_1 = R_1 c_1 + R_2 c_2 + R_3 c_3$$

1) One RC should dominate

2) One pole should dominate

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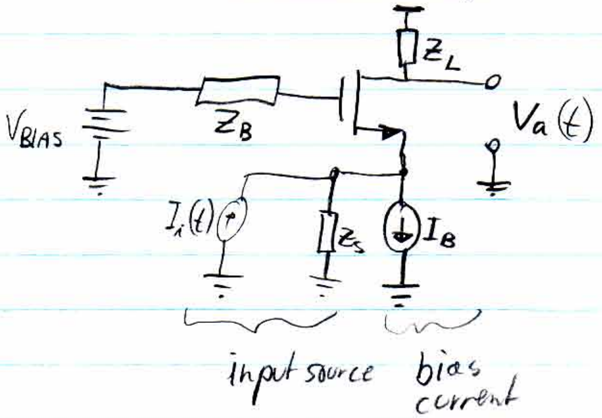
ECE 264A Jan 24, 2008

(o.h. tomorrow with TA 11:00~12³⁰ in EBU1,3329)

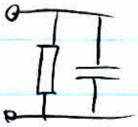
Freq resp of common gate (Cascode) stage

assume Body to ground (Lowest potential)

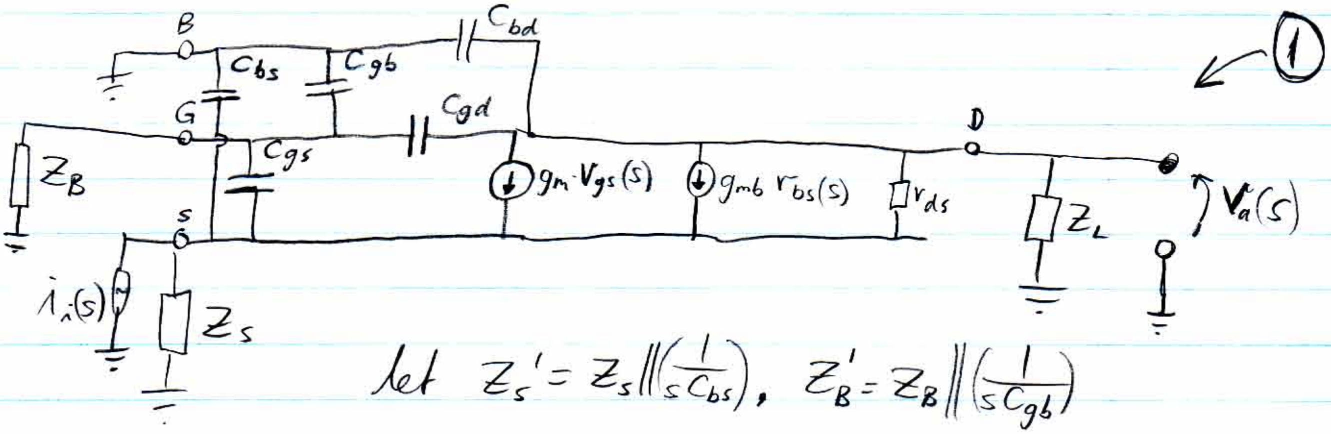


assume

$Z_B, Z_S, Z_L \equiv$



SSM



let $Z_s' = Z_s \| \left(\frac{1}{sC_{bs}}\right)$, $Z_B' = Z_B \| \left(\frac{1}{sC_{gb}}\right)$

$$Z_L' = Z_L \| \frac{1}{sC_{bd}}$$

KCL: (at G) $$\frac{V_g}{Z_B'} + (V_g - V_d)sC_{gd} + (V_g - V_s)sC_{gs} = 0$$

$$\therefore v_g = \underbrace{\left[\frac{1}{Z_B'} + s(C_{gd}\ C_{gs})\right]}_{\text{call it } a(s)}^{-1} \cdot \left[sC_{gs}v_s + sC_{gd}v_d\right] \quad ②$$

KCL: (at s) $$\therefore v_s = \left[\underbrace{\frac{1}{Z_s'} + g_m + g_{mb} + \frac{1}{r_{ds}} + sC_{gs}}_{\text{call it } b(s)}\right]^{-1}\left[i_i + (g_m + sC_{gs})v_g + \frac{1}{r_{ds}}v_d\right] \quad ③$$

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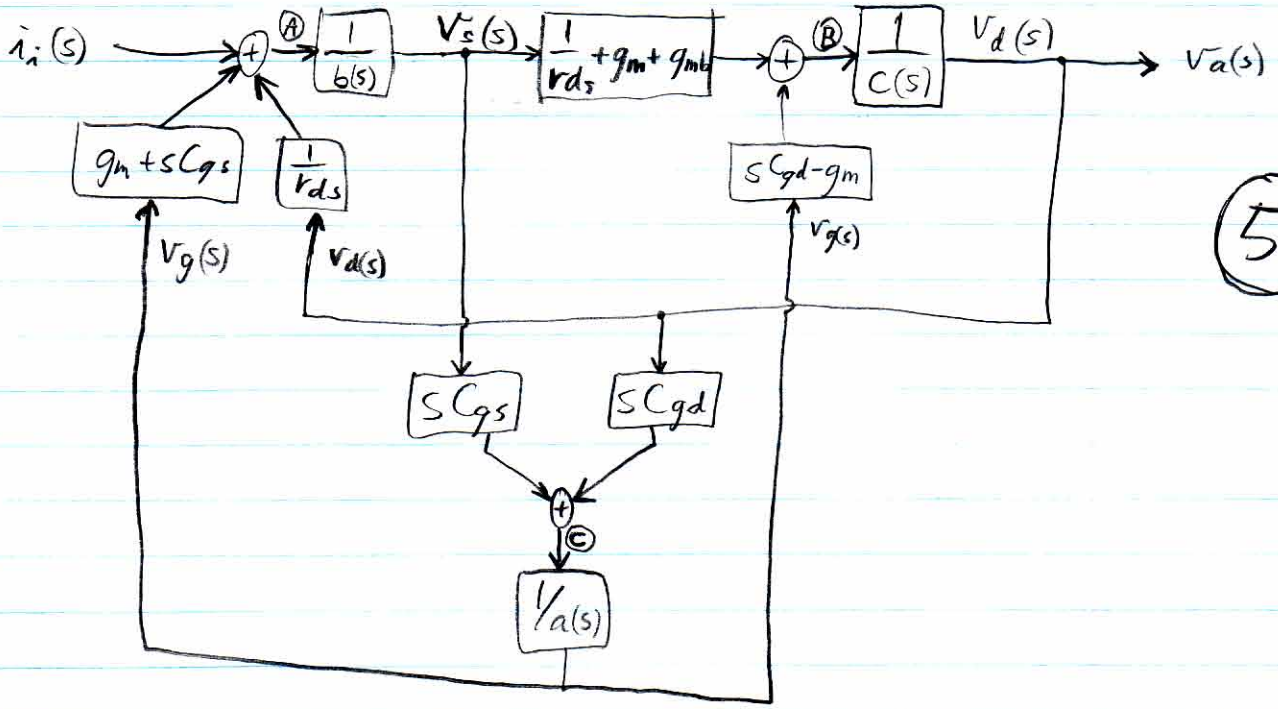
KCL (at D) : ...

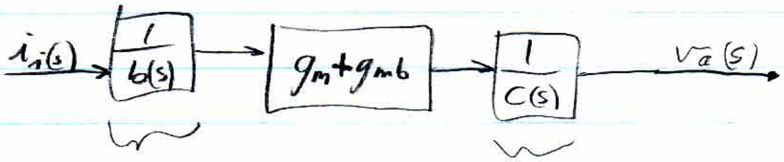
$$v_d = \Big[\underbrace{\frac{1}{Z'_L} + \frac{1}{r_{ds}} + sC_{gd}}_{\text{call it } C(s)}\Big]^{-1}\Big[\Big(\frac{1}{r_{ds}} + g_m + g_{mb}\Big)v_s + \big(sC_{gd} - g_m\big)v_g\Big] \quad ④$$

can generate "block diagram" from ②, ③, ④:



⑤

Observations
1) all blocks are unidirectional (in dir. of the arrows)
2) all feedback paths arise from parasitic elements (i.e. if $C_{xy}=0$, $r_{ds}=\infty$, then no feedback loops)
with $C_{xy}=0$, $r_{ds}=\infty$, ⑤ reduces to



⑥

$$= \frac{1}{1/Z'_s + g_m + g_{mb}} \qquad\qquad = Z'_L$$

$$\therefore \left.\frac{v_a(s)}{i_i(s)}\right|_{C_{xy}=0,\, r_{ds}=\infty} = \frac{Z'_L(g_m+g_{mb})}{1/Z'_s + g_m + g_{mb}} = \frac{Z'_L Z'_s (g_m+g_{mb})}{1+(g_m+g_{mb})Z'_s} \quad ⑦$$

$$\rightarrow Z'_L \text{ as } |Z'_s| \rightarrow \infty$$

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Inspection of ⑥ ⇒ ⑦

Q: Can we find $\left.\frac{v_a(s)}{i_i(s)}\right|_{C_{xy}\neq 0,\ r_{ds}\neq\infty}$ from ⑤ by inspection?

A: Yes. Using Mason's Gain Formula (M.G.F.)

Application of MGF to ⑤

$$\text{"Loops":}\quad \left.\begin{array}{ll} 1: & \textcircled{A}\to\textcircled{B}\to\textcircled{A} \\ 2: & \textcircled{A}\to\textcircled{C}\to\textcircled{A} \\ 3: & \textcircled{A}\to\textcircled{C}\to\textcircled{B}\to\textcircled{A} \\ 4: & \textcircled{A}\to\textcircled{B}\to\textcircled{C}\to\textcircled{A} \\ 5: & \textcircled{B}\to\textcircled{C}\to\textcircled{B} \end{array}\right\}\ \text{Note: } \underline{\text{All loops touch each other}}$$

$$\text{"Forward Paths":}\quad \left.\begin{array}{ll} 1: & i_i\to\textcircled{A}\to\textcircled{B}\to v_d \\ 2: & i_i\to\textcircled{A}\to\textcircled{C}\to\textcircled{B}\to v_d \end{array}\right\}\ \begin{array}{l}\text{Note: deleting either} \\ \text{path breaks all loops}\end{array}$$

$$\text{"Loop gains":}\quad \left.\begin{aligned} L_1 &= \left(\frac{1}{r_{ds}}+g_m+g_{mb}\right)\frac{1}{r_{ds}\,b(s)\,c(s)} \\ L_2 &= sC_{gs}\left(g_m+sC_{gs}\right)\cdot\frac{1}{a(s)\cdot b(s)} \\ L_3 &= sC_{gs}\left(sC_{gd}-g_m\right)\frac{1}{r_{ds}\cdot a(s)\,b(s)\,c(s)} \\ L_4 &= sC_{gd}\left(g_m+sC_{gs}\right)\left(\frac{1}{r_{ds}}+g_m+g_{mb}\right)\cdot\frac{1}{a(s)\,b(s)\,c(s)} \\ L_5 &= sC_{gd}\left(sC_{gd}-g_m\right)\frac{1}{a(s)\,c(s)} \end{aligned}\right\}\ ⑧$$

$$\text{"Path gains"}\quad \left.\begin{aligned} P_1 &= \left(\frac{1}{r_{ds}}+g_m+g_{mb}\right)\cdot\frac{1}{b(s)\,c(s)} \\ P_2 &= sC_{gs}\left(sC_{gd}-g_m\right)\cdot\frac{1}{a(s)\,b(s)\,c(s)} \end{aligned}\right\}\ ⑨ \leftarrow (\text{units}: \Omega)$$

Fact M.G.F. (this case) $\Rightarrow$ $$\frac{v_a(s)}{i_i(s)} = \frac{P_1+P_2}{1-L_1-L_2-L_3-L_4-L_5} \qquad ⑩$$

(Both paths touch all the loops)

∴ Can easily find $G(s) = \frac{v_a(s)}{i_i(s)}$ but it's really messy to look at

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For negligible capacitance in Z_L, Z_B and Z_s then

$$G(s) \cong \frac{\text{second order poly.}}{\text{third order poly.}}$$ ($a(s)$, $b(s)$ & $c(s)$ are 1st order polys.) } conductance (Siemens)

(Call full version of $G(s)$ ⑪)

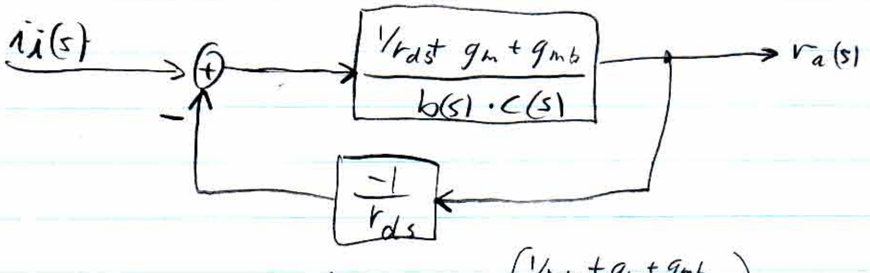
$\Downarrow$

have third-order system with {2 zeros, 3 poles}

The good news is ⑪ = "exact" expr.
The bad news is ——— " ———
(exact expr. is too complicated to give insight)

Q So what now?
A Plug in numbers (i.e freq. and comp. values) into boxes in ⑤ and eliminate highly attenuated paths.

Ex Z_B usually arrises from parasitics
-often $Z_B \mp 0$ causes stability problems in feedback applications
V_{BIAS} = DC, so we can use bypass cap to reduce $|Z_B|$
∴ Suppose $|Z_B| \cong 0$ Then $\left|\frac{1}{a(s)}\right| \approx 0$ and ⑤ becomes ≈



Now, MGF gives $$G(s)\Big|_{|Z_B|=0} = \frac{\left(\frac{1/r_{ds} + g_m + g_{mb}}{b(s) \cdot c(s)}\right)}{1 - \frac{1}{r_{ds}}\left[\frac{(1/r_{ds} + g_m + g_{mb})}{b(s)\ c(s)}\right]}$$

let $g_m' = g_m + g_{mb}$ $(\approx 1.2 \cdot g_m)$ assume $g_m \gg \frac{1}{r_{ds}}$

then

$$b(s) = \frac{1}{Z_s'} + g_m' + sC_{gs}, \quad \underbrace{c(s) = \frac{1}{Z_L'} + \frac{1}{r_{ds}} + sC_{gd}}_{\text{(as before)}}$$

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$$\therefore \quad G(s)\Big|_{Z_B=0} = \frac{g_m'}{\left(\frac{1}{Z_s'} + g_m' + sC_{gs}\right)\left(\frac{1}{Z_i'} + \frac{1}{r_{ds}} + sC_{gd}\right) - g_m'/r_{ds}} \qquad ⑬$$

Sanity check: 1) units V/I ☑

2) $A(j0)\Big|_{r_{ds}, Z_s=\infty} = Z_i'$ ☑

⑬ ⇒ 2^{nd} order behaviour: 2 poles, no zeros

⇒ always stable denom. of ⑬ has form $\alpha_0 + \alpha_1 s + \alpha_2 \cdot s^2$

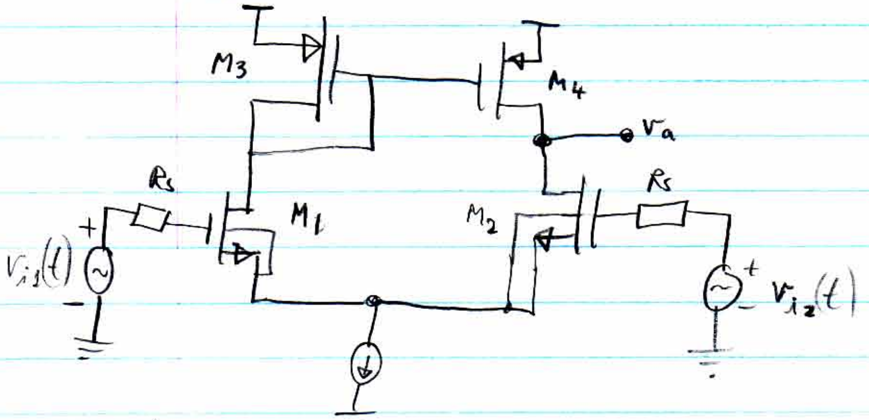
$$\text{with poles} = -\frac{\alpha_1}{2\cdot\alpha_2} \pm \frac{1}{2\cdot\alpha_2}\cdot \underbrace{\sqrt{\alpha_1^2 - 4\alpha_0\alpha_2}}_{<\alpha_1}$$

$\therefore \operatorname{Re}\{\text{poles}\} < 0$

closed book/notes (bring calc.)
↳ papir + penn (blyant)
} mid-term

Block diagrams & MGF (continued)

Ex: Diff to single-ended OTA → Operational transconductance amplifier

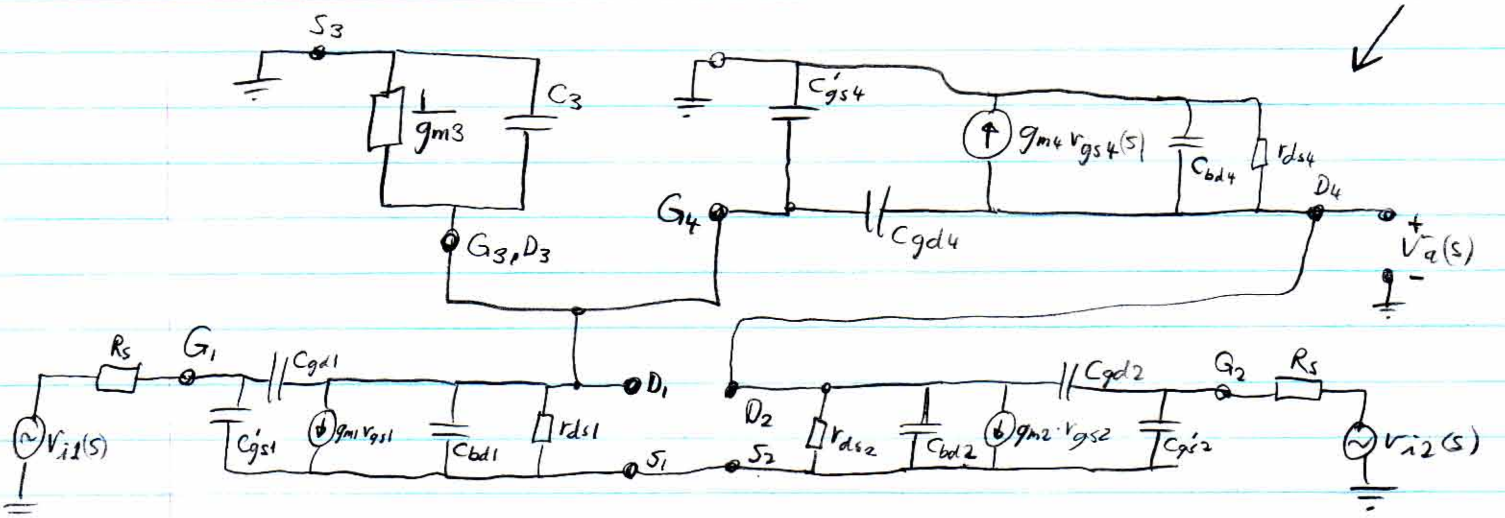


①

$M_1 = M_2$, $M_3 = M_4$

SSM of ①

②



where $C'_{gsi} = C_{gsi} + C_{gbi}$, $C_3 = C'_{gs3} + C_{bd3}$

See textbook pp. 180 ~ 181

Want to analyze differential mode (DM) operation: $v_{i1} = \frac{v_{id}}{2}$ $v_{i2} = -\frac{v_{id}}{2}$

Assume R_S = **negligible** for analysis

Want BD. containing nodes: $\underset{\text{Input}}{v_{id}}$, $\underset{\text{Drain 1}}{v_{d1}}$, $\underbrace{v_s \equiv v_{s1} \equiv v_{s2}}_{\text{Sources}}$, $\underset{\text{output}}{v_a}$

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KCL at D_1

$$v_{d1}\left[g_{m3}+s\left(C_3+C_{gs4}'\right)\right]+\frac{v_{d1}-v_s}{r_{ds1}\|C_{bd1}}+\left(v_{d1}-v_a\right)sC_{gd4}+\left(v_{d1}-\frac{v_{id}}{2}\right)sC_{gd1}$$

$$+\left(\frac{v_{id}}{2}-v_s\right)g_{m1}=0$$

where "$r_x \| C_y$" $\equiv \left(\frac{1}{r_x}+sC_y\right)^{-1}$

$$\underbrace{v_{d1}\left[g_{m3}+\frac{1}{r_{ds1}}+s\left(C_3+C_{gs4}'+C_{bd1}+C_{gd4}+C_{gd1}\right)\right]}_{(\text{call it } a(s))}-v_s\underbrace{\left[g_{m1}+\frac{1}{r_{ds1}\|C_{bd1}}\right]}_{\approx g_{m1}+sC_{bd}}$$

$$-v_a s C_{gd4}+\frac{1}{2}v_{id}\left(g_{m1}-sC_{gd1}\right)=0$$

$$\therefore\; v_{d1}=\frac{1}{a(s)}\cdot\left[v_s\left(g_{m1}+sC_{bd1}\right)+v_a\cdot s\cdot C_{gd4}-\frac{1}{2}v_{id}\left(g_{m1}-sC_{gd1}\right)\right] \quad ③$$

KCL at D_2

...

$$\therefore\; v_a \cong \frac{1}{b(s)}\left[v_s\left(g_{m2}+sC_{bd2}\right)+v_{d1}\left(sC_{gd4}-g_{m4}\right)+\frac{1}{2}v_{id}\left(g_{m2}-sC_{gd2}\right)\right] \quad ④$$

where $b(s)=\frac{1}{r_{ds4}\|r_{ds2}}+s\left(C_{bd4}+C_{bd2}+C_{gd4}+C_{gd2}\right)$ (used $g_{m2}\gg\frac{1}{r_{ds2}}$)

KCL at S_1, S_2

...

$$\therefore\; v_s=\frac{1}{c(s)}\left[\frac{v_{d1}}{r_{ds1}\|C_{bd1}}+\frac{v_a}{r_{ds2}\|C_{bd2}}\right] \quad ⑤$$

where $c(s)=g_{m1}+g_{m2}+s\left(C_{gs1}'+C_{gs2}'\right)$

$\left.\right\}$ used $M_1=M_2$ & $M_3=M_4$

③~⑤ ⟹ block diagram

$$\boxed{\left[\frac{1}{a(s)}\right]=\left[\frac{1}{b(s)}\right]=\left[\frac{1}{c(s)}\right]=\Omega} \leftarrow 1^{\text{st}} \text{ order polynomials}$$

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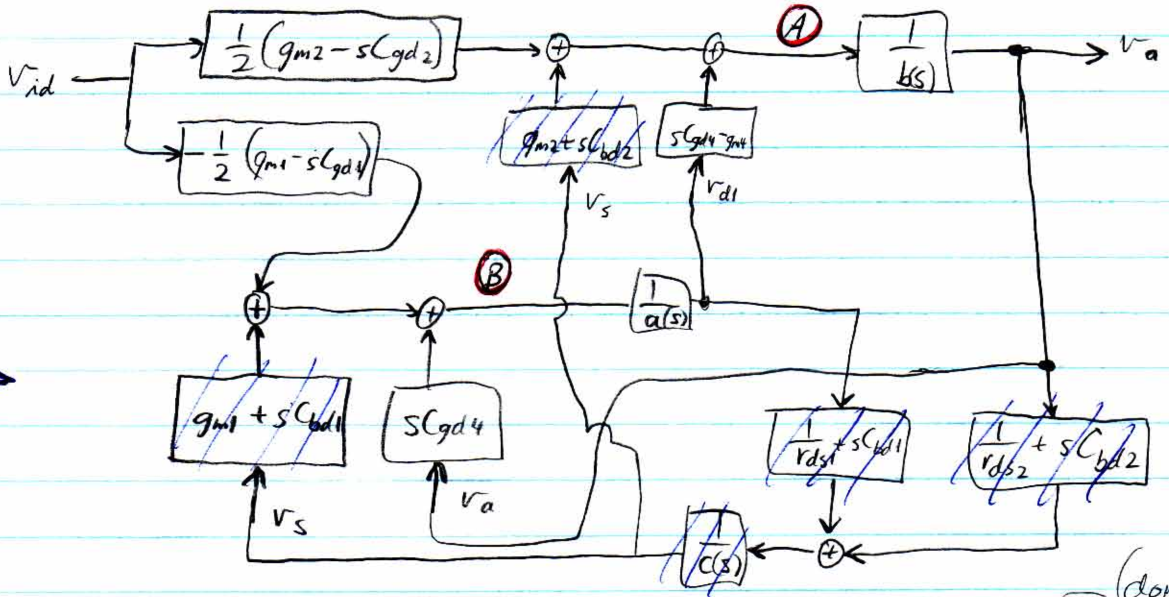

⑥

(dominant)

Let $p_1, p_2, \ldots$ be the poles $A_v(s) = \frac{v_a(s)}{v_{id}(s)}$ with $|p_1| \le |p_2| \le \ldots$

Provided $\left|\frac{1}{r_{ds1,2}} + j\omega C_{bd1.2}\right|$ is sufficiently small that

$$|v_s(j\omega)| << |v_{d1}(j\omega)|, \ |v_a(j\omega)| \text{ for } |\omega| \le |p_2|$$ ⑦

can eliminate the v_s feedback paths in ⑥
(asymmetry ⇒ $v_s \neq 0$)

We'll assume ⑦ holds for now. Can later check the assumption by testing with resulting value of p_2.

Sanity check: Find $A_{v_0} = A_v(j\omega)\Big|_{\omega=0}$ $\quad \omega=0 \Rightarrow s=0$

$\therefore \ a(0) = g_{m3} \quad b(0) = \frac{1}{r_{ds2}||r_{ds4}}$

$\therefore$ ⑥ $\Rightarrow v_a = \left(\frac{1}{2} g_{m2} r_{ds2}||r_{ds4} - \frac{1}{2} g_{m1} \cdot \frac{-g_{m4}}{g_{m3}} \cdot r_{ds2}||r_{ds4}\right) v_{id}$

$\therefore \ A_{v_0} = g_{m1} \cdot r_{ds2}||r_{ds4}$ $\boxed{\checkmark}$

(4/6.)

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Now, apply MGF:

Loop: Ⓐ → Ⓑ → Ⓐ

Forward paths
1. $v_{id} \to$ Ⓐ $\to v_a$
2. $v_{id} \to$ Ⓑ $\to$ Ⓐ $\to v_a$

⑥ $\Rightarrow P_1 = \dfrac{g_{m2} - sC_{gd2}}{2 \cdot b(s)}$, $P_2 = \dfrac{(g_{m1} - sC_{gd1})(g_{m4} - sC_{gd4})}{2 \cdot a(s) \cdot b(s)}$

$$L_1 = \frac{sC_{gd4}(sC_{gd4} - g_{m4})}{a(s)\ b(s)}$$

MGF $\Rightarrow A_v(s) = \dfrac{P_1 + P_2}{1 - L_1}$

$$= \frac{1}{2} \cdot \frac{a(s)(g_{m2} - sC_{gd2}) + (g_{m1} - sC_{gd1})(g_{m4} - sC_{gd4})}{a(s)\,b(s) - sC_{gd4}(sC_{gd4} - g_{m4})} \quad ⑧$$

$\Rightarrow$ 2 zeros, 2 poles

2nd order $\Rightarrow$ can easily find poles & zeros, but results give little insight

Instead, note:
- Feedback depends on $s \cdot C_{gd4}$ (no Miller effect, why?) ↳ $Z = 0$ (source)
- Usually $C_{gd4} \ll C'_{gsi}, C_{bdi}$
- ∴ Taking $s \cdot C_{gd4} \cong 0$ in ⑧ = reasonable approximation

Now, MGF $\Rightarrow A_v(s) = P_1 + P_2$ $(L_1 \approx 0)$ $= \dfrac{\text{2nd order poly}}{a(s)\ b(s)}$

$$= g_{m1}(r_{ds2} \| r_{ds4}) \cdot \frac{(1 - s/z_1)(1 - s/z_2)}{(1 - s/p_1)(1 - s/p_2)}$$

where $p_1 = \dfrac{-1}{(r_{ds2} \| r_{ds4})\, C_2}$, $p_2 = \dfrac{-g_{m3}}{C_1}$

Where $C_1 = C'_{gs3} + C_{bd3} + C'_{gs4} + C_{bd1} + C_{gd1}$

$C_2 = C_{bd2} + C_{bd4} + C_{gd2}$

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$$\left.\begin{array}{l}C_1, C_2 \text{ have same order of mag} \\ r_{ds2} \| r_{ds4} \gg 1/g_{m3} \text{ (typically)}\end{array}\right\} \Rightarrow \begin{array}{l} p_1 = \text{dom pole} \\ p_2 = \text{non dom. pole (next most dominant)}\end{array}$$

Observation

$\frac{1}{g_{m3}}$ = resistence to small-signal ⏚ at D_1 in ②

C_1 = cap ——— " ———

$r_{ds2} \| r_{ds4}$ = res ——— " ——— D_2 —"—

C_2 = cap ——— " ———

∴ In ② could have found p_1 and p_2 by calculating R_i, C_i
where R_i = res to gnd. at node i
C_i = cap. —— " ——

Then $p_1 = \frac{-1}{\text{largest}\{R_i \cdot C_i\}}$, $p_2 = \frac{-1}{\text{next largest}\{R_i \cdot C_i\}}$

Q: Coincidence?
A: No. This method is reasonable approx in many cases (HW 2)

Slang: Because of this, people often say that the dominant pole occurs at node D_2 and the non-dominant pole occurs at node D_1.

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Full version of MGF

Block diagram defs.:

1) "Path" ≡ route through B.D. (in dir. of arrows) connecting a pair of nodes
2) "Path gain" ≡ product of block gains along path
3) "Loop" ≡ path which starts and ends at same node with no node along path encountered more than once
4) "Loop gain" ≡ path gain of loop
5) "Determinant" ≡ $1-\sum$ (all loop gains)
   $+\sum$ (products of loop gains of all loop pairs with no common nodes)
   $-\sum$ (——— ″ ——— triples ——— ″ ———)
   $+\sum$ (. . .)
   . . .
6) "Forward path" ≡ path from input to output containing no full loops

Let $H = \frac{X_{out}}{X_{in}}$ where X_{in} = input of B.D.
X_{out} = output of B.D.

Then $H = \frac{1}{\Delta} \cdot \sum_{k=1}^{L} p_k \cdot \Delta_k$ (MGF)

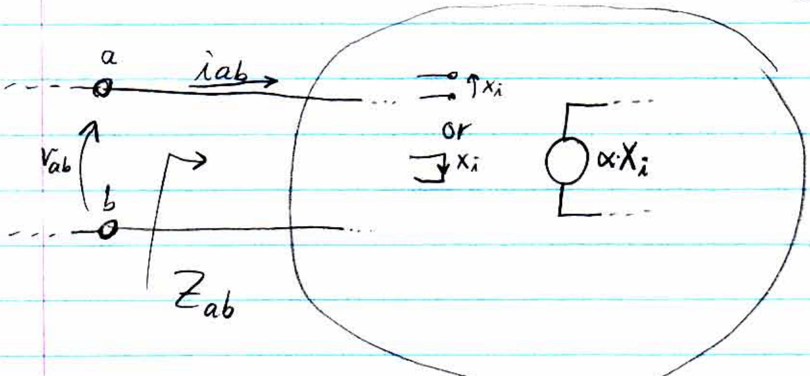
where Δ = determinant
L = # of forward paths
P_k = k^{th} forward path
Δ_k = determinant of B.D. that remains after deleting k^{th} path, P_k

"cofactor" = Δ with loops touching the k^{th} path removed.

ECE 264A Jan 31, 2008 E.G.

Blackman's Impedance Relation

arbitrary SSM circuit
containing ≥1 controlled sources



x_i = voltage or current

αX_i = —— " ——

(e.g. $X_i = v_{gs}$, $\alpha = g_m$
∴ αx_i = current)

BIR $$Z_{ab} = Z_{ab}^{0} \cdot \frac{1+T_{sc}}{1+T_{oc}}$$ ①

$$\left(\equiv \frac{v_{ab}(s)}{i_{ab}(s)} \right)$$

nullstilt!

where $Z_{ab}^{0} \equiv$ Impedance between a and b with controlled source removed ($\alpha = 0$)

$T_{sc} \equiv -\alpha \cdot \frac{x_i}{X_x}$ when $v_{ab} = 0$ (a, b shorted)

and αx_i is replaced by an independent "test source", X_x

$T_{oc} \equiv$ Same as T_{sc} except with $i_{ab} = 0$ (a, b open circuited)

Note: if have more than one controlled sources, pick any one of them (call it the reference source).

T'ene er "loop gain"

See note of March 18, 2008

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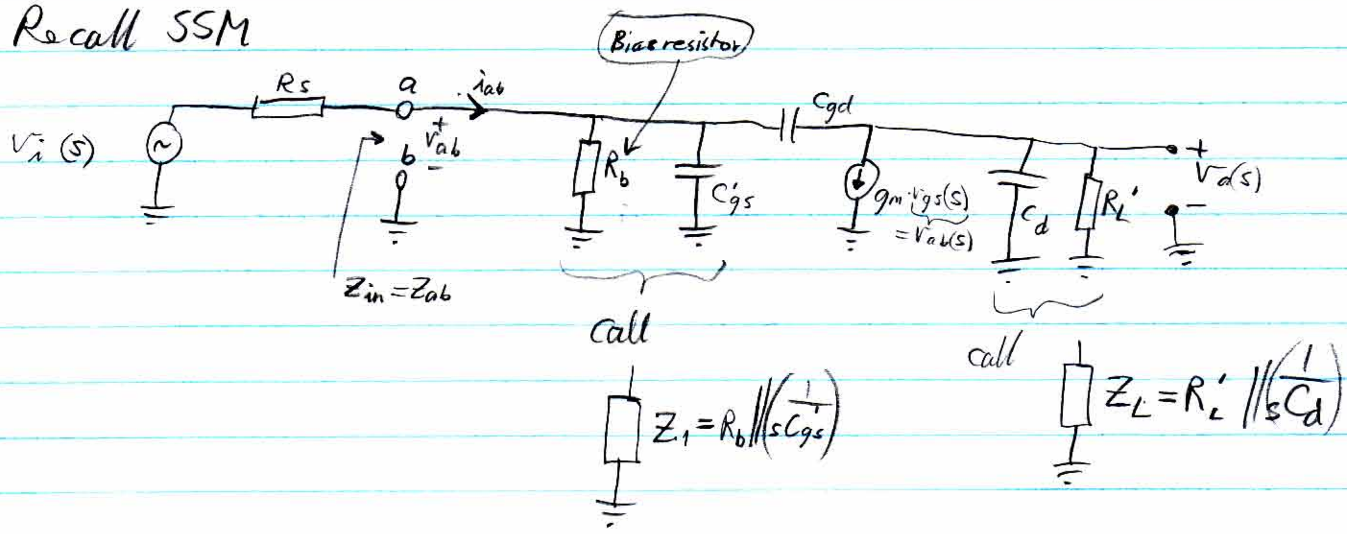
Ex 1 C.S. Amp

Recall SSM



BIR: $\alpha = g_m$, $X_i = v_{ab}$, $x_x = i_x$

$$Z_{ab}^{0} = Z_1 \| \left(\frac{1}{sC_{gd}} + Z_L\right) = \frac{Z_1 \cdot (1 + Z_L \cdot s \cdot C_{gd})}{1 + (Z_1 + Z_L) \cdot s \cdot C_{gd}}$$

$$T_{sc} = 0 \quad \left(\text{because } X_i = V_{ab} = 0\right)$$

$$T_{oc} = -g_m \cdot \frac{1}{i_x} \cdot \left[\underbrace{-i_x \left(Z_L \| \left(Z_1 + \frac{1}{sC_{gd}}\right)\right)}_{V_a(s)} \cdot \frac{Z_1}{Z_1 + \frac{1}{sC_{gd}}} \right]$$

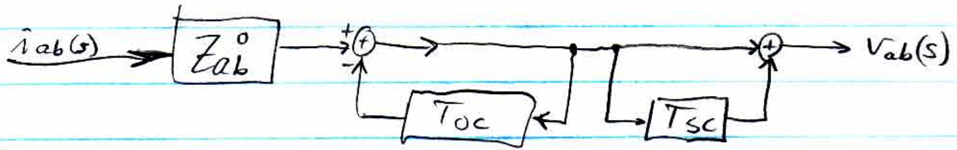
$$= \frac{g_m Z_1 Z_L}{Z_L + Z_1 + \frac{1}{sC_{gd}}} \quad \left(\text{note} \rightarrow 0 \text{ as } C_{gd} \rightarrow 0\right)$$

$$\therefore \; ① \Longrightarrow \quad Z_{in} = Z_{ab}^{0} \cdot \frac{1}{1 + T_{oc}} = Z_1 \cdot \frac{1 + sC_{gd} \cdot Z_L}{1 + [Z_L + \underbrace{Z_1 (1 + g_m Z_L)}_{\text{Miller effect}}] \cdot s \cdot C_{gd}} \qquad ②$$

Observations

1) T_{oc} represents a feedback path around the reference source
Why? def ⇒ If output of ref. source has no connection to it's controlling voltage (or current) then $T_{oc} = 0$

Also, MGF and ① give:



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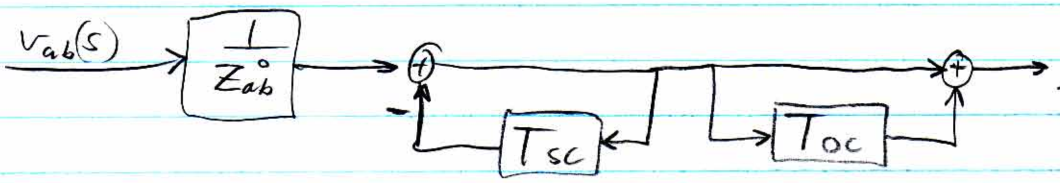
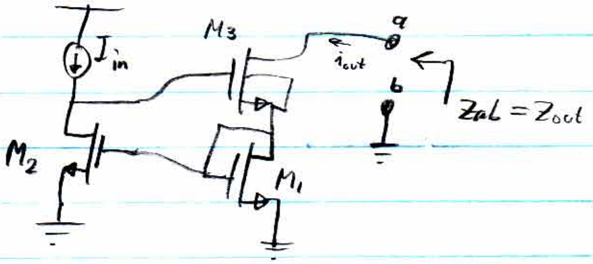
or...

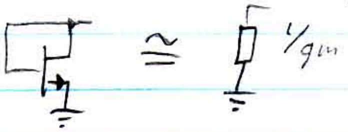


∴ T_{sc} also arises from a feedback path within the circuit

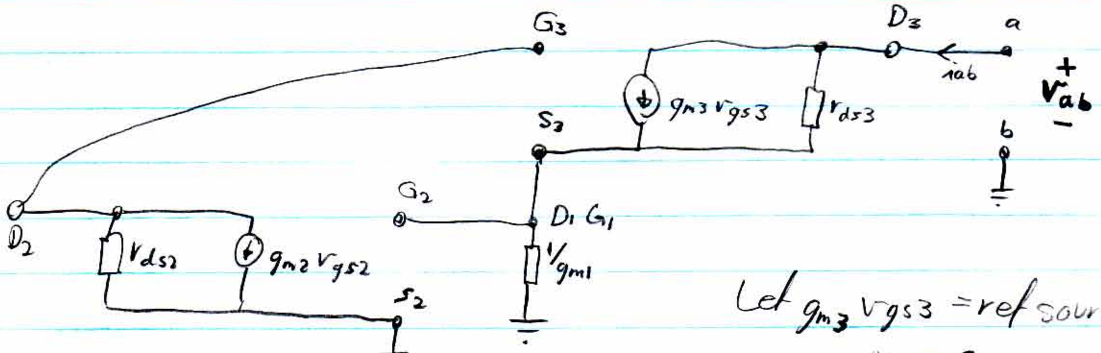
2) Feedback can either increase or decrease Z_{ab}

Ex 2 Wilson current mirror (low freq analysis)



Low freq. SSM (using  $\cong$ $1/g_m$)

(Set $i_{in} = 0$ to find Z_{out})



③

Let $g_{m3} v_{gs3}$ = ref source ∴

$\alpha = g_{m3}$

$x_i = v_{gs3}$

Inspection of ③ $\Rightarrow Z_{ab}^{\circ} = r_{ds3} + \frac{1}{g_{m1}} \approx r_{ds3}$

④/6

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Toc : $i_{ab}=0 \Rightarrow v_{gs2}=0 \Rightarrow v_{gs3}=0 \Rightarrow x_i=0 \;\therefore T_{oc}=0$

Tsc : with $g_{m3}v_{gs3}$ replaced by i_x and $v_{ab}=0$ have

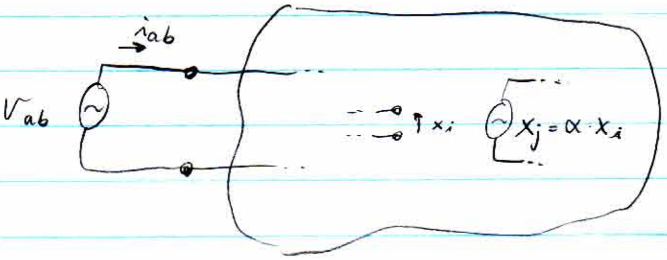
$$\left.\begin{aligned} v_{gs2} &= i_x\left(r_{ds3} \,\|\, \left(\frac{1}{g_{m1}}\right)\right) \approx \frac{i_x}{g_{m1}} \\ \text{KVL} \Rightarrow\ v_{gs2}+v_{gs3} &= -g_{m2}\cdot v_{gs2}\cdot r_{ds2} \end{aligned}\right\} \Rightarrow v_{gs3} = -\frac{i_x}{g_{m1}}\left(1+g_{m2}r_{ds2}\right)$$

$$\therefore T_{sc} = \frac{g_{m3}}{g_{m1}}\left(g_{m2}\cdot r_{ds2}+1\right) = g_{m3}r_{ds2}+1$$

(for $M_1=M_2=M_3$)

$$\therefore ① \Rightarrow Z_{ab} = r_{ds3}\left(2+g_{m2}\cdot r_{ds2}\right)$$ ∴ Feedback increased Z_{ab}

Proof of BIR:



SSM circuit as before (all indep. sources set to zero)

Linearity ⇒ can write

$$\left.\begin{aligned} v_{ab} &= A\cdot i_{ab} + B\cdot x_j \\ x_i &= C\cdot i_{ab} + D\cdot x_j \end{aligned}\right\} ④$$

(two-port representation)
$A(s), B(s), C(s), D(s) \equiv$ transfer functions

Solving ④ for $Z_{ab}=\frac{v_{ab}}{i_{ab}}$ gives $Z_{ab}=A\cdot\dfrac{1-\frac{\alpha(AD-BC)}{A}}{1-\alpha D}$ ⑤

④ ⇒ $A=\left.\frac{v_{ab}}{i_{ab}}\right|_{x_j=0}$ = def of Z^{0}_{ab} ⑥

Now suppose $v_{ab}=0$ and $x_j=x_x$ (indep. of x_i)
Then ④ ⇒ $i_{ab} = -\frac{B}{A}\cdot x_x$
$x_i = C\cdot i_{ab} + D\cdot x_x$

$$\Rightarrow \underbrace{-\alpha\frac{x_i}{x_x}}_{\equiv T_{sc}} = -\frac{\alpha(AD-BC)}{A} \quad ⑦$$

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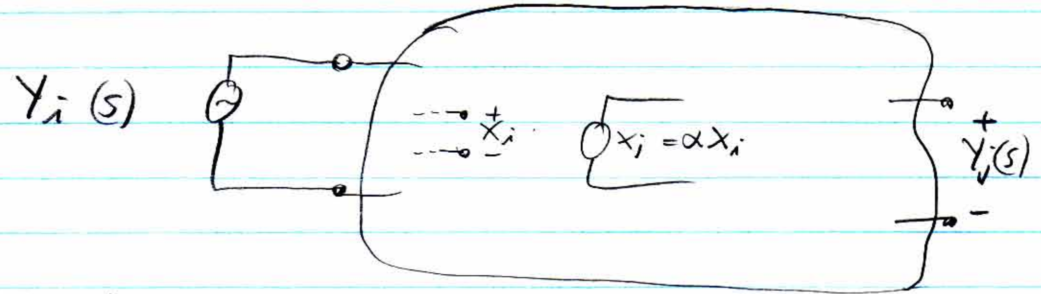
Now suppose $i_{ab} = 0$ and $x_j = x_x$ (indep. of x_i)
Then ④ similarly $\Rightarrow$ $T_{oc} = -\alpha D$ ⑧

$$\therefore \; ⑤ \sim ⑧ \Rightarrow Z_{ab} = Z_{ab}^{0} \cdot \frac{1+T_{sc}}{1+T_{oc}} \quad ☑$$

□

Asymptotic Gain Relation

⑨



arbitrary SSM circuit with at least 1 controlled source

$X_i \neq Y_i$ required

any controlled source in circuit (≡ "ref. source")

$\left.\begin{matrix} x_i, x_j \\ Y_i, Y_j \end{matrix}\right\}$ voltages or currents

$$\text{AGR:} \quad A(s) = A_\infty \cdot \frac{T}{1+T} + A_0 \cdot \frac{1}{1+T} \qquad ⑩$$

$$\left(= \frac{Y_j(s)}{Y_i(s)} \right)$$

where $\underbrace{T \equiv -\alpha \cdot \frac{x_i}{x_x}}_{\text{"loop gain"}}$ where $Y_i = 0$ and X_j replaced by an independent "test source", X_x

$$A_\infty(s) \equiv \left. \frac{Y_j(s)}{Y_i(s)} \right|_{\alpha \to \infty} \equiv \text{"asymptotic gain"}$$

$$A_0(s) \equiv \left. \frac{Y_j(s)}{Y_i(s)} \right|_{\alpha = 0} \equiv \text{"direct transmission term"}$$

⑥/6

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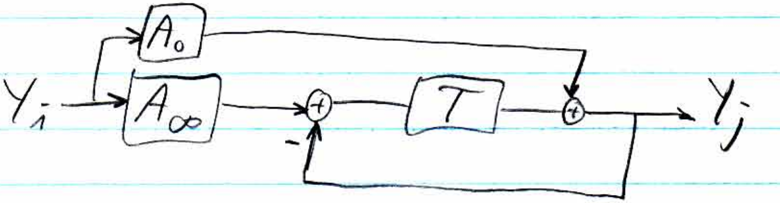
Observations

MGF & ⑩ ⇒ can redraw ⑨ as



⇒ large T ⇔ feedback dominates behavior ⇒ $A(s) \cong A_\infty(s)$

$A_0(s)$ arises from forward path through feedback network

"Reference source" in BIR & AGF:

See note of March 18, 2008

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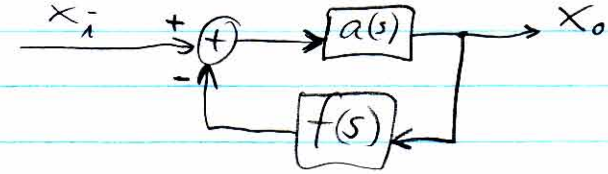
Avg.: 68
Med.: 70 } Mid-term

Feedback

Let $x_i(s)$ = input signal (V or I)
$x_o(s)$ = output —— " ——

Then a circuit incorporates feedback if its B.D. can be reduced to:

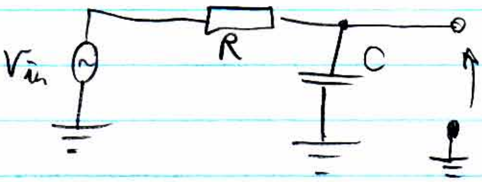


①

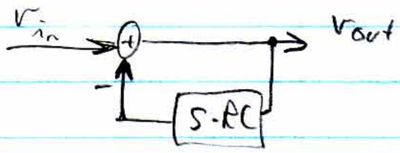
MGF + ① ⇒

$$A(s) = \frac{a(s)}{1+a(s)f(s)} \quad ② \quad \text{where } A(s) \equiv \frac{x_o(s)}{x_i(s)}$$

Ex 1 "Passive feedback"



⇒



∴ $a(s) = 1$
$f(s) = sRC$

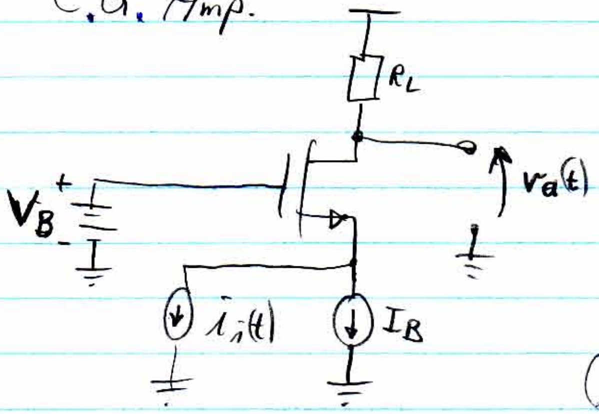
(more generally, poles ⇒ feedback)

$$\frac{v_{out}}{v_{in}} = \frac{\frac{1}{sC}}{\frac{1}{sC}+R} = \frac{1}{1+sRC} \Rightarrow v_{out} = v_{in} - v_{out} \cdot s \cdot RC$$

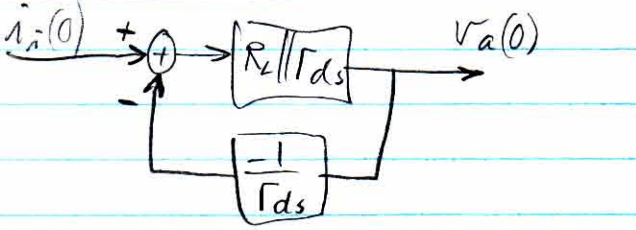
Feb 12, 2008

Ex 2 "Parasittic feedback"

C.G. Amp.

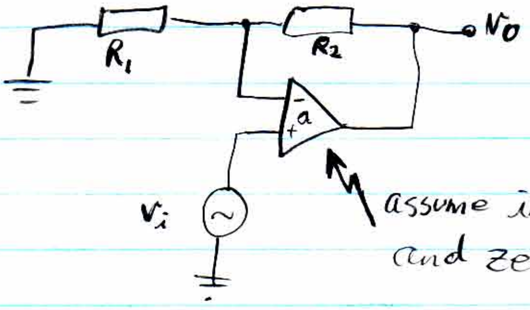


⇒ (found prev.) (low freq. analysis)



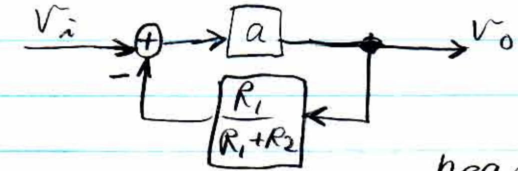
Note: In Ex1 feedback reduces the magnitude of X_o ≡ "neg. feedback"
In Ex2 —"— increases ——"—— ≡ "pos. feedback"

Ex 3 "Intentional active feedback"



assume infinite inp. impedance and zero output —"—

$$v_o = a\left[v_i - \frac{R_1}{R_1+R_2}\cdot v_o\right]$$



$$A = \frac{a}{1+a\left(\frac{R_1}{R_1+R_2}\right)} \rightarrow 1+\frac{R_2}{R_1} \text{ as } a\to\infty$$

neg. feedback

$\therefore \underbrace{\text{large } a}_{\text{easy in IC's}} \Rightarrow A$ depends only of the $\underbrace{\text{ratio of } R\text{'s}}_{\text{accurate in IC's}}$

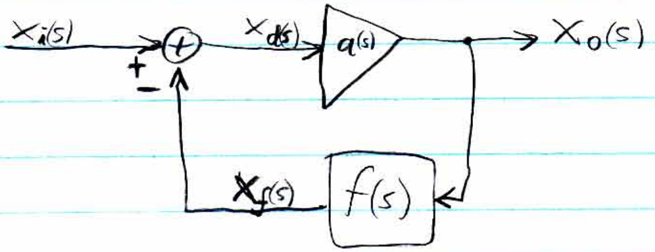
3/6

Feb. 12, 2008

Intentional active feedback ⇒ feedback around high gain amp
⇒ many benefits (e.g. ③)

Negative Intentional Active Feedback

Defs.: In the following feedback system



④

$a(s) \equiv$ "open loop gain", $f(s) \equiv$ "feedback factor"

$T(s) \equiv a(s)\cdot f(s) \equiv$ "loop gain", $A(s) \equiv$ "closed loop gain"
(recall $A(s) = \frac{a(s)}{1+a(s)\cdot f(s)}$)

Let $A_{ideal} = \lim_{a\to\infty} A = \frac{1}{f}$ (e.g. $A_{ideal} = 1+\frac{R_2}{R_1}$ in Ex 3)

Asymptotic gain-relation: jan. 31

$A = A_{ideal}\cdot\frac{T}{1+T}$

$\boxed{A = \frac{1}{f}\left(1-\frac{1}{1+a\cdot f}\right) = \frac{1}{f}\cdot\frac{af}{1+af} = \frac{a}{1+af}}$

∴ ② ⇒ $A = A_{ideal}\left(1-\underbrace{\frac{1}{1+T}}\right)$ fractional deviation of A from Aideal

Also, MGF & ④ ⇒

$$x_d = \frac{1}{1+T}\cdot x_i \to 0 \text{ as } T\to\infty \quad (a\to\infty)$$

$$x_f = \frac{1}{1+1/T}\cdot x_i \to x_i \text{ as } T\to\infty \quad (a\to\infty)$$

∴ x_f "tracks" x_i with "error signal" x_d (d = difference)

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Benefits: (i) gain desensitivity
(ii) non-linear distortion reduction
(iii) in-loop noise suppression
(iv) broadbanding
(V) input/output impedance adjustment

(Potential) Drawbacks (i) $A < a$
(ii) excessive phase shift can cause instability

$\left(\frac{U}{V}\right)' = \frac{U'V - UV'}{V^2}$

(i) Gain desensitivity

② $\Rightarrow \frac{dA}{da} = \frac{1}{(1+af)^2} \Rightarrow \frac{\Delta A}{\Delta a} \cong \frac{1}{(1+af)^2} \rightarrow = \frac{A}{a} \cdot \frac{1}{1+a \cdot f}$ small Δa

$$\therefore \frac{\Delta A}{A} \cong \left(\frac{1}{1+T}\right) \cdot \frac{\Delta a}{a} \Rightarrow$$ large variation in a $\Rightarrow$
small —"— A

$\Rightarrow$ A is "stable" WRT variations in a

② $\Rightarrow \frac{dA}{df} = -A^2$

$$\therefore \frac{\Delta A}{\Delta f} \cong -A \cdot \overbrace{\frac{1}{f}}^{\rightarrow A_{ideal}} \cdot \frac{T}{1+T}$$

$$\therefore \frac{\Delta A}{A} \cong -\underbrace{\left(\frac{T}{1+T}\right)}_{\cong 1 \text{ for for } |T| \gg 1} \frac{\Delta f}{f}$$ $\Rightarrow$ A is not "stable" WRT variations

$\Rightarrow$ want high gain amplifier (not nec. well defined) and precise feedback factor.

$$\boxed{\frac{\Delta A}{A} = \frac{-T}{1+T} \cdot \frac{\Delta f}{f} \quad \Bigg| \quad \frac{\Delta A}{A} = \frac{1}{1+T} \cdot \frac{\Delta a}{a}}$$

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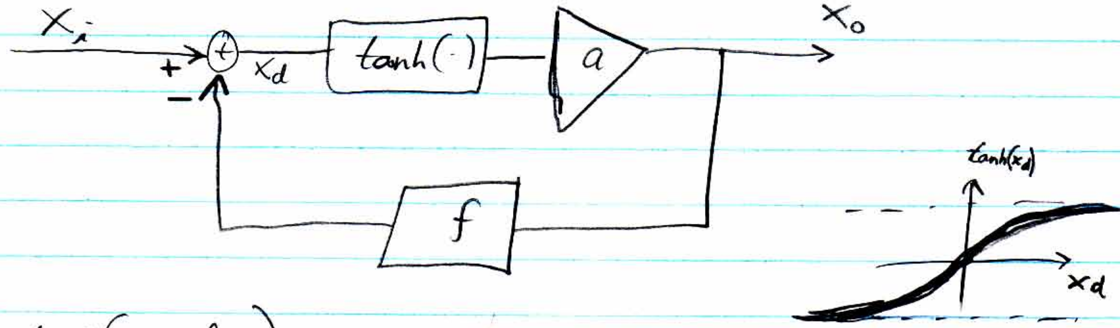
(ii) Non-linear distortion reduction

Ex 4



$$x_o = a \cdot \tanh(x_i - f . x_o)$$

$$\therefore \tanh^{-1}\left(\frac{x_o}{a}\right) = x_i - f \cdot x_o$$

$$\therefore \frac{x_o}{a} + \frac{x_o^3}{3a^3} + \frac{x_o^5}{5a^5} + \dots = x_i - f x_o$$

$$\therefore X_o + \underbrace{\frac{x_o^3}{3a^2} + \frac{x_o^5}{5a^4} + \dots}_{\rightarrow 0 \text{ as } a \rightarrow \infty} = a(x_i - f X_o)$$

$$\therefore X_o \cong A \cdot x_i \text{ for large } a \text{ and } X_o \rightarrow \underbrace{\frac{1}{f} \cdot x_i}_{\text{no distortion}} \text{ as } a \rightarrow \infty$$

Heuristics: 1) Follows from gain desensitivity
(distortion prior to a ⇔ input dependent gain variation)
2) Large a ⇒ small x_d ⇒ small portion of non-linear curve is traversed ⇒ ≈ linear

⑥/⑥

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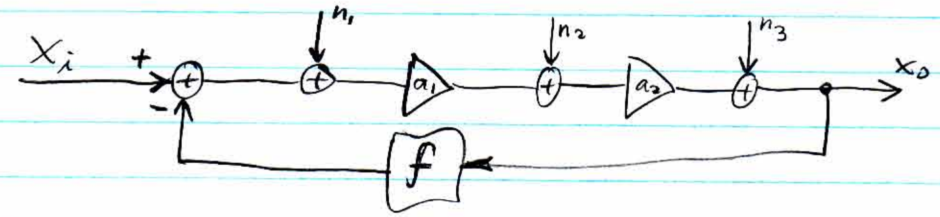
(iii) In-loop noise suppression



Open loop gain:

$a = a_i a_2$

$$MGF \Rightarrow x_o = \frac{a}{1+af}\left(x_i + h_1 + \frac{n_2}{a_1} + \frac{n_3}{a_1 a_2}\right)$$

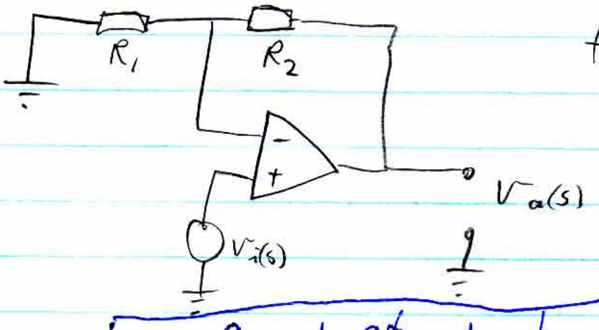
not suppressed | somewhat suppressed | most suppressed

(iv) Broadbanding

Ex 5 suppose $a \cong \frac{a_o}{1-j\,\omega/\omega_{po}}$ (dom. pole approx.)

$F \cong$ indep. of ω

e.g.



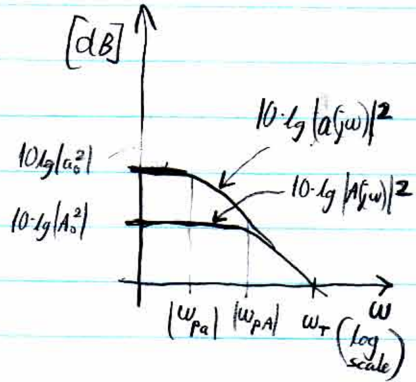
$$f = \frac{R_1}{R_1+R_2}$$

Then ② $\Rightarrow$ $\boxed{A = \frac{a}{1+af} = \frac{1}{f}\cdot\frac{af}{1+af} = \frac{1}{f}\cdot\frac{1}{1+\frac{1}{f}\cdot\frac{1}{a}} = A_{ideal}\cdot\frac{1}{1+1/T}}$

$$A(j\omega) = \underbrace{\left(1+\frac{R_2}{R_1}\right)}_{1/f}\frac{1}{1+\left(1+R_2/R_1\right)\left(1-j\,\omega/\omega_{p0}\right)/a_o}$$

algebra $\Rightarrow A(j\omega) = \frac{A_o}{1-j\,\omega/\omega_{pA}}$ where $A_o = \left(1+\frac{R_2}{R_1}\right)\frac{1}{1+\left(1+\frac{R_2}{R_1}\right)/a_o}$ ⑤

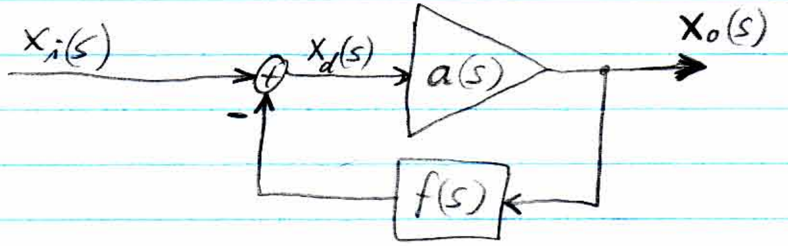
and $\omega_{pA} = \omega_{po}\left(1+a_o\cdot\frac{R_1}{R_1+R_2}\right)$ ⑥



Feb. 14, 2008 ECE 264A E.G.

Negative intentional active feedback (cont. from pp. $\frac{3\sim 6}{6}$ last time)

Recall: Can allways write the block diagram of a circuit with feedback as:



①

$a(s) \equiv$ "open loop gain"
$f(s) \equiv$ "feedback factor"
$T(s) \equiv a(s) \cdot f(s) \equiv$ loop gain ← (i M.GF.: $T = \div a \cdot f$ ulike definisjoner!)

$A(s) = \frac{x_o(s)}{x_i(s)} \equiv$ "closed loop gain"

$$MGF \Rightarrow A(s) = \frac{a(s)}{1 + a(s) \cdot f(s)} \quad ②$$

Broadbanding (cont.)

$$\text{Suppose: } a(j\omega) = \underbrace{a_o}_{\text{DC-gain}} \cdot \frac{1}{1 - j\,\omega/\omega_{pa}} \quad ③$$

↳ dominant pole approx.

$$f(j\omega) = \text{const.}$$

$$②, ③ \Rightarrow A(j\omega) = A_o \cdot \frac{1}{1 - j\,\omega/\omega_{pA}} \quad \text{where}$$

$$A_o = \frac{a_o}{1 + a_o \cdot f} \quad (\text{DC closed loop gain})$$

$$\& \quad \omega_{pA} = (1 + a_o \cdot f) \cdot \omega_{pa}$$

④

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$|A_0 \cdot \omega_{pA}| \equiv$ "closed loop gain bandwidth product", $GBW_{closed\ loop}$

$|a_0 \cdot \omega_{pa}| \equiv$ "open loop gain bandwidth product", $GBW_{open\ loop}$

④ $\Rightarrow \quad A_0 \cdot \omega_{pA} = a_0 \cdot \omega_{pa}$

$\therefore \quad GBW_{open\ loop} = GBW_{closed\ loop}$ in this case.

$\therefore$ Increasing f (between 0 and 1) decreases A_0, but increases the 3dB BW.

"Unity Gain Frequency"

$\equiv \omega_t$ such that $a(j\omega_t) = 1$, $\omega_t \in \mathbb{R}$

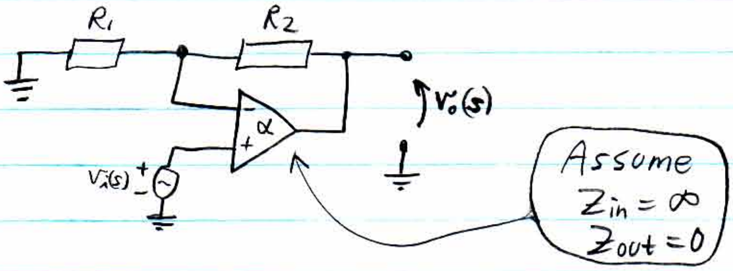
$\therefore$ ③ $\Rightarrow \quad \left| a_0 \cdot \frac{1}{1 - j\,\omega_t/\omega_{pa}} \right| = 1$

$\Rightarrow (a_0^2 - 1) \cdot \omega_{pa}^2 = \omega_t^2$

$\Rightarrow \omega_t \approx a_0 \cdot |\omega_{pa}| \qquad (a_0 \gg 1)$

$\therefore$ GBW $= \omega_t$ in this case (not allways in other cases)

Ex 1 (from last time)



Assume
$Z_{in} = \infty$
$Z_{out} = 0$

$\Rightarrow$ ① if $a = \alpha$, $f = \frac{R_1}{R_1 + R_2}$

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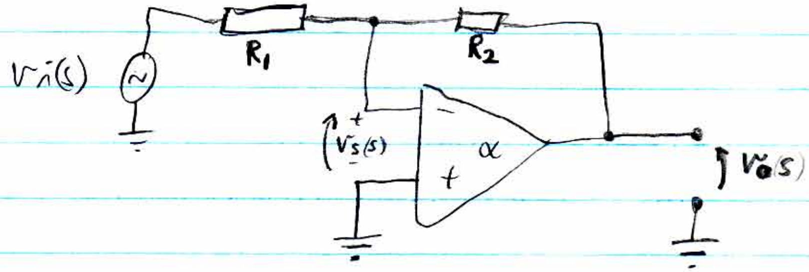
Ex2 (Same except different input location)



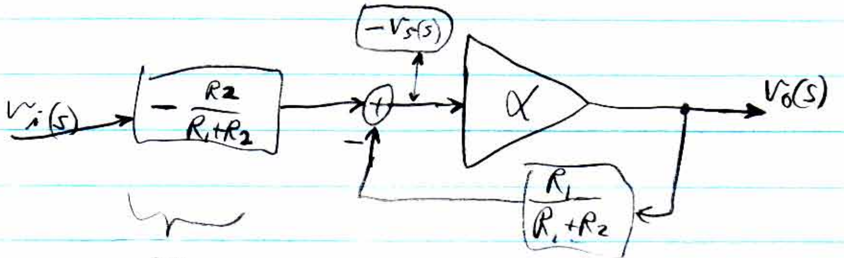
⑥

$$v_o(s) = \alpha(s)\cdot\left[-v_s(s)\right]$$

$$\therefore\ v_s(s) = v_i(s) - R_1\cdot\left[\frac{v_i(s)-v_o(s)}{R_1+R_2}\right]$$

$$\therefore\ -v_s(s) = -v_i(s)\underbrace{\left[1-\frac{R_1}{R_1+R_2}\right]}_{=\frac{R_2}{R_1+R_2}} - v_o(s)\cdot\left[\frac{R_1}{R_1+R_2}\right]$$

∴ BD of ⑥ is:



BW = ∞

$\equiv$ (1) with $a = \alpha$ & $f = \frac{R_1}{R_1+R_2}$ (Same as Ex. 1.)

$\Rightarrow$ BW = same as Ex 1

But GBW of ⑥ less than that of Ex 1 by $\frac{R_2}{R_1+R_2}$

Also, $GBW_{\text{closed loop}} = \frac{R_2}{R_2+R_1}\cdot\omega_t = \frac{R_2}{R_1+R_2}\cdot\underbrace{a_0\,|\omega_{pa}|}_{GBW_{\text{open loop}}}$

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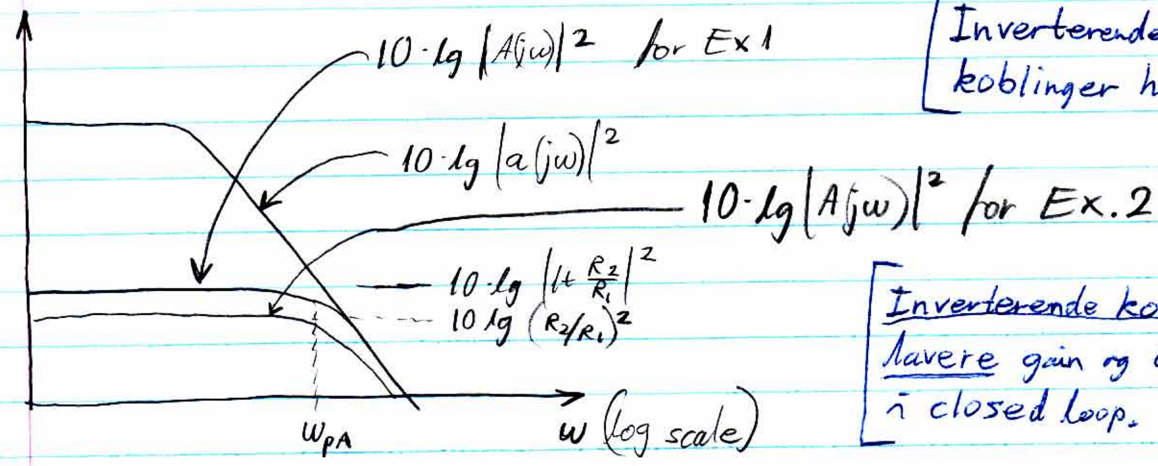
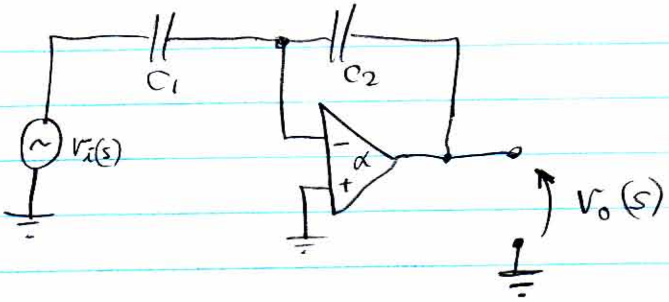


[Inverterende og ikke-inv. koblinger har samme B.W.]

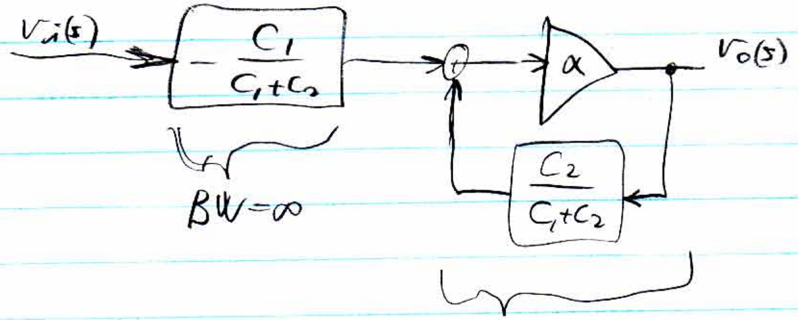
[Inverterende kobling gir lavere gain og GBW produkt i closed loop.]

Ex 3 Same as Ex 2 with R's replaced by C's (important later in switched cap. circuits)



⑦

Exercise show BD of ⑦ is



$$\equiv ① \text{ with } \begin{array}{l} a=\alpha \\ f=\frac{C_2}{C_1+C_2} \end{array} \Rightarrow f=\text{const} \, (\in \mathbb{R})$$

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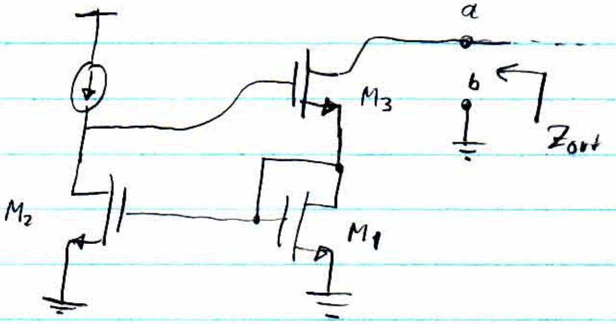
$$\therefore \text{④} \Rightarrow BW = \left(1 + a_0 \frac{C_2}{C_2 + C_1}\right) \cdot |\omega_{pa}|$$

Also $$GBW_{\text{closed loop}} = \frac{C_1}{C_1+C_2} \cdot \omega_t = \frac{C_1}{C_1+C_2} \cdot a_0 \cdot |\omega_{pa}|$$

(V) Impedance transformation

Ex. 4 Wilson current mirror



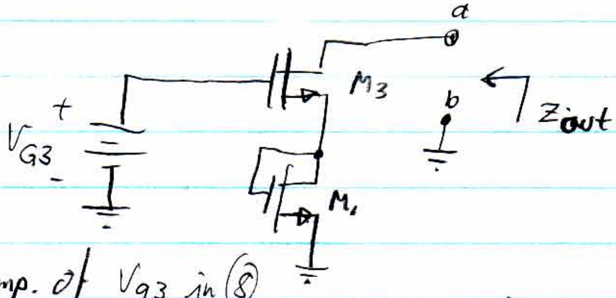
⑧

previously found:

$Z_{out} \approx r_{ds3} \cdot g_{m2} \cdot r_{ds2}$

(low freq. analysis, neglecting body effect)

Now consider



⑨

Where V_{g3} = DC comp. of V_{g3} in ⑧

Can show: Z_{out} of ⑨ $\cong r_{ds3} \left(2 + \frac{1}{g_{m1} \cdot r_{ds3}}\right)$

$$\approx 2 \cdot r_{ds3}$$

Note: ⑨ is equiv. to ⑧ with feedback disabled

$\therefore$ feedback increases Z_{out} by $\left(\frac{1}{2} g_{m2} \cdot r_{ds2}\right)$ factor

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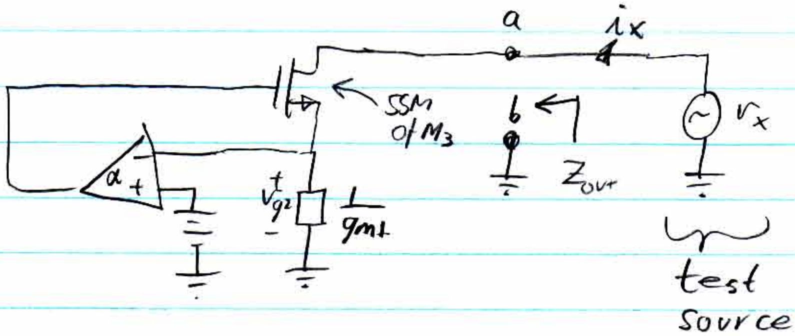
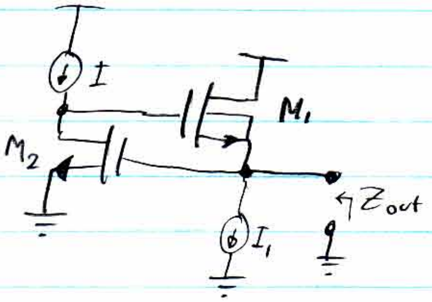
Heuristics:

(8) ≡



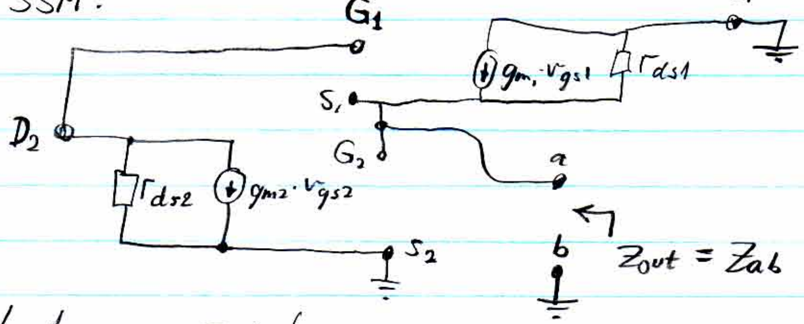
If v_x increases, i_x increases ⇒ v_{g2} increases
⇒ v_{g3} decreases
⇒ feedback acts to reduce i_x

Ex 5 Voltage source



(10)

Low freq SSM:



(11)

BIR: Let $g_{m1}\cdot v_{g1} \equiv$ ref. source
$\therefore v_{gs1} = x_i,\ \alpha = g_{m1},\ Z_{ab}^{\circ} = r_{ds1}$

Exercise: Show $T_{sc} = 0$, $T_{oc} = g_{m1}\cdot g_{m2}\cdot r_{ds1}\cdot r_{ds2}$

$$\therefore Z_{ab} = r_{ds1}\cdot\frac{1}{1+g_{m1}\, g_{m2}\cdot r_{ds1}\cdot r_{ds2}} \approx \frac{1}{g_{m1}\, g_{m2}\, r_{ds2}} = \text{small!}$$

Feedback reduced output imp. by a factor of $g_{m2}\cdot r_{ds2}$

(1/4) ECE 264A Feb. 19, 2008 E.G.

Stability

Let $$A(s) = \frac{a_0' + a_1' \cdot s + a_2' s^2 + \ldots + a_m' s^m}{b_0' + b_1' s + b_2' s^2 + \ldots + b_n' \cdot s^n} \quad ①$$

If $A(0) \neq 0$ or ∞, can write ① as

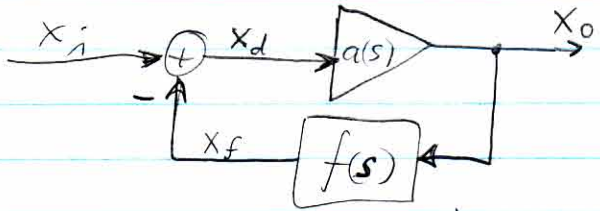
...then:

$$A(s) = A_0 \cdot \frac{1 + a_1 s + \ldots + a_m \cdot s^m}{1 + b_1 s + b_2 \cdot s^2 + \ldots + b_n \cdot s^n} \quad ②$$

(other choises of $T(s)$ are possible if we allow $a(s)$ has poles)

$$\therefore A(s) = \frac{a(s)}{1 + a(s) \cdot f(s)}$$ where $a(s) \cdot f(s) = T(s)$
$a(s) = A_0 (1 + a_1 s + \ldots + a_m \cdot s^m)$

∴ ② can be impl. as:



Def.: A system is stable if (and only if) it satisfies the following property:

For each bounded input signal, $X_i(t)$, the output signal, $X_o(t)$, must also be bounded

($X(t)$ is bounded means $\exists B \in \mathbb{R}$ s.t. $|X(t)| < B \; \forall t$)

"such that"

∴ "Stability" ≡ $\underbrace{\text{"bounded input, bounded output}}_{\text{"BIBO"}}$, stability"

April 18, 2007

ECE 171A

Bounded input: $R(s) = \frac{A_R(s)}{B_R(s)}$ [redacted]

...hvor $B_R(s)$ sine røtter har negativ realdel eller, om de er på den vertikale aksen, multiplisitet maks 1 (ellers får vi terms med rampe-form)

⇒ Stabilt sys. må ha neg. realdel på polene for å garantere bounded output.

②/4

Feb. 19, 2008

Claim 1 Let $h(t)$ be the impulse response of an LTI system and let $H(s)$ be its transfer function. Then the system is stable iff either of the following hold:

i) All poles of $H(s)$ are in the LHP (not including imag. axis) (ie. if s_0 is a pole then $Re\{s_0\} < 0$)

ii) $\int_{-\infty}^{\infty} h(\tau)\, d\tau < \infty$

iff ≡ "if and only if"

proof Exercise (review material)

Claim 2 If an LTI has a transfer function given by ②, then it is stable iff

$$T(s_0) = -1 \Rightarrow Re\{s_0\} < 0$$

proof: Exercise.

Def. "Nyquist plot" ≡ plot of $Im\{T(j\omega)\}$ vs. $Re\{T(j\omega)\}$ as ω increases from $-\infty$ to ∞

Nyquist Criterion:

Provided there are no RHP pole-zero cancellations in $T(s) = a(s) \cdot f(s)$ an LTI system is stable iff the net no. of CCW encirclements of the point $(-1, 0)$ by the Nyquist plot equals the no of RHP poles in $T(s)$

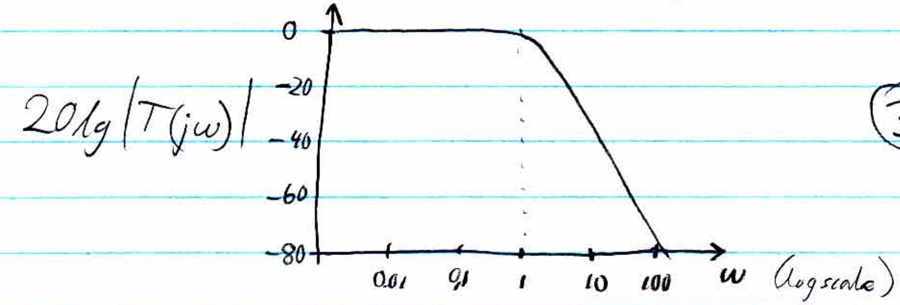
∴ Nyquist crit. ⇒ stability test ⟷ {not so useful these days (because of computers)}

will soon see: Nyquist crit. ⇒ insight

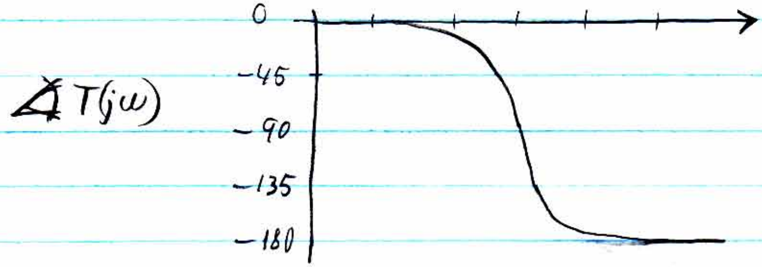
⇒ concepts of phase & gain margin } very useful

Feb. 19, 2008

Ex1 Suppose $T(s) = \frac{1}{(1+s)(1+s/2)}$ poles: -1, -2

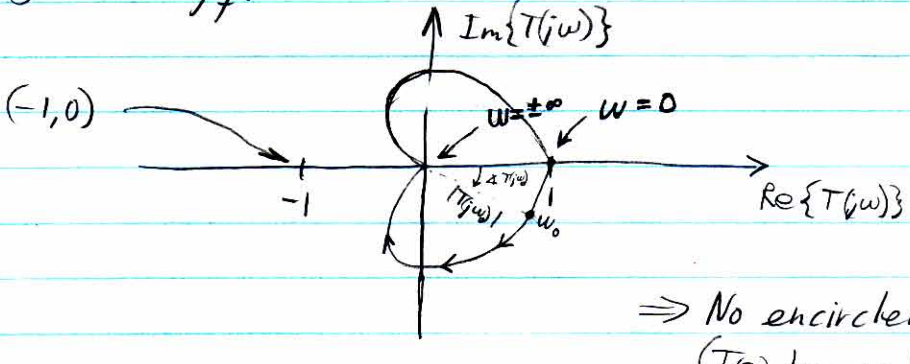


③



④

③, ④ ⇒ Nyq. Plot



⇒ No encirclements of $(-1,0)$
($T(s)$ has no RHP poles by inspection)

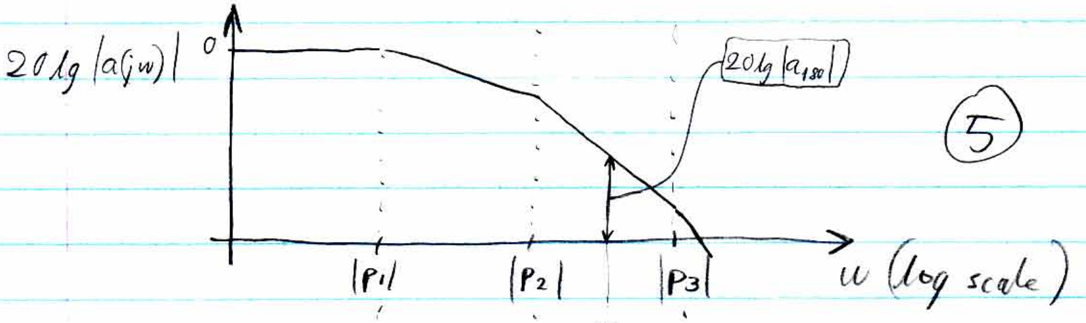
Nyq. crit ⇒ system with t.f. $A(s) = \frac{a(s)}{1+T(s)}$
is stable (provided there are no pole-zero cancellations in $a(s)\,f(s)$) (why?)

Feb. 19, 2008

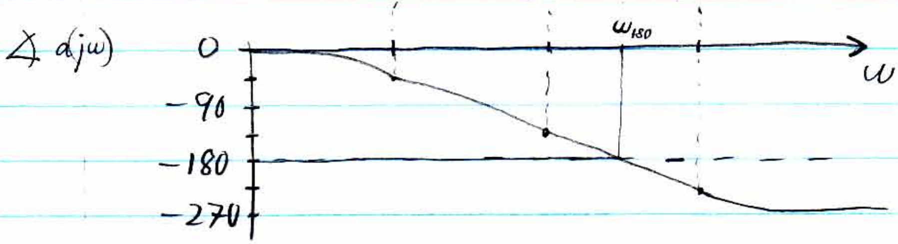
Ex 2 Suppose $a(s) = \dfrac{a_o}{(1-s/p_1)(1-s/p_2)(1-s/p_3)}$ and $f = \text{const.}$

$$\therefore T(j\omega) = f \cdot a(j\omega)$$

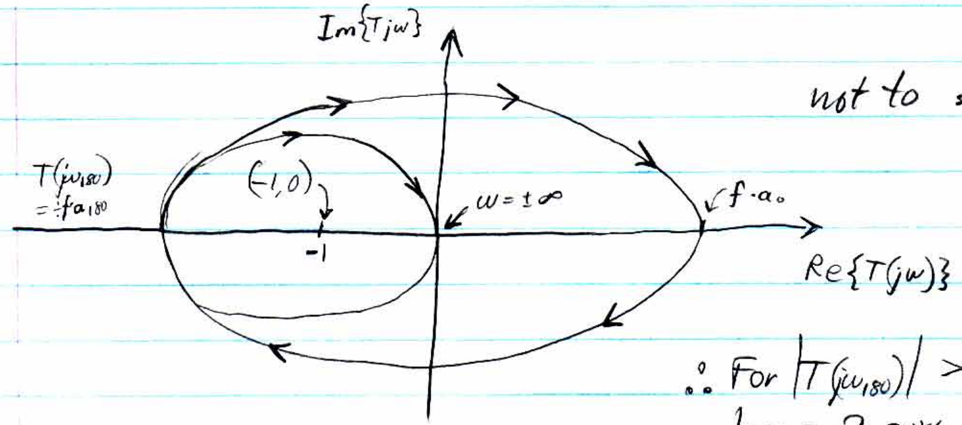
For $|p_1| \approx \frac{1}{10} \cdot |p_2| \approx \frac{1}{100} |p_3|$ and $Re\{p_i\} < 0 \quad i = 1,2,3$



⑤



⑥



not to scale

$\therefore$ For $|T(j\omega_{180})| > 1$ (the case shown) have 2 c.w. encirclements, but $T(s)$ has no RHP poles

$\therefore$ Nyquist crit $\Rightarrow$ unstable if $|T(j\omega_{180})| > 1$
$\Rightarrow$ stable if $|T(j\omega_{180})| < 1$

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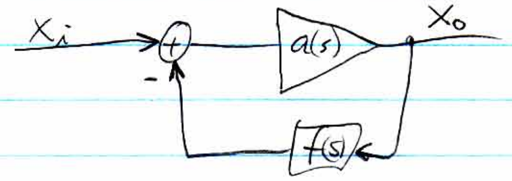
Nyquist Criterion (cont.)

Let $A(s) = \frac{a(s)}{1 + a(s)\cdot f(s)}$ $\Leftrightarrow$



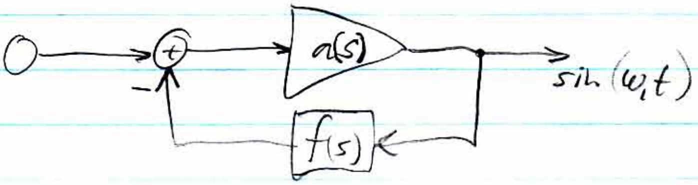
$T(s) = a(s)\cdot f(s)$

Assume: No zero of $f(s)$ is a pole of $a(s)$
Then $T(s_0) = -1 \Leftrightarrow s_0$ is a pole of $A(s)$

s.t. = "such that"

Ex. 1 Suppose $T(j\omega_1) = -1$ for some $\omega_1 \in \mathbb{R}$
$\Rightarrow A(s)$ has pole on imag. axis $\Rightarrow$ oscillation
$\Rightarrow$ unstable

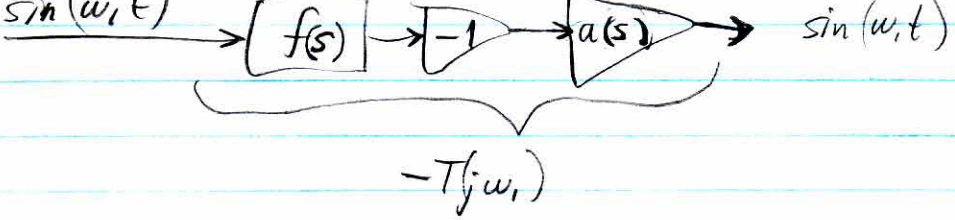
$\therefore$ can choose initial cond. s.t.



①

Occurs.

This happens because ① is equiv. to



$T(j\omega_1) = -1 \Rightarrow |T(j\omega_1)| = 1$

and

$\measuredangle T(j\omega_1) = \pi$

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Ex 2 Suppose $T(s_0) = -1$ for $s_0 = \sigma_0 + jw$, $\sigma_0 \geq 0$

$\Rightarrow$ have ① with $\sin(\omega_1 t)$ replaced by $\underbrace{e^{\sigma_0 t}}_{\to\infty \text{ as } t\to\infty} \cdot e^{j\omega_0 t}$

$\Rightarrow$ 0 input $\Rightarrow$ unbounded output

Ex3 Suppose $\underbrace{A(s) \text{ is stable}}_{\text{slang}}$, but for some $\omega_1 \in \mathbb{R}$,

$$|T(j\omega_1)| = 1 - |\varepsilon| \text{ where } |\varepsilon| = \text{small } \#$$

and $\angle T(j\omega_1) = \pi$

Then have ringing for any input step

e.g.: $X_i =$  $X_o =$ 

In many systems, ringing is undesirable because it slows the settling time.

$\Rightarrow$ We need to design with "gain margin" s.t. $|T(j\omega)| \neq 1$ when $\angle T(j\omega) = \pi$ with sufficient "margin" to minimize ringing.

Nyq. crit. = method of evaluating "marginal stability" using simulated or calculated freq. response plots

Recall Nyq. Plot $\equiv$ Plot of $\text{Im}\{T(j\omega)\}$ vs. $\text{Re}\{T(j\omega)\}$ as ω increases from $-\infty$ to ∞

Nyq. Crit $\equiv$ An LTI system is stable iff the net number of CCW encirclements of $(-1, 0)$ by the Nyq. plot equals the number of RHP poles of $T(s)$.

③/⑤

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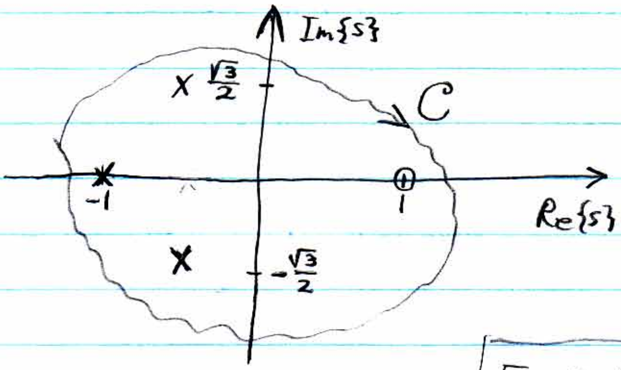
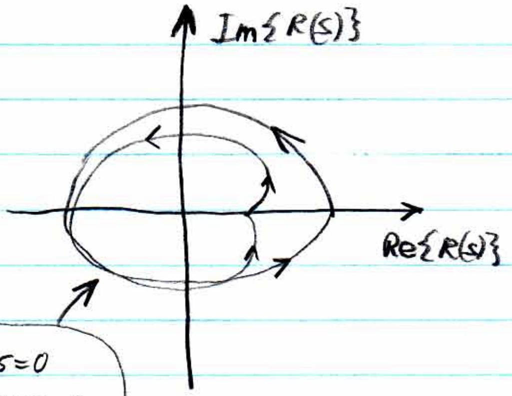
The Nyquist crit. is based on The Encirclement Property (E.P.).

E.P.: Let $R(s)$ be any rational fn. The plot of $R(s)$ along a closed CW path, C, in the complex s-plane encircles the point $s=0$ in a clockwise direction a net no. of times equal to the no. of zeros ÷ no. of poles within the contour.

Ex. 4 Suppose $R(s) = \frac{s-1}{(s+1)(s^2+s+1)}$ poles: $-1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2} j$

zero: 1

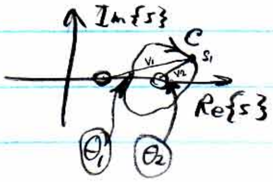




Encircles s=0 twice in ccw. dir.

Why does this work?

Ex.5 Suppose $R(s)$ has only 2 (RHP) zeros and no poles



$(V_1, V_2 \in \mathbb{R}, >0)$

$\therefore R(s_1) = V_1 V_2 e^{j(\theta_1+\theta_2)}$

As C is traversed: $\begin{cases} \text{net change in } \theta_1 \text{ is } 0 \\ \text{" ——— " } \theta_2 \text{ is } -2\pi \end{cases}$

② $\Rightarrow \measuredangle R(s) = \theta_1 + \theta_2$

$\therefore$ As C is traversed, the net change in $\measuredangle R(s)$ is -2π

$\Rightarrow R(s)\big|_{s \in C}$ circles $\underset{(s=0)}{\text{origin}}$ once (in cw dir.)

④/5

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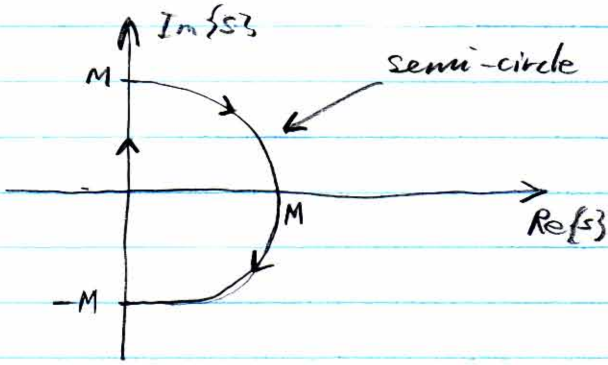
Proof of Nyq. crit.

Restrictions of proof:
1) Rational $T(s)$
2) $|T(j\omega)| < \infty \;\forall\, \omega \in \mathbb{R}$
3) $T(s)$ has (# poles) $\geq$ (# of zeros)

(But Nyq. crit also applies without restrictions 1) & 3) and can be modified to handle poles on the imag axis.)

Consider contour C as follows:



As $M \to \infty$, C contains all RHP poles of $A(s)$

$$1), 3) \Rightarrow T(s) = \frac{c_0 + c_1 \cdot s + \ldots + c_{N_1} \cdot s^{N_1}}{d_0 + d_1 \cdot s + \ldots + d_{N_2} \cdot s^{N_2}}$$

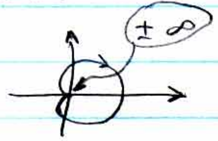
Where $N_1 \leq N_2$

$$\therefore \lim_{|s|\to\infty} T(s) = \begin{cases} \frac{c_{N_1}}{d_{N_1}} & \text{if } N_1 = N_2 \\ 0 & \text{if } N_1 < N_2 \end{cases} = \text{const } (\in \mathbb{R})$$

$\Rightarrow$ The plot of $T(s)$ along the semi-circular part of C approaches a single point on real axis as $M \to \infty$

$\Rightarrow$ Nyq. plot is equiv. to plot of $T(s)$ along C as $M \to \infty$

e.g., recall: Nyq. plot of $T(s) = \frac{1}{(1+s)(1+s/2)}$ is



$\Rightarrow$ Can apply encirclement property to Nyq. plot

Let $R(s) = 1 + T(s)$. Then plot of $R(s)$ along C encircles the origin exactly as many times as that of $T(s)$ encircles $(-1, 0)$.

$\therefore$ E.P. $\Rightarrow$ net no. of cw. encirclements of $(-1, 0)$
= # of zeros of $(1+T(s))$ minus # of poles of $1+T(s)$ inside C.

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But:
- zeros of $1+T(s) \equiv$ poles of $A(s)$
- poles of $1+T(s) \equiv$ poles of $T(s)$
- inside C = all RHP as $M \to \infty$

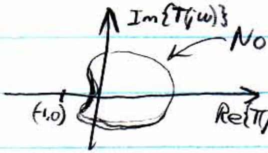
$\Rightarrow$ Nyq. crit.

$\square$

Important special case of Nyquist criterion:

Suppose $\measuredangle T(0) = 0$ and $T(s)$ has no RHP poles
If $|\measuredangle T(j\omega)| = \pi$ for only one pos. value, $\omega = \omega_\pi$,
Then $A(s)$ is stable iff $|T(j\omega_\pi)| < 1$

eg. picture:



No encirclements are possible

Note:

In general, $|T(j\omega_\pi)| > 1 \not\Rightarrow$ instability(!)

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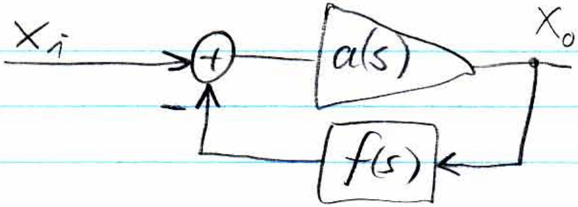
OH: $1 \sim 2^{30}$ this Friday (2/29)

Gain & Phase Margin



$$A(s) = \frac{a(s)}{1+\underbrace{a(s)\cdot f(s)}_{T(s)}}$$

Assume no pole of $a(s)$ is a zero of $f(s)$.

Nyq. crit ⇒ 2 special cases:

i) Suppose $\measuredangle T(0) = 0$, $T(s)$ has no RHP poles and $|\measuredangle T(j\omega)| = \pi$ has no more than one positive solution, $\omega = \omega_\pi$ } ①

Then $A(s)$ is stable iff $|T(j\omega_\pi)| < 1$

ii) Suppose $\measuredangle T(0) = 0$, $|T(0)| > 1$, $T(s)$ has no RHP poles, and $|T(j\omega)| = 1$ has no more than one positive solution, $\omega = \omega_u$ } ②

Then $A(s)$ is stable iff $-\pi < \measuredangle T(j\omega_u) < \pi$

---

i) ⇒ Basis of "Gain margin" (GM) definition (soon)

ii) ⇒ Basis of "Phase margin" (PM) definition (soon)

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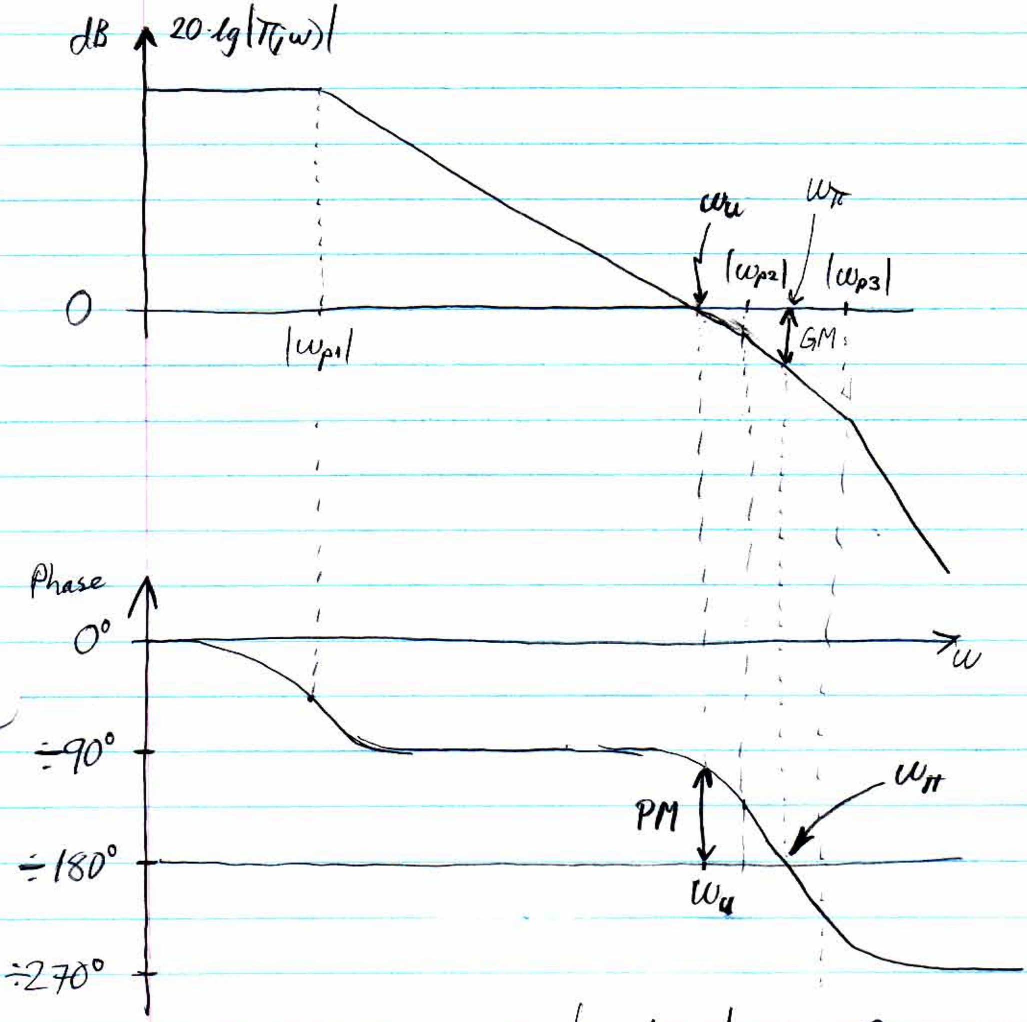
Def. of PM & GM (via 3-pole T(s) example)



ie. $GM \equiv 20\,lg\left|\frac{1}{T(j\omega_\pi)}\right|$ (pos. as shown)

$PM \equiv 180° - |\measuredangle T(j\omega_u)|$ (pos. as shown)

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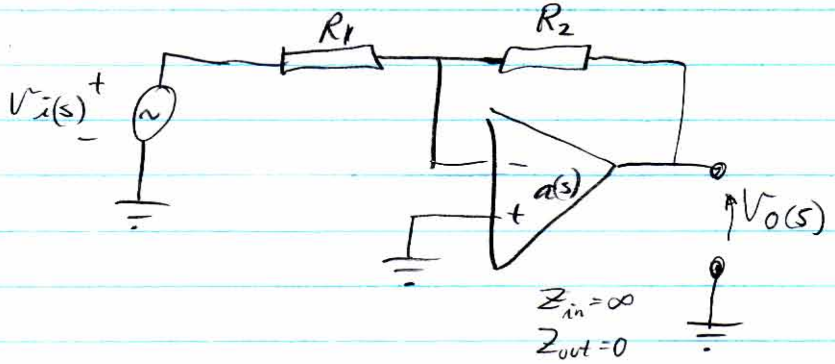
∴ ①, $GM > 0 \Leftrightarrow$ stable (closed loop)

②, $PM > 0 \Leftrightarrow$ stable (closed loop)

Also, larger GM (PM) ⇔ better marginal stability (i.e. step response has less ringing; can tolerate larger component errors without instab.)

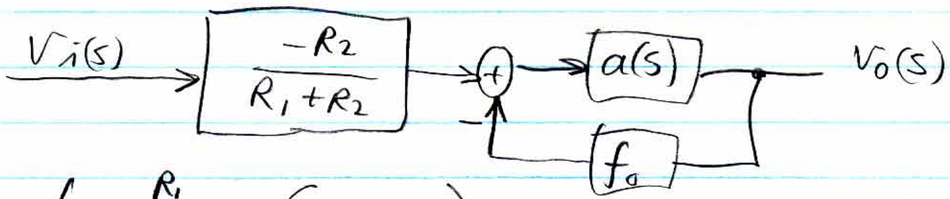
Consider:



③

Previously found:

③ ≡



where $f_0 = \frac{R_1}{R_1+R_2}$ $(0 < f_0 < 1)$

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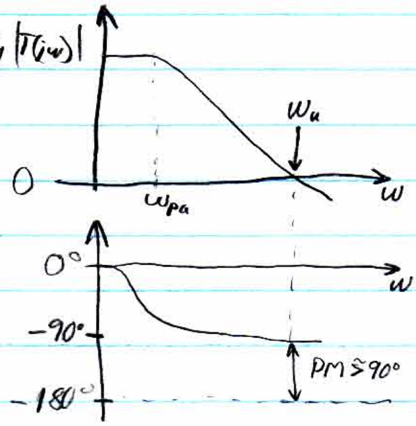
Ex 1 ③ with $a(s) = \frac{a_0}{1 - s/\omega_{pa}}$, $\omega_{pa} < 0$ $(\in \mathbb{R})$

$a_0 > 0$ $(\in \mathbb{R})$

$\therefore T(j\omega) = \frac{a_o \cdot f_o}{1 - j\omega/\omega_{pa}}$



Depending on $a_o f_o$

$90° < PM < 180°$

e.g. $\lim_{a_o f_o \to 1} PM = 180°$

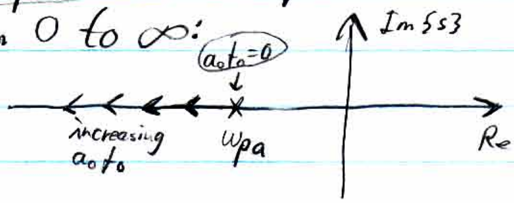
$\lim_{a_o f_o \to \infty} PM = 90°$

Feb. 14

Recall: $A(s) \equiv \frac{v_o(s)}{v_i(s)} = A_o \cdot \frac{1}{1 - s/\omega_{pA}}$ where $A_o \in \mathbb{R}$

$\omega_{pA} = (1 + a_o f_o) \cdot \omega_{pa}$

Plot of ω_{pA} position in s-plane as $a_o f_o$ increases from 0 to ∞:



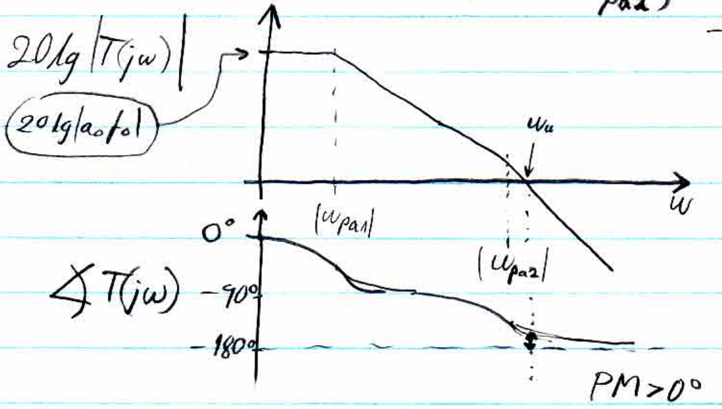
$\}$ ≡ "root locus" plot

∴ as expected, it's stable for any choise of $a_o f_o > 0$

Ex 2 ③ with $a(s) = \frac{a_0}{(1 - s/\omega_{pa1})(1 - s/\omega_{pa2})}$, $\omega_{pai} < 0$ $(\in \mathbb{R})$ $i = 1,2$

$a_0 > 0$ $(\in \mathbb{R})$

e.g.



⇒ $0° < PM < 180°$

⇒ always stable (at least barely)

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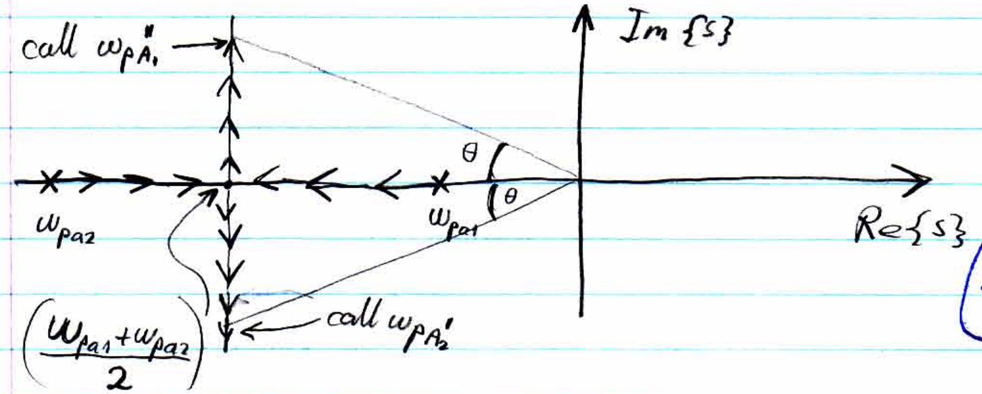
Root Locus plot (verify) (pos. of ω_{pA_i})



$$\frac{a(s)}{1+f\cdot a(s)} = \frac{a_0 \cdot \omega_{pa1} \cdot \omega_{pa2}}{s^2-(\omega_{pa1}+\omega_{pa2})\cdot s+(1+a_0\cdot f)\cdot \omega_{pa1}\omega_{pa2}}$$

Recall $\theta = \cos^{-1}\zeta$ where ζ = damping ratio

$\therefore$ $\zeta = \cos\theta$, so

$\zeta \to 0$ as $\theta \to 90°$

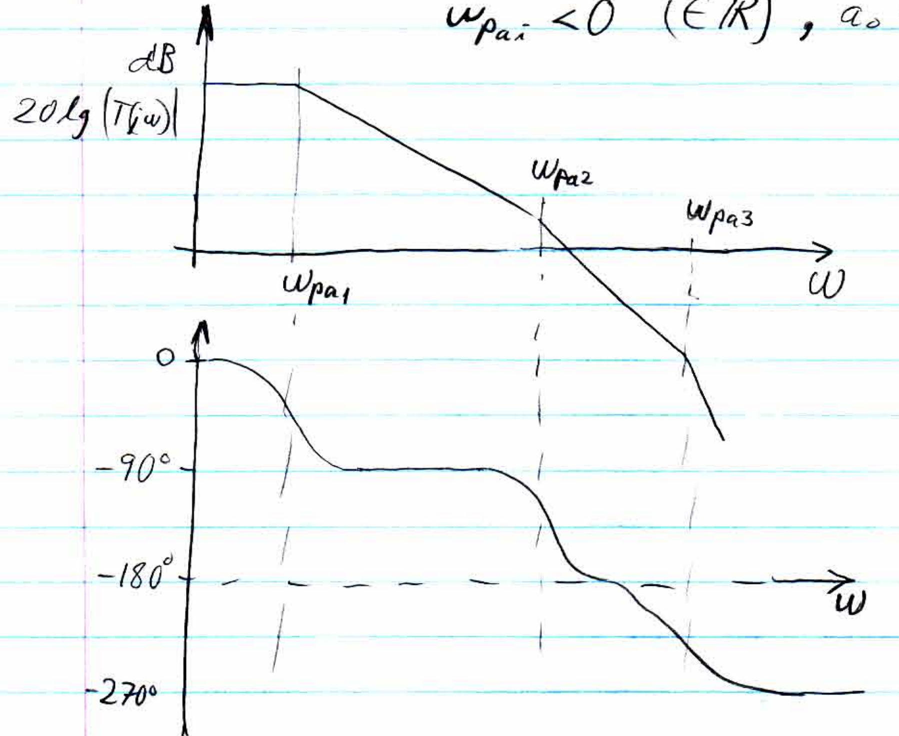
(i.e. as $a_0 f_0 \to \infty$)

(i.e. as PM $\to 0$)

$\therefore$ Smaller PM ⇒ more ringing.

Ex. 3 ③ with $a(s) = \dfrac{a_0}{(1-s/\omega_{pa1})(1-s/\omega_{pa2})(1-s/\omega_{pa3})}$

$\omega_{pai} < 0$ $(\in \mathbb{R})$, $a_0 > 0$ $(\in \mathbb{R})$



⇒ $-90° < PM < 180°$

⇒ can be unstable

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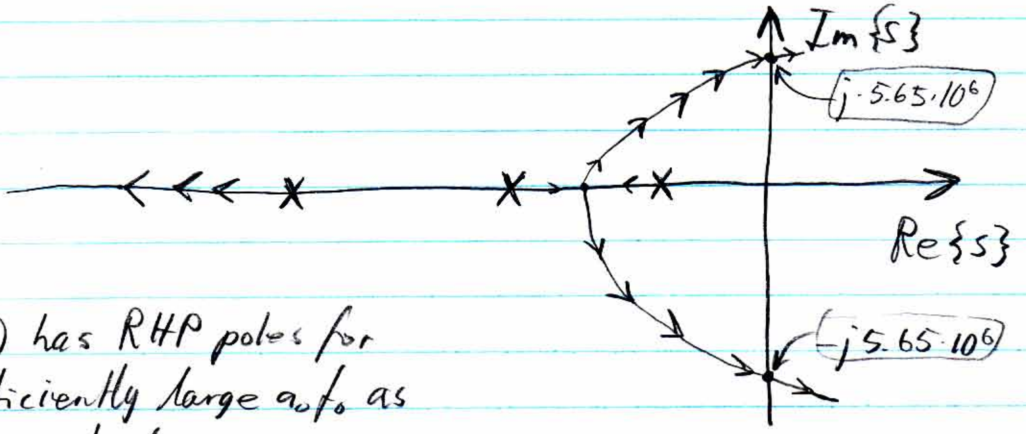
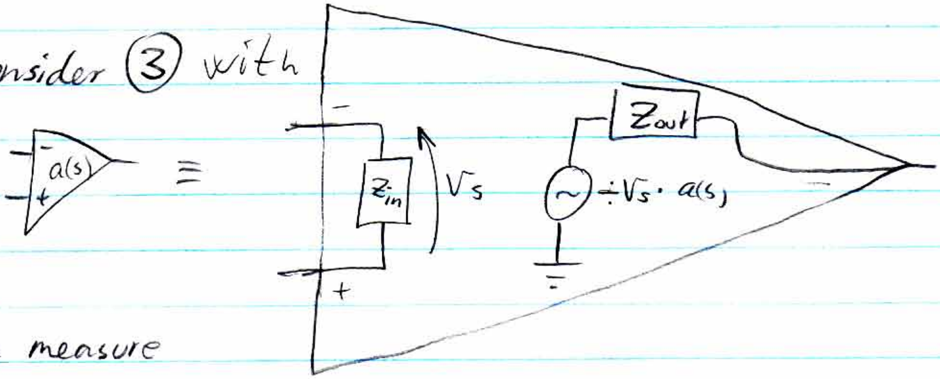
e.g. Suppose $\omega_{pa1} = -10^6 \frac{rad}{sec}$, $\omega_{pa2} = -2 \cdot 10^6 \frac{rad}{sec}$, $\omega_{pa3} = -10^7 \frac{rad}{sec}$
Then the root locus plot is as follows:



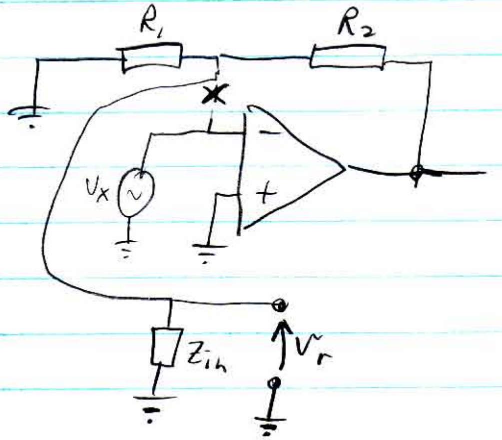
∴ A(s) has RHP poles for sufficiently large $a_o f_o$ as expected.

Using SPICE to estimate T(jω)

Ex4 Consider ③ with



Suppose we measure T(jω) using:



④

have:
1) Broken feedback loop
2) Applied test source after break
3) Loaded node prior to break as if it were not broken

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First suppose $Z_{in} = \infty$, $Z_{out} = 0$

Inspection $\Rightarrow$ $\frac{v_r}{v_x} = -a(s) \cdot \frac{R_1}{R_1 + R_2} = \div a(s) \cdot f_o = \div T(s)$

$$\therefore \; T(s) = -\frac{v_r(s)}{v_x(s)}$$

$\therefore$ Can use this approach to "measure" $T(j\omega)$
Often works, but must be careful in general.

Suppose $Z_{in} \neq \infty$, $Z_{out} \neq 0$

AGR $\Rightarrow$ $A(s) = A_\infty \cdot \frac{T}{1+T} + A_0 \cdot \frac{1}{1+T}$

Before $A_0 = 0$ because $Z_{out} = 0$
Now, $A_0 \neq 0$

We still get $T(j\omega)$ using ④ but analysis will miss poles contributed by $A_0(s)$

$$N^{(1+T)}_{CW,(0,0)} = Z_{(1+T)} - P_{(1+T)} =$$

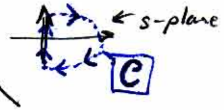
$$= P_{cl.l.} - P_{T} = N^{T}_{CW,(-1,0)}$$

$$N^{T}_{CCW,(-1,0)} = P_{T} - P_{cl.l.}$$

<u>RHP</u> poles & <u>RHP</u> Zeros in general

(map from s to either T(s) or to (1+T(s)).

SEC:



)

For stability, $P_{cl.l} = 0$

$$\Rightarrow N^{T}_{CCW,(-1,0)} = P_{T}$$

$N^{T}_{CCW,(-1,0)}$ = "# RHP poles, open loop" ÷ "#RHP poles, closed loop"

(pg. 471 ~ 479 in Dorf & Bishop)

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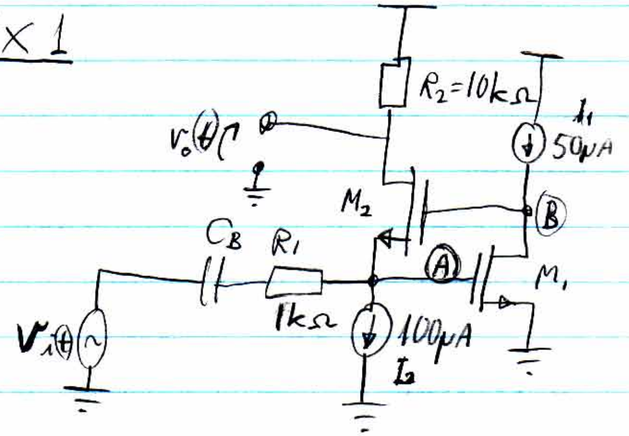
Using SPICE to estimate $T(j\omega)$

Ex 1



①

$M_1 : \frac{W}{L} = 15\mu m / 1\mu m$
$M_2 : \frac{W}{L} = 20\mu m / 1\mu m$
C_B = large DC-blocking cap (assume $C_B \approx \infty$)

Observations

(i) Ⓐ is a low impedance node:

- inside the loop bandwidth ⇒ Ⓐ = almost virtual ground (HW)
- even with feedback disabled, impedance of Ⓐ $\approx 1k\Omega \,||\, \left(\frac{1}{g_{m2}}\right)$

(ii) Similar reasoning ⇒ driving point resistance between nodes Ⓐ and Ⓑ is relatively small ($\approx 1/g_{m1}$)

(ii) ⇒ Reasonable to neglect C_{gd1}, C_{gs2} ②

Apply A.G.R. with $g_{m1} \cdot v_{gs1} \equiv$ ref. source

$$\therefore \quad A(s) = A_\infty(s) \cdot \frac{T(s)}{1+T(s)} + \underbrace{A_0(s)}_{} \cdot \frac{1}{1+T(s)}$$

→ gain of C.G. amplifier formed by M_2 (but with high impedance gate connection)

$\frac{1}{g_{m2}} \approx 1k\Omega$, so $A_0(0) \approx 5$

The point is: $T(s)$ does not give the whole stability "picture" because $A_0(s)$ might have marginal stability.

2/5

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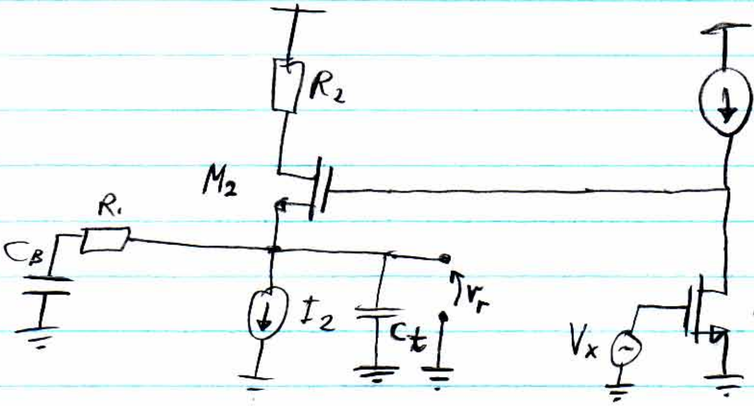
Good approach
1) Measure $T(s)$ to estimate the degree of stab.
2) If simulations show more ringing than expected, consider $A_0(s)$

② ⇒ Can "measure" $T(j\omega)$ using:



③

where
$C_t \equiv C_{gs1}$ (termination impedance),
DC-value of V_x = DC-value of node Ⓐ in ①

⇒

have "broken" feedback loop and applied test signal

③ ⇒ $\underbrace{T(j\omega)} = -\frac{v_r(j\omega)}{v_x(j\omega)}$

→ (same $T(j\omega)$ as in A.G.R. if ② holds - verify)

⇒

simple test to find PM, GM, or Nyquist plot

simulations ⇒ PM ≈ 90°

In general, this allways works, but must:

1) Bias input node following break point as in the closed loop circuit.
2) Must terminate the node (prior to breakpoint) as in the closed loop circuit.

HW6 ⇒ Method of avoiding need to terminate output node.

$\frac{3}{5}$

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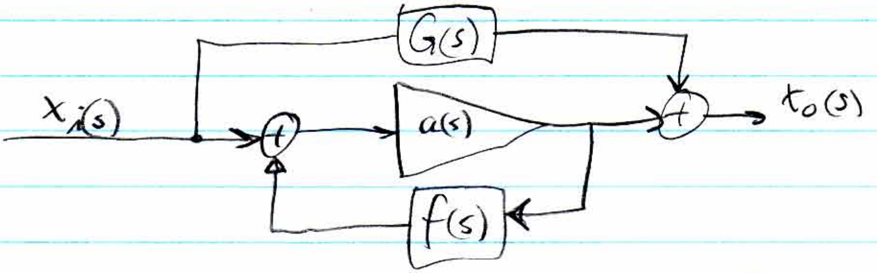
Theory behind breaking feedback loops

Ex



④

$$H(s) \equiv \frac{x_o(s)}{x_i(s)} = \underbrace{\frac{a(s)}{1+a(s)f(s)}}_{\text{call } A(s)} + G(s)$$

Both $G(s)$ and $A(s)$ } • have poles (in general)
• affect stability

In circuits, often } • $A(s) \Rightarrow$ desired behavior (dominant)
• $G(s) \Rightarrow$ parasitic signal-path (non-dominant)

$\therefore$ Feedback loop in ④ $\Rightarrow A(s)$

Knowing $A(s) \not\Rightarrow$ insight into how $a(s)$ or $f(s)$ can be modified to improve performance.

But

Knowing $T(j\omega) \Longrightarrow$ " ———||——— ...
... ———||——— "
(via Nyq. crit., etc..)

Can "break" any point of feedback loop in ④ and "measure" $T(j\omega)$ via simulation.

The problem: Usually have a circuit, not a B.D. to analyze. Previously have used A.G.R. for specific circuits to justify measuring $T(j\omega)$ directly from circuit. (i.e. without first conv. to a B.D.)

Does this work in general?

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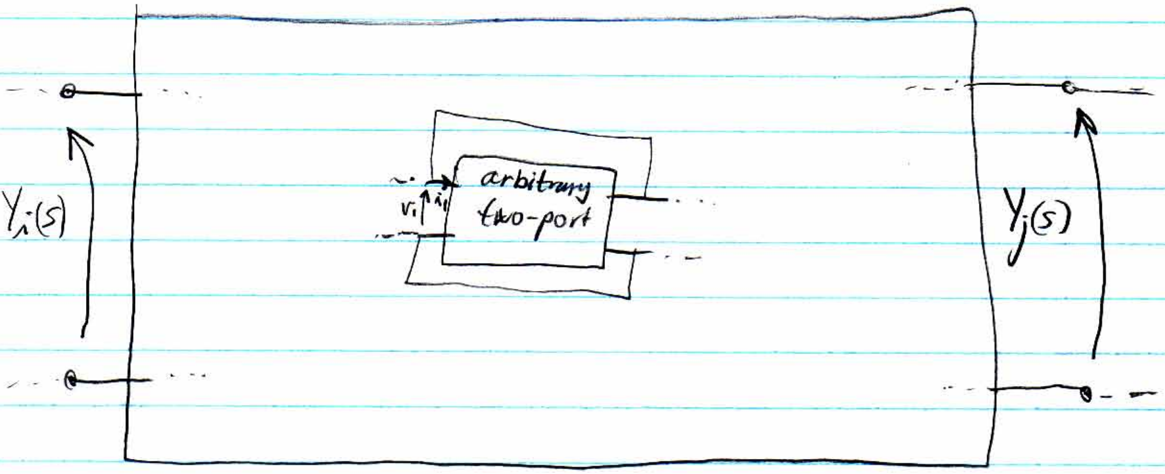
Arbitrary LTI circuit containing a feedback loop



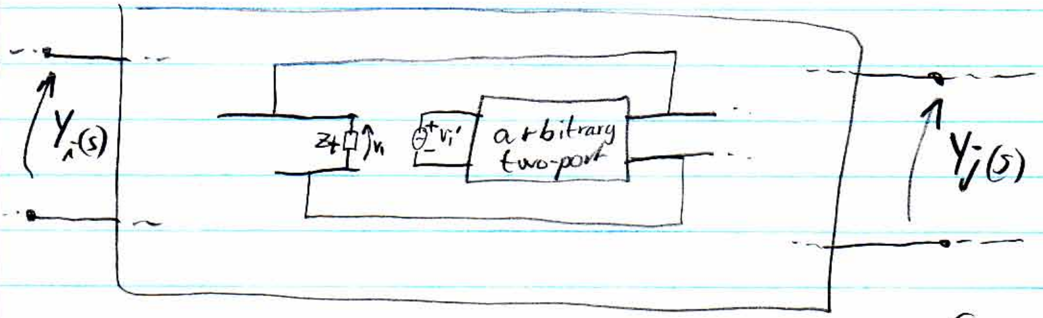
⑤

(Y is voltage, but could be current!)

Let $H(s) = \frac{Y_j(s)}{Y_i(s)}$

Note: Can draw ⑤ s.t. $Y_i(s)$ = current and/or $Y_j(s)$ = current

Can redraw ⑤ as:



⑥

where $v_1' = \alpha \cdot v_1$ with $\alpha \equiv 1$, $Z_t \equiv \frac{v_1(s)}{i_1(s)}$ (input imp. of two-port depends on loading of $Y_i(s)$ & $Y_j(s)$ ports)

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Now, suppose $v_i'(s)$ replaced by independent source, v_x
LTI-system ⇒ $v_i = C(s)\cdot Y_i(s) + D(s)\cdot v_x(s)$

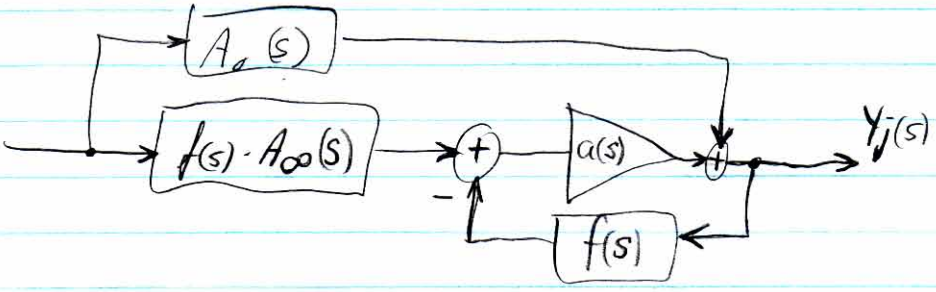
Feedback loop "enabled" ⇔ $\underbrace{D(s) \not\equiv 0}_{⑦}$ (eg. $v_i \equiv Y_i$ ⇒ ⑦ violated)

Provided ⑦ holds, can apply A.G.R.
Recall A.G.R. ⇒ $H(s) = A_\infty(s)\cdot\frac{T(s)}{1+T(s)} + A_0(s)\cdot\frac{1}{1+T(s)}$ ⑧

where $T(s) = -\frac{v_i(s)}{v_x(s)}$ for $Y_i(s)=0$ and $v_i'(s)$ replaced by $v_x(s)$.

$A_0(s) = Y_j(s)/Y_i(s)$ with $\alpha = 0$, $A_\infty = \lim_{\alpha\to\infty}\frac{Y_j(s)}{Y_i(s)}$

∴ Block Diagram of ⑥:



⑨

Where $a(s)$, $f(s)$ are any functions s.t. $a(s)\cdot f(s) = T(s)$

Observations

1) $T(j\omega)$ in ⑧ same as that found by breaking feedback loop in ⑤, term. the break with $Z_t(s)$, and injecting v_x (or i_x) at input of broken loop
⇒ Method always works provided ⑦ holds
2) Can use Nyq. crit. (or P.M., G.M.) to assess relative stability of F.B. loop in ⑧ (If F.B. loop has marginal stab. then ⑨ (ie ⑤) "——— " ——— ")

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Note: For the version of the Nyquist Plot and Criterion presented previously, must have at least as many poles as zeros.

(More zeros $\overset{\text{than poles}}{}$ ⇒ inf. B.W. !)

Compensation

The problem: Given an amplifier and feedback network, how do we adjust or add components to achieve a desired stability margin?

Most often $T(s)$ satisfies:

(i) $|\measuredangle T(0)| < \pi$ (neg. feedback at DC)
(ii) $T(s)$ has no RHP (incl. $j\omega$ axis) poles
(iii) $|T(j\omega)| = 1$ has one pos. solution, $\omega = \omega_u$

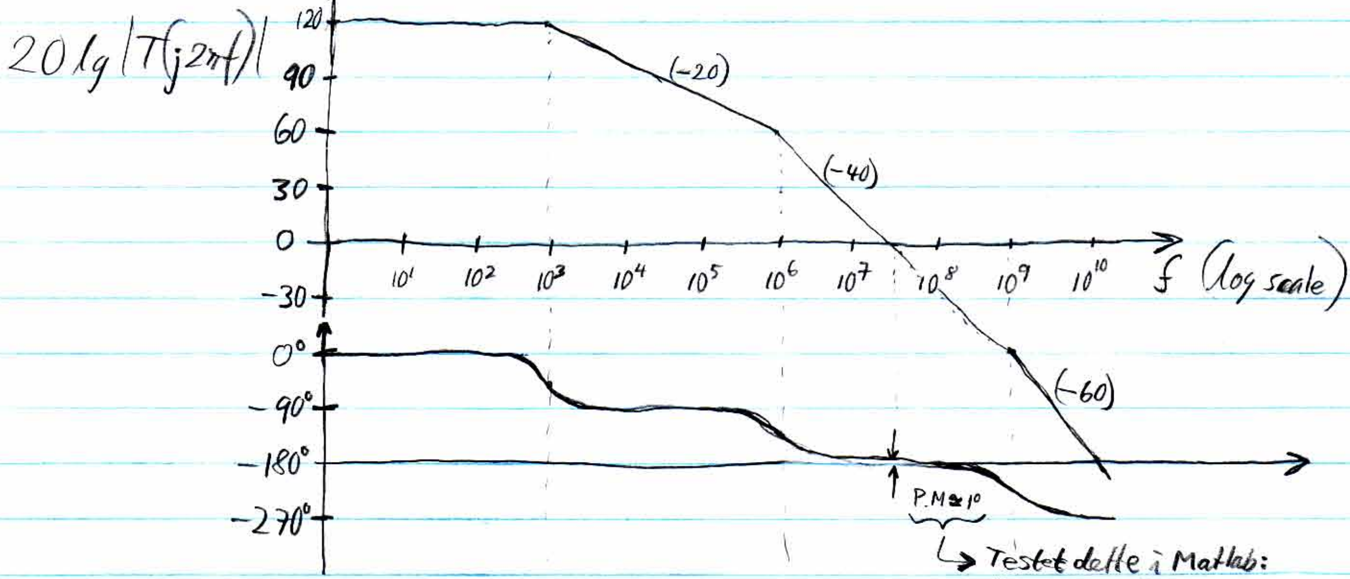
①

① ⇒ PM, GM valid indicators of stab. margin.

Ex 1: $$T(s) = T_0 \cdot \frac{1}{(1-s/\omega_{p1})(1-s/\omega_{p2})(1-s/\omega_{p3})}$$ ②

with

$T_0 = 10^6$, $f_{p_1} = \frac{\omega_{p1}}{2\pi} = -10^3 Hz$, $f_{p2} = -10^6 Hz$, $f_{p3} = -10^9 Hz$



↳ Testet dette i Matlab: Fikk 0.004°

$\frac{2}{4}$

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Observations

i) Both $\measuredangle T(j\omega)$ and $|T(j\omega)|$ decrease **monotonically**

ii) PM $\gtrsim 45°$ requires $|f_{p2}| > f_u$

$\therefore$ Only 3 ways (compensation methods) to increase P.M. given $T(s)$ has the form of ②

1) Reduce T_0 $\Rightarrow$ ω_u decreases, but $\measuredangle T(j\omega)$ unchanged
$\Rightarrow$ PM increases

But: Trades C.L. accuracy for PM
(feedback benefits requires large T_0)

2) Reduce $|f_{p1}|$
$\Rightarrow$ ω_u decreases, but $\measuredangle T(j\omega)$ almost unchanged for $\gtrsim 4 \cdot |f_{p1}|$ (2 octaves = 0.6 decades)
$\Rightarrow$ P.M. increases

But: Trades C.L. BW for PM

3) Increase $|f_{p2}|$
$\Rightarrow$ ω_u increases until $|f_{p2}| \geq f_u$, then remains unchanged but $\measuredangle T(j\omega_u)$ increases
$\Rightarrow$ P.M. increases (up to 90°)

But: Usually not possible without simultaneously reducing $|f_{p1}|$.

Jargon: 1) $\equiv$ "gain compensation"
2) $\equiv$ "dom. pole compensation"
2)+3) $\equiv$ "pole splitting compensation" or "miller compensation"

See ③/⑤ Jan. 17

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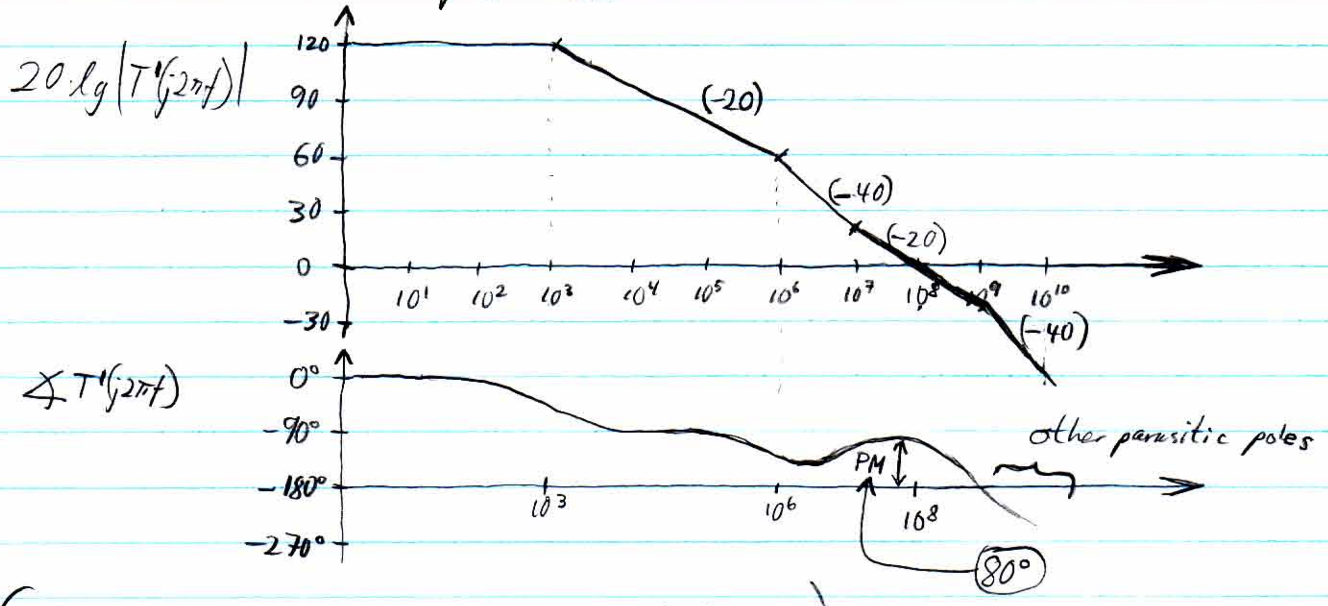
Ex 2 Suppose we can add LHP zero to $T(s)$ given by ②

e.g. let $T'(s) = \underbrace{T(s)}_{\text{same as in Ex 1.}} \cdot (1 - s/\omega_{z1})$

with $f_{z1} = \frac{\omega_{z1}}{2\pi} = -10^7 Hz$



(zero approx at $1.2 \cdot \omega_u$ ← rule of thumb)

Observations

i) LHP zero added pos. phase shift without significantly **changing f_u** (f_u gikk fra $10^{7.5}$ til 10^8 Hz)

⇒ Increased PM (fra ~0° til 80°)

⇒ Adding LHP zero ≡ compensation strategy, called "lead compensation"

ii) A RHP zero would have decreased PM → to −89.4° ⇒ unstable!

Lead compensation is used in 2-stage CMOS op-amps (soon).

Vil nesten ikke endre ω_u men likevel påvirke fasen til en viss grad. Denne tommelfinger-regelen er ikke brukt her. (Her er $|z| < \omega_u$.)

④/④

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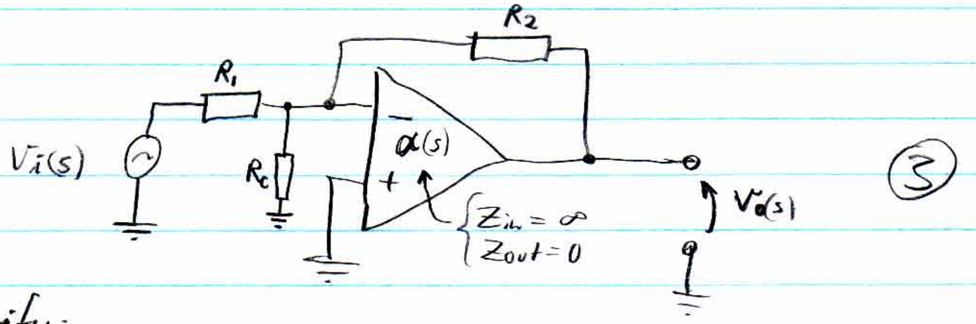
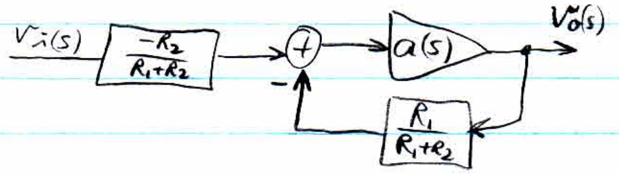
Ex 3   Gain compensation



Can verify:

③ ⇒



Where

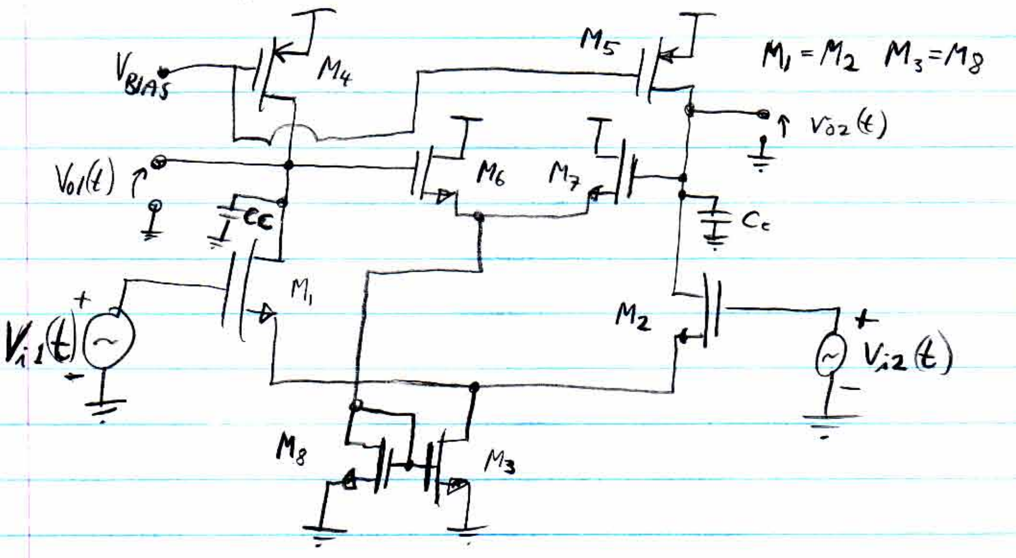
$$a(s) = \frac{R_1 /\!/ R_2 /\!/ R_C}{R_1 /\!/ R_2} \cdot \alpha(s)$$

e.g. Suppose $\alpha(s)$ = 3-pole, no-zero transfer fcn.
⇒ $T(s)$ given by ②
⇒ decreasing R_c ⇒ smaller T_0 but same $\angle T(j\omega)$ as before

e.g. suppose with $R_c = \infty$, $T(s)$ = same as Ex 1
Then $PM \cong 4°$ (very poor relative stability)   { Jeg fikk 0.004° PM i Matlab...

e.g. suppose $R_1 = R_2$ and $R_c = \frac{R_1}{1000}$
Then $PM \simeq 35°$
But went from $T_0 = 120\,dB$ to $T_0 = 66\,dB$

Ex 4   Dom. pole compensation in CMFB



$M_1 = M_2$ $M_3 = M_8$ $M_4 = M_5$
$M_6 = M_7$

Exercise: Consider how C_c implements dom. pole compensation!

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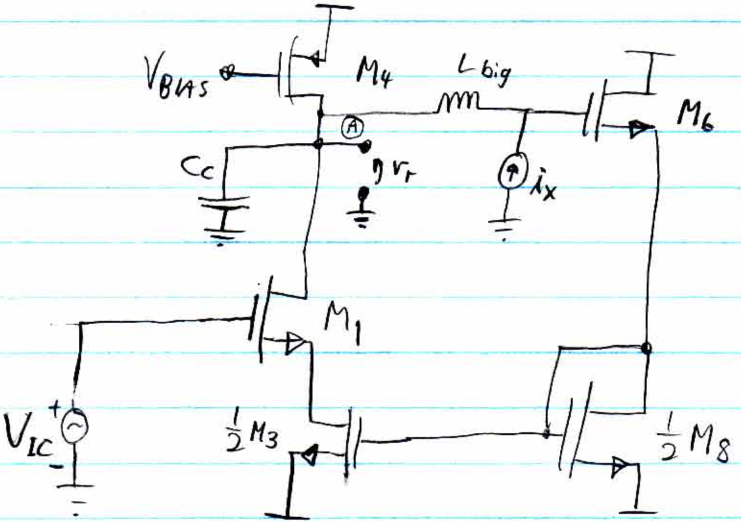
Ex 4 from last time

(recall diff pair with CMFB diagram)

CM ½-circuit & T(jω) measurement config



(L_{big} ⇒ zero-freq voltages at two terminals of L_{big} are same, but L_{big} blocks non-zero frequency components)

valid if $C_{gs6} \ll C_c$

Note: trans impedance from node Ⓐ to all other nodes is small *

⇒ $-\frac{1}{C_c \cdot R_{\textcircled{A}}}$ = pole of $T(s)$ (C_c has little effect on other poles of $T(s)$)

where $R_{\textcircled{A}}$ = small-sig. res. from node Ⓐ to small-sig ground

⇒ Ⓐ = dom. pole node

⇒ can reduce $|f_{p1}|$ (only) by increasing C_c

Strøm injisert i node Ⓐ vil i liten grad påvirke spenningene på andre noder i kretsen. $\frac{V_{andre}}{I_{\textcircled{A}}}$ = liten = $Z_{trans.}$

(2/5)

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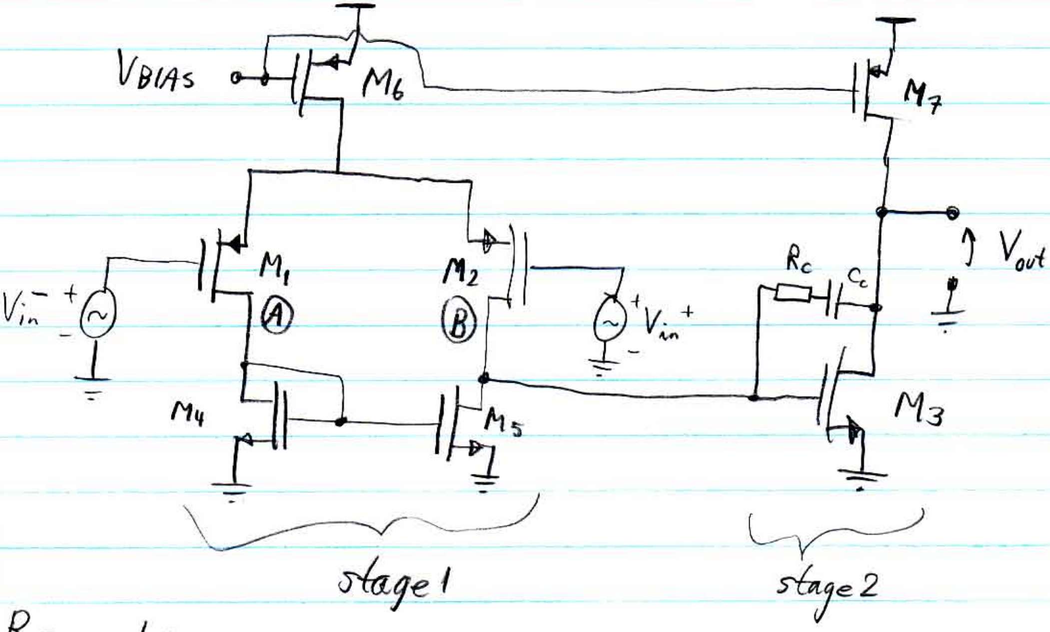
Ex Two-stage op-amp with lead compensation



①

Remarks:

1) Typ. designed s.t. pole assoc. with Ⓐ

i.e $p_{\textcircled{A}} = \frac{-g_{m4}}{C_{\textcircled{A}}}$ ← cap from Ⓐ to small-sig. ground

is s.t. $|p_{\textcircled{A}}| \gg \omega_u$ ← unity-gain freq.

2) A source-follower buffer stage is sometimes added to ①

3) Could replace pMOSs by nMOSs and vice versa, but pMOSs have better 1/f noise performance so ① as shown has better 1/f noise (why?).

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First consider with $R_c = 0$

Using prev. results and Remark 1):

① has 2 significant poles, p_1 & p_2 and 1 significant zero, z_1 given by:

$$p_1 \cong -\frac{1}{g_{m3} R_1 R_2 C_c} \quad ② \quad \text{where } R_1 \equiv r_{ds2} \| r_{ds5}, \; R_2 \equiv r_{ds3} \| r_{ds7}$$

and $C_c \gg C_{gd}$ is assumed

$$p_2 \cong -\frac{g_{m3} C_c}{C_{d3} C'_{gs3} + C_c (C_{d3} + C'_{gs3})} \quad ③$$

where C_{d3} = total cap. from output to ground from M_3, M_7, and any load.

$$\cong -\frac{g_{m3}}{C_{d3}} \quad \text{(often)}$$

$$z_1 \cong \frac{g_{m3}}{C_c} \quad ④$$

p_1 = dom. pole (sets open loop BW)

p_2 = non-dom pole (with p_1 sets open-loop phase shift at unity gain freq.)

z_1 = RHP-zero (adds neg. phase shift - just like a LHP pole)

eg. $g_{m3} = 1 \cdot 10^{-3}\,\Omega^{-1}$, $R_1 = R_2 = 100\text{k}\Omega$, $A_{V01} = 70$ (DC-gain of stage 1), $C_{d3} = C'_{gs3} = 350\text{fF}$, $C_c = 1\text{pF}$ ⑤

② - ⑤ $\Rightarrow$

$p_1 = -100 \cdot 10^3 \frac{rad}{sec}$ (-16kHz)

$p_2 = -1.2 \cdot 10^9 \frac{rad}{sec}$ (-191MHz)

$z_1 = 10^9 \frac{rad}{sec}$ (159MHz)

A_{V02} ($\equiv$ DC-gain of stage 2) $= -g_{m3} \cdot R_2 = -100$

$$\therefore \; A_v(s) \cong 7000 \cdot \frac{(1 - s/10^9)}{(1 + s/10^5)(1 + s/1.2 \cdot 10^9)}$$

$\therefore \; \omega_u \cong 7.1 \cdot 10^8 \frac{rad}{sec}$ (113 MHz) (verify)

$$\measuredangle A_v(j\omega_u) = \tan^{-1}\left(-\frac{7.1\cdot10^8}{10^9}\right) - \tan^{-1}\left(\frac{7.1\cdot10^8}{10^5}\right) - \tan^{-1}\left(\frac{7.1\cdot10^8}{1.2\cdot10^9}\right)$$

$$\cong \underbrace{-35.4^\circ}_{\text{zero}} \quad \underbrace{-\ 90^\circ}_{\text{dom. pole}} \quad \underbrace{-\ 30.6^\circ}_{\text{non-dom pole}} = -156^\circ$$

If $f(s) = 1$, then $T(s) = A_v(s)$ so

$$P.M. = 180^\circ - 156^\circ = \underline{24^\circ} \quad (\text{low } P.M.)$$

Note: 1) Increasing C_c reduces neg. phase shift from p_2 at ω_u but increases —〃— z_1 at ω_u (because ω_u decreases but so does z_1)

2) Situation is actually worse than calculated because of pole at Ⓐ

Heuristics: As ω increases, feed fwd. path through C_c begins to dominate the gain path through M_3. Gain path through M_3 has negative polarity, but feed-fwd. path through C_c has phase $\to 0$ as $\omega \to \infty$ $\Rightarrow$ As $\omega \to \infty$, have pos. feedback

Now consider with $R_c > 0$

Can show, $R_c \neq 0 \Rightarrow$ have $z_1 \cong \dfrac{1}{C_c(1/g_{m3} - R_c)}$ ⑥

have a 3rd pole, p_3

Usually:

1) $|p_3| \gg |p_1|, |p_2|, |z_1|$, so can ignore p_3 ⎫
2) p_1, p_2 are hardly affected by R_c ⎭ ⑦

⑥ $\Rightarrow R_c > \frac{1}{g_{m3}} \Rightarrow$ z_1 "moves" to LHP

$\Rightarrow$ z_1 contributes <u>pos</u> phase shift

Fact: ⑦ breaks down for very large R_c.

Typical rule of thumb: Choose R_c s.t. $|z_1| \cong 1.2 \cdot \omega_u$ ⑧

⑥, ⑧ $\Rightarrow$ $1.2 \cdot \omega_u \cong \left| \frac{1}{C_c(1/g_{m3} - R_c)} \right|$

$R_c = \frac{1}{g_{m3}} + \frac{1}{1.2 \cdot \omega_u \cdot C_c}$ ⑨ (for $R_c > \frac{1}{g_{m3}}$)

e.g. (using ⑤) ⑨ $\Rightarrow$ $R_c = 2.17\,k\Omega$

Now $\measuredangle A_v(j\omega) = \tan^{-1}\left(\frac{\omega_u}{1.2 \cdot \omega_u}\right) - 90° - 30.6° \cong -80.8°$

$\therefore$ new P.M. for $f(s) = 1$: P.M. $\cong 99.2°$

$\Rightarrow$ Can afford to reduce C_c ($\Rightarrow$ increases BW) to reduce P.M. (typ. want P.M. > 65°)

Closed loop system:

- critically damped if P.M. = **76°**
- gain peaking if P.M. < 66°

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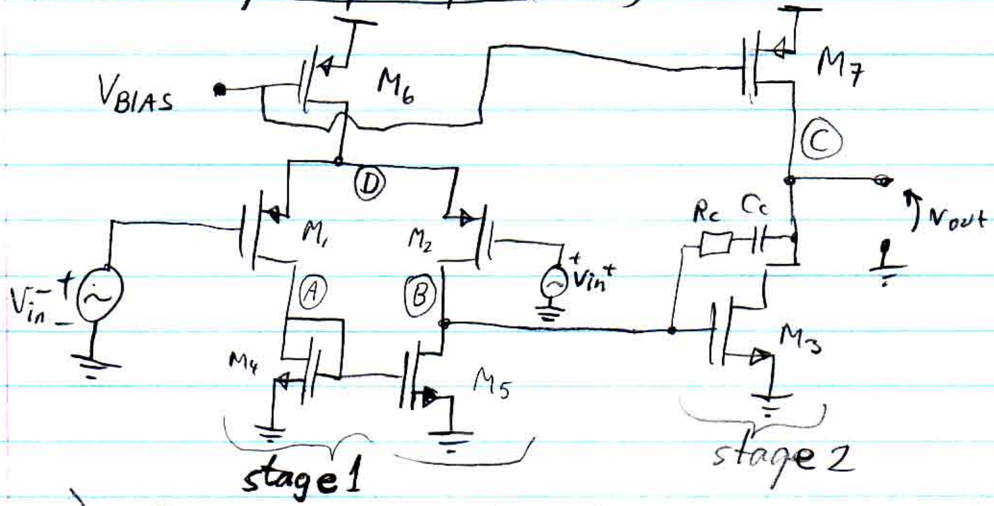
Final exam: WLH 2204 (Thurs, March 20)

Extra Office Hours on Wed. of next week

Two-stage Op-Amp (cont.)



①

1) Common mode rejection of stage 1 ⇒ usually can neglect **pole** associated with node **Ⓓ** when considering diff. **mode** of op-amp.

2) Usually designed s.t. pole associated with Ⓐ i.e. $p_Ⓐ \cong -\frac{g_{m4}}{C_Ⓐ}$ is s.t. $|p_Ⓐ| \gg \omega_u$ } ②

Then $M_2, M_5, M_3, M_7, R_c, C_c$ determine P.M. when ① is used in feedback system.

3) Prev. found:

$$A_v(j\omega) \equiv \frac{V_{out}(j\omega)}{V_{in}^+(j\omega)-V_{in}^-(j\omega)} \cong \underbrace{g_{m1}\cdot R_1}_{\text{DC-gain of stage 1}} \cdot \underbrace{g_{m3}\cdot R_2}_{\text{DC-gain of stage2}} \frac{(1-j\omega/z_1)}{(1-j\omega/p_1)(1-j\omega/p_2)} \quad ③$$

where $R_1 \equiv r_{ds2} \| r_{ds5}$, $R_2 \equiv r_{ds3} \| r_{ds7}$

$$p_1 \cong -\frac{1}{g_{m3}R_1R_2C_c} \quad \text{(dom. pole)} \quad ④$$

$$p_2 \cong \frac{-g_{m3}}{C_{d3}+C'_{gs3}} \quad \text{(first non-dom. pole)} \quad ⑤$$

and

$$z_1 \cong \frac{1}{C_c(1/g_{m3}-R_c)} \quad \text{(dom. zero)} \quad ⑥$$

Note: ④–⑥ hold for typ. design parameters

②/④

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general:
Zero is placed near ω_u

4) $p_1 \Rightarrow$ 3dB BW and introduces $-90°$ phase at ω_u

$p_2, z_1 \Rightarrow$ PM $\underbrace{\text{(i.e. P.M. varies with } p_2, z_1\text{; not with } p_1\text{)}}_{\text{assumes } |p_2|, |z_1| \gtrsim 10\cdot|p_1|}$

5) Rc chosen s.t. z_1 in LHP $\Rightarrow$ introduces pos. phase shift at ω_u

Jargon: PM of an open-loop op-amp $\equiv 180° + \angle A_v(j\omega_u)$
why? When connected as voltage follower, i.e. $f = 1$, (hardest non-attenuating config to compensate) loop gain $\equiv T(j\omega) \equiv A_v(j\omega)$

Usually, ① designed s.t. $\underbrace{|p_2|, |z_1|}_{\text{same order of. mag.!}} > \omega_u$

$$\therefore \text{③} \Rightarrow |A_v(j\omega_u)| \cong \left| g_{m1} g_{m3} R_1 R_2 \cdot \frac{1}{1 - j\,\omega_u/p_1} \right|$$

$$\cong g_{m1} g_{m3} R_1 R_2 \cdot \left| \frac{p_1}{j\omega_u} \right|$$

$$\therefore \text{④} \Rightarrow \cong \left| \frac{g_{m1}}{C_c \cdot j \cdot \omega_u} \right|$$

$$\therefore |A_v(j\omega_u)| = 1 \Rightarrow \omega_u \cong \frac{g_{m1}}{C_c} \qquad \text{⑦}$$

$$P.M. = 180° - \tan^{-1}\frac{\omega_u}{z_1} + \tan^{-1}\frac{\omega_u}{p_2} - \underbrace{90°}_{\text{contrib. by } p_1} \qquad \text{⑧}$$

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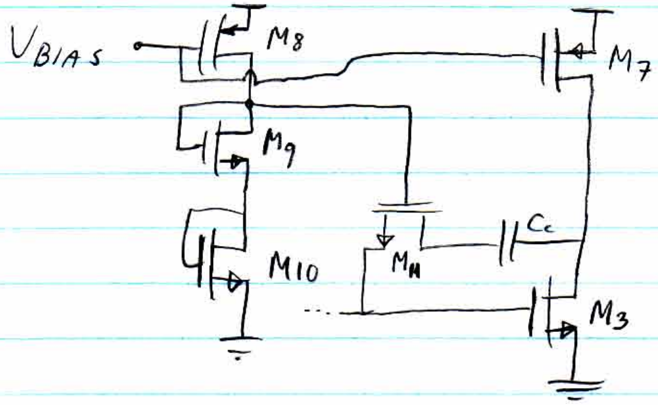
Problem: How to maintain constant P.M. across temp./process/supply voltage variations?

$$\left\{ ⑤ - ⑧ \Rightarrow \text{P.M. depends on } \underbrace{g_{m1}, g_{m3}, C_{d3}, C_{gs3}, C_c, R_c}_{\text{These parameters vary and dont track each other}} \right.$$

A Solution: Replace stage 2 of ① by:



with $\frac{W_7/L_7}{W_3/L_3} = \frac{W_8/L_8}{W_{10}/L_{10}}$ ⑩

⑨

M_{11} biased by M_8-M_{10} ≡ R_C

No DC-current ⇒ M_{11} in triode

$$\Rightarrow R_C \cong \frac{1}{\mu_n C_{ox} (W_{11}/L_{11}) \underbrace{(V_{GS_{11}} - V_{T_n})}_{\text{call } V_{eff\,11}}}$$

$$g_{m3} = \mu_n C_{ox} (W_3/L_3) \cdot V_{eff\,3}$$

$$\therefore R_C\, g_{m3} = \frac{(W_3/L_3)\, V_{eff\,3}}{(W_{11}/L_{11})\, V_{eff\,11}} \quad ⑪$$

$$\therefore ⑥ \Rightarrow z_1 = \frac{g_{m3}}{C_c \cdot \left[1 - \left[\frac{(W_3/L_3)\, V_{eff\,3}}{(W_{11}/L_{11})\, V_{eff\,11}}\right]\right]} \quad ⑫$$

$$⑦, ⑫ \Rightarrow \frac{\omega_u}{z_1} = \frac{g_{m1}}{g_{m3}} \cdot \left[1 - \left[\frac{(W_3/L_3)\, V_{eff\,3}}{(W_{11}/L_{11})\, V_{eff\,11}}\right]\right]$$

$$⑤, ⑦ \Rightarrow \left|\frac{\omega_u}{p_2}\right| = \frac{g_{m1}}{g_{m3}} \cdot \frac{C_{d3} + C_{gs3}}{C_c}$$

$$\boxed{\mu_0 \cdot C_{ox} \approx K'}$$

$$\frac{g_{m1}}{g_{m3}} = \frac{\sqrt{2 \cdot \mu_p \cdot C_{ox} \cdot (W_1/L_1) \cdot I_{D1}}}{\sqrt{2 \mu_n \cdot C_{ox} (W_3/L_3) I_{D3}}}$$

Facts:

1) $\mu_p/\mu_n \cong$ const. for a given process

2) $I_{D1}/I_{D3} \cong$ const. because derived from a common bias network

} ⇒ $\frac{g_{m1}}{g_{m3}} \cong$ const.

3) $\frac{C_{d3} + C_{gs3}}{C_c} \neq$ const., but does not vary much over process & temp. because dominated by oxide capacitor

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$\therefore$ P.M. $\cong$ const. provided $V_{eff3}/V_{eff11} \cong$ const.

⑩ $\Rightarrow$ $V_{eff10} = V_{eff3}$

$V_{eff9} = V_{eff11}$

$$\therefore \frac{V_{eff3}}{V_{eff11}} = \frac{V_{eff10}}{V_{eff9}} = \frac{\sqrt{\dfrac{2 \cdot I_{D10}}{\mu_n \cdot C_{ox}(W_{10}/L_{10})}}}{\sqrt{\dfrac{2 I_{D9}}{\mu_n \cdot C_{ox}(W_9/L_9)}}} = \sqrt{\frac{W_{10}/L_{10}}{W_9/L_9}} = \left\{\begin{array}{l}\text{ratio of}\\ \text{like quantities}\end{array}\right.$$

$\therefore$ P.M. $\cong$ indep. of process & temperature

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The "reference source" in Blackman's Impedance Relation (BIR)
and the Asymptotic Gain Formula (AGF):

The values of the variables in BIR (Z^0_{ab}, T_{sc}, T_{oc}) or in AGF (A_∞, A_0, T) depend on which reference source we use.

Of course, the resulting impedance (Z_{ab}) or gain (A) is the same, independent of our choise of reference source.

BIR & AGF was covered on Jan. 31 in class.