

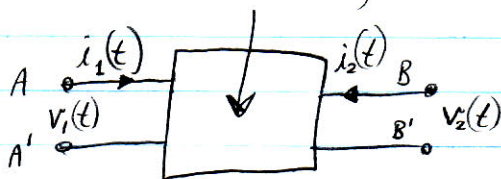
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Poles Zeros & Freq. Response

SSM (small signal model) of circ. containing trans. and RLC networks



(1)

$A, A' \equiv$ any two nodes of the circuit

$B, B' \equiv$ " " " "

Def. Let $x(t)$ be any real signal
 then $\mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-st}dt \equiv$ Bilateral Laplace transform
 lowercase $\rightarrow x(s)$

Let $G(s) = \frac{x(s)}{y(s)}$ where $x(t) = v_1(t), i_1(t), v_2(t), i_2(t)$
 $y(t) =$ " " " "

e.g. $G(s) = \frac{v_2(s)}{v_1(s)} =$ Laplace trans. of gain from A, A' to B, B'

or $G(s) = \frac{v_1(s)}{i_1(s)} =$ " " " impedance at A, A' terminals.

Fact $G(s)$ is always rational with real coefficients.

$$\text{i.e. } G(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0} \quad (2)$$

where $a_k, b_k \in \mathbb{R}$

rational

ratio of two polynomial functions

Q Do $\exists$ practical circ. for which $G(s) \neq$ rational

A Yes! Ex: Delay element: $G(s) = e^{-sT} \equiv T$ second delay

2/3

Can write (2) as

$$G(s) = G' \frac{(s-z_1)(s-z_2)\dots(s-z_n)}{(s-p_1)(s-p_2)\dots(s-p_m)} \quad (3) \quad \text{where } G', z_k, p_k \in \mathbb{C}$$

$$z_k = \text{zeros of } G(s)$$

$$p_k = \text{poles of } G(s)$$

Fact $\left\{ \begin{array}{l} G(s), s \in \mathbb{C} \\ G(j\omega), \omega \in \mathbb{R} \text{ (if it exists)} \\ g(t) = \mathcal{L}^{-1}\{G(s)\} \end{array} \right\}$ contain equiv. info. about (1)

"impulse response"
freq. response
transfer fcn.

Also:
Magnitude response & phase response

Goal: Find simple way to deduce "shape" of $G(j\omega)$ from poles & zeros of $G(s)$

Claim 1 real coeff $\Leftrightarrow$ If $s_0 = \alpha + j\beta$ ($\alpha, \beta \in \mathbb{R}$) is a pole (or zero) of $G(s)$ Then $s_0^* = \alpha - j\beta$ is also a pole (or zero) of $G(s)$

Proof: exercise

$$\text{Ex } (s-s_0)(s-s_0^*) = s^2 - s(\overbrace{s_0+s_0^*}^{2 \cdot \text{Re}\{s_0\}}) + \overbrace{s_0 s_0^*}^{|s_0|^2} = s^2 + s(2\zeta \omega_0) + \omega_0^2$$

where $\omega_0 \equiv |s_0| \equiv \text{resonant frequency}$
 $\zeta \equiv -\text{Re}\{s_0\}/\omega_0 \equiv \text{damping ratio}$ (Claim 1 $\Rightarrow \zeta, \omega_0 \in \mathbb{R}$)

$\therefore$ Can group conjugate poles & zeros together

$$G(s) = G' \left[\prod_{\text{real zeros}} (s-z_k) \cdot \prod_{\text{complex zero pairs}} (s^2 + s(2\zeta_i \omega_{0i}) + \omega_{0i}^2) \right] \quad (4)$$
$$\cdot \left[\prod_{\text{real poles}} \left(\frac{1}{s-p_k} \right) \cdot \prod_{\text{complex pole pairs}} \left(\frac{1}{s^2 + s(2\zeta_i \omega_{0i}) + \omega_{0i}^2} \right) \right]$$

$\frac{3}{3}$

Let $w_{zi}, i=1, 2, \dots N$ $\equiv$ the set of (non-zero) real zeros
 $w_{pi}, i=1, 2, \dots N$ $\equiv$ "—————" poles

$$\therefore G(s) = G(0) s^{n_0} \left[\prod_{\text{real zeros}} \left(1 - s/\omega_{zk} \right) \right] \left[\prod_{\text{complex zero pairs}} \left(\frac{s^2}{\omega_{zi}^2} + \frac{2\zeta_{zi}}{\omega_{zi}} s + 1 \right) \right]$$

$$\cdot \left[\prod_{\text{real pole}} \left(\frac{1}{1 - s/\omega_{pk}} \right) \right] \left[\prod_{\text{complex pole pairs}} \left(\frac{1}{\frac{s^2}{\omega_{pi}^2} + \frac{2\zeta_{pi}}{\omega_{pi}} s + 1} \right) \right]$$

Where $G(0) = \frac{a_0}{b_0}$, $n_0 = \# \text{ of DC-zeros} \div \# \text{ of DC-poles}$

Usually interested in $10 \lg(|G(j\omega)|^2)$

and $\angle G(j\omega) = \tan^{-1} \left(\frac{\text{Im}\{G(j\omega)\}}{\text{Re}\{G(j\omega)\}} \right)$ gain in dB — dB: viser større range, på en målestokk i decibel

Gain (assume $h_o = 0$ for now)

$$10 \cdot \lg |G(j\omega)|^2 = 10 \cdot \lg |G(0)|^2 \quad (6)$$

$$+ \sum_{\substack{\text{real} \\ z_k \neq 0}} \log(1 + \omega^2 / \omega_{z_k}^2) \quad (7)$$

$$- \sum_{\text{real poles}} 10 \cdot \lg \left(1 + \frac{\omega^2}{\omega_{pk}^2} \right) \quad (8)$$

$$+ \sum_{\substack{\text{complex} \\ \text{Zeropairs}}} 10 \cdot \lg \left(\left(1 - \frac{\omega^2}{\omega_{0Zi}^2} \right)^2 + 4 \frac{\zeta_{Zi}^2}{\omega_{0Zi}^2} \cdot \omega^2 \right) \quad (9)$$

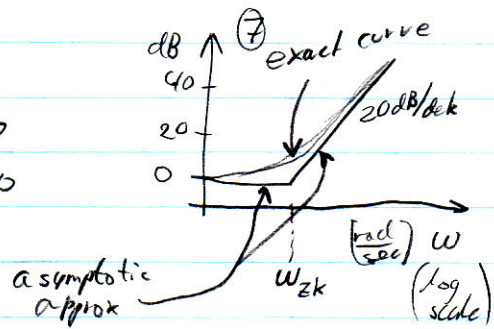
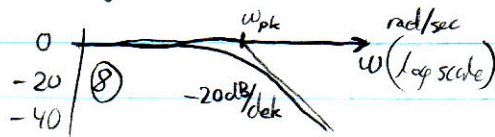
$$- \sum_{\text{complex pole pairs}} (\text{and so on...}) \quad (10)$$

$\Rightarrow$ only 2 types of curves summed together

$\textcircled{7}, \textcircled{8} \rightarrow 0$ as $w \rightarrow 0$

⑦ $\rightarrow 20 \cdot \lg(w) + \text{const}$ as $w \rightarrow \infty$

⑧ $\rightarrow -20 \lg(w) - \text{const as } w \rightarrow \infty$



①

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New room: Tuesday '15 (website)

Homework on Tuesday. Gjør tutorial for Cadence

cont.:

recall ⑩ ... $-\sum_{\text{complex pole pairs}} 10 \cdot \lg \left[\left(1 - \frac{\omega^2}{\omega_{pi}^2}\right)^2 + 4 \frac{\zeta_{pi}^2}{\omega_{pi}^2} \omega^2 \right]$

Claim 2 The max departure of the exact curve for real pole and zero factors from the asymptotic approx is a factor of 0.707 in both ⑦ and ⑧.

Proof exercise

⑨, ⑩ $\rightarrow 0 \text{ dB}$ as $\omega \rightarrow 0$

⑨ $\rightarrow 40 \cdot \lg \omega - \text{const}$ as $\omega \rightarrow \infty$

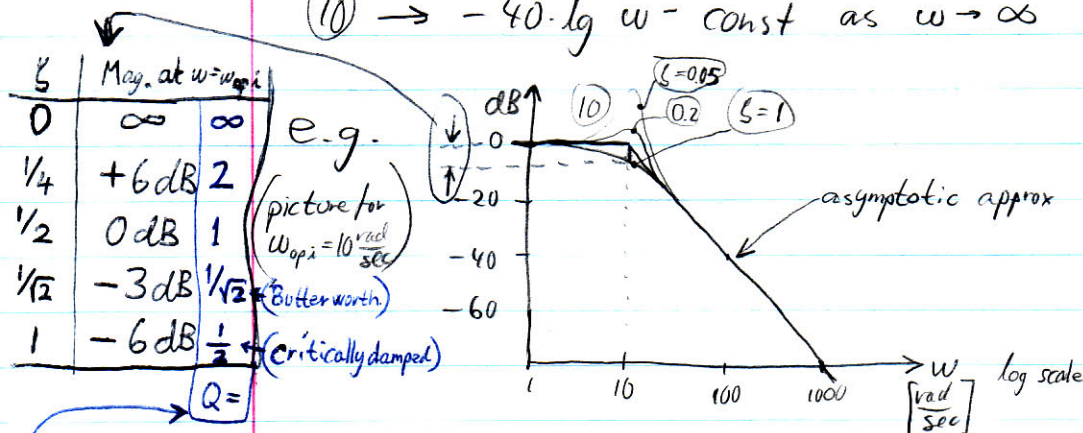
⑩ $\rightarrow -40 \cdot \lg \omega - \text{const}$ as $\omega \rightarrow \infty$

$Q = \frac{1}{2\zeta}$

Peaking for $0 \leq \zeta < \frac{1}{\sqrt{2}}$

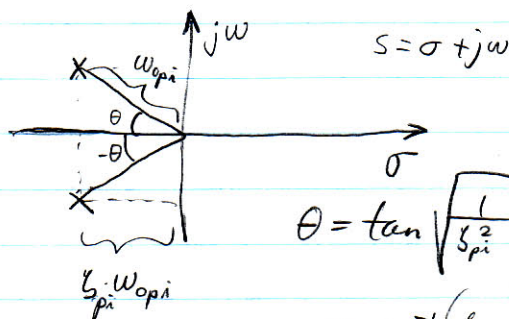
Peak @ DC for $\zeta \geq \frac{1}{\sqrt{2}}$

$Q > \frac{1}{\sqrt{2}}$



$\therefore$ asymptotic approx $\neq$ good approx near $\omega = \omega_{pi}$ / or $\zeta \approx 0.2$

poles associated with ⑩ (verify)



Voltage amplitude at the peak freq.:

$\frac{1}{2\zeta \cdot \sqrt{1-\zeta^2}}$

or

$\frac{Q^2}{\sqrt{Q^2 - 1/4}}$

or

$\frac{1}{\sin(2\theta)}$

RLC: Parallel Series

damping:	$\frac{L}{4RC}$	R	} Ω
critical damping:	$\frac{1}{2} \sqrt{\frac{L}{C}}$	$2 \cdot \sqrt{\frac{L}{C}}$	
damping ratio ζ :	$\frac{1}{2R} \cdot \sqrt{\frac{L}{C}}$	$\frac{R}{2} \sqrt{\frac{C}{L}}$	
Q-factor	$R \sqrt{\frac{C}{L}}$	$\frac{1}{R} \sqrt{\frac{L}{C}}$	

$\omega_r = \omega_0 \cdot \sqrt{1 - 2\zeta^2}$

$\omega_d = \omega_0 \cdot \sqrt{1 - \zeta^2}$

(2)

(start of "New" lecture)

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EG.

$$G(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{b_n s^n + \dots + b_1 s + b_0} \quad (1)$$

where $a_k, b_k \in \mathbb{R}$ and $s \in \mathbb{C}$

Let $\{\omega_{zk} : k=1, 2, \dots, N\} = \text{non-zero real zeros of (1) (if any)}$

$\{\omega_{pk} : k=1, 2, \dots, N'\} = \text{poles}$

$\{s_{zk}, s_{zk}^* : k=1, \dots, N''\} = \text{non-real conj. zeros of (1)}$

$\{s_{pk}, s_{pk}^* : k=1, \dots, N'''\} = \text{poles}$

$\omega_{0xk} = |s_{xk}| \quad (x=z \text{ or } p) \quad (\equiv \text{natural freq. when } x=p)$

$\zeta_{xk} = -\frac{\text{Re}\{s_{xk}\}}{\omega_{0xk}} \quad (\equiv \text{damping ratio when } x=p)$

Using (2), (1) becomes

$$G(s) = G(0) \cdot s^{n_0} \left[\prod_{k=1}^N \left(1 - s/\omega_{zk}\right) \right] \left[\prod_{k=1}^{N''} \frac{1}{\omega_{0zk}^2 \left(s^2 + 2\zeta_{zk}\omega_{0zk}s + \omega_{0zk}^2\right)} \right] \cdot \left[\prod_{k=1}^{N'} \left(1 - s/\omega_{pk}\right)^{-1} \right] \left[\prod_{k=1}^{N'''} \frac{1}{\omega_{0pk}^2 \left(s^2 + 2\zeta_{pk}\omega_{0pk}s + \omega_{0pk}^2\right)^{-1}} \right] \quad (3)$$

where $G(0) = \frac{a_0}{b_0}$, $n_0 = \begin{pmatrix} \# \text{ zero-freq zeros} \\ -\# \text{ poles} \end{pmatrix}$

$$G(j\omega) = \underbrace{|G(j\omega)|}_{\text{magnitude}} e^{j \underbrace{\angle G(j\omega)}_{\text{phase}}}$$

(3)

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Phase responseExercise: verify $\angle G(j\omega) = \sum \angle (\text{each factor in (3)})|_{s=j\omega}$

where

$$(\text{factor in (3)}) \rightarrow \angle (\text{factor in (3)})|_{s=j\omega}$$

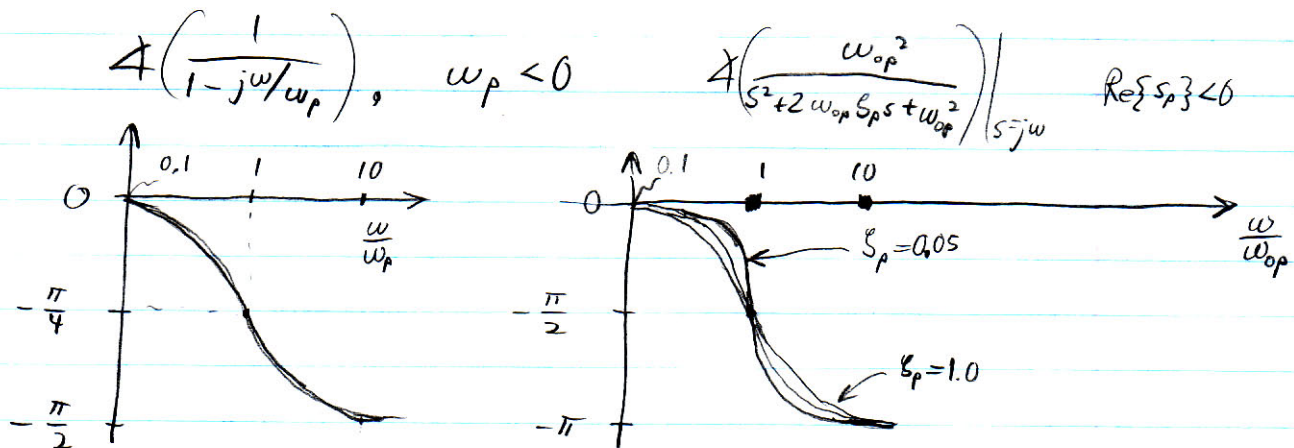
$$G(0) = \frac{a_0}{b_0} \rightarrow \begin{cases} 0 & \text{if } G(0) > 0 \\ \pi & \text{if } G(0) < 0 \end{cases}$$

$$s^{n_0} \rightarrow \frac{\pi}{2} \cdot n_0$$

$$\left(1 - \frac{s}{\omega_{xk}}\right)^{\pm 1} \rightarrow \mp \tan^{-1}\left(\frac{\omega}{\omega_{xk}}\right) \quad (x=p \text{ or } z)$$

$$\left[\frac{1}{\omega_{oxk}^2} (s^2 + 2\zeta_{xk}\omega_{oxk}s + \omega_{oxk}^2)\right]^{\pm 1} \rightarrow \mp \tan^{-1}\left(\frac{2\zeta_{xk}\omega_{oxk}\omega}{\omega_{oxk}^2 - \omega^2}\right) \quad x=p \text{ or } z$$

Picture (for LHP poles; only the sign changes for LHP zeros, same for RHP zeros)



(4)

Poles & Zeros - time response

$$\left. \begin{aligned} \text{Let } p(s) &= a_m s^m + a_{m-1} s^{m-1} + \dots + a_0 \\ q(s) &= b_n s^n + b_{n-1} s^{n-1} + \dots + b_0 \end{aligned} \right\} \Rightarrow G(s) = \frac{p(s)}{q(s)}$$

Recall "Partial Fraction Expansion" (PFE)

$$= \frac{P(s)}{b_1 (s-p_1) (s-p_2) \dots (s-p_n)}$$

Ex 1 $m < n$ $p_i \neq p_j$ for $i \neq j$

$\Leftrightarrow$ roots of $q(s)$ are all "first order"

$$\text{Then } G(s) = \sum_{k=1}^n \frac{A_k}{s-p_k}, A_k \in \mathbb{R} \quad \text{where } A_k = \lim_{s \rightarrow p_k} [(s-p_k) G(s)]$$

Ex 2 $m < n$ $p_1, p_2, p_3, \dots, p_n = 1^{\text{st}}$ order roots of $q(s)$ (4)

where $n' \leq n-2$
 $4 \leq 7-2$

and $p_{n'+1} = p_{n'+2} = \dots = p_n$

r "multiple roots" of $q(s)$ where $r = n - n'$
 $4 \rightarrow n'$ $3 = 7-4$

} minst 2 like
rotter
(example no. 3)

$$\text{Then } G(s) = \sum_{k=1}^{n'} \left(\frac{A_k}{s-p_k} \right) + \sum_{k=1}^r \frac{B_k}{(s-p_n)^k}$$

where $A_k \equiv$ (4)

$$B_k = \lim_{s \rightarrow p_n} \frac{1}{(r-k)!} \frac{d^{r-k}}{ds^{r-k}} \left[(s-p_n)^r G(s) \right] \quad \text{(5)}$$

Similar results hold for any comb. of 1st order and multiple roots of $q(s)$.

Can extend for $m \geq n$

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E.G.

Cont. from last time

(Mid-term calc.: maybe)

$$\text{let } g(t) = \mathcal{L}^{-1}\{G(s)\}$$

PFE $\Rightarrow g(t) = \text{weighted sum of terms of types}$

$$\begin{cases} \mathcal{L}^{-1}\left\{\frac{1}{s-p_k}\right\} = e^{p_k t} \cdot u(t) & (6) \end{cases}$$

$$\begin{cases} \mathcal{L}^{-1}\left\{\frac{1}{(s-p_k)^r}\right\} = \frac{t^{r-1}}{(r-1)!} \cdot e^{p_k t} \cdot u(t) & (7) \end{cases}$$

$$\left(\text{Note: } (7) = \underbrace{(6) * (6) * (6) * \dots * (6)}_{r \text{ times}} \right)$$

In circuits, rarely have multiple roots of $g(s)$
(i.e., rarely have multiple equal poles)
 $\Rightarrow$ only concerned with (6) usually

For real non-zero poles, (6) $\propto \mathcal{L}^{-1}\left\{\frac{1}{1 - s/\omega_{pk}}\right\}$

For non-real poles, we can group conj. pairs for which (6) results in terms of form

$$\mathcal{L}^{-1}\left\{\frac{\omega_{pk}^2}{s^2 + s(2\zeta_{pk}\omega_{pk}) + \omega_{pk}^2}\right\}$$

ω_{pk} is an absolute value
(Jan 10, pg. 2)

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Step Response

$$\mathcal{L}\{u(t)\} = \frac{1}{s}$$

Response of "output" with the input $= u(t) \equiv$ "step response"

$$\therefore g_{\text{step}} = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot G(s)\right\}$$

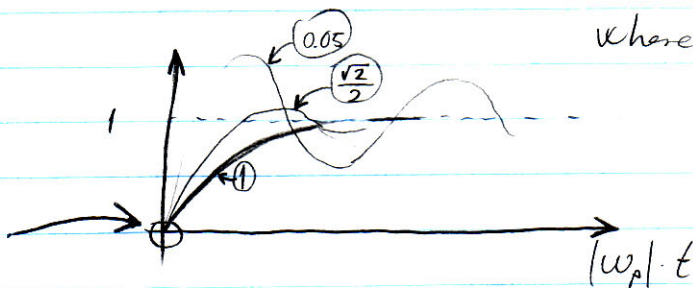
$$\mathcal{L}^{-1}\left\{\frac{1}{s} \cdot \frac{1}{1-s/\omega_p}\right\} = u(t) \cdot (1 - e^{\omega_p t})$$



$$\mathcal{L}^{-1}\left\{\frac{1}{s} \cdot \frac{\omega_{op}^2}{s^2 + s(2\zeta_p \omega_{op}) + \omega_{op}^2}\right\} = u(t) \cdot \left[1 - \frac{1}{\sqrt{1-\zeta_p^2}} e^{-\zeta_p \omega_{op} t} \cdot \sin(\omega_{op} \sqrt{1-\zeta_p^2} \cdot t + \theta)\right]$$

$$\text{where } \theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta_p^2}}{\zeta_p}\right)$$

Den deriverte
i starten av
step-responsen
er 0 dersom
vi ikke har
zeros i t.f.



For $\zeta_p = 0$, poles on imag. axis $\Rightarrow$ oscillation.

For $\zeta_p < 0$ poles in RHP $\Rightarrow$ oscillation with exp. increasing amplitude

For $0 < \zeta_p < 1$ "overshoots" but settles to 1.

$$= u(t) \cdot \left[1 - \frac{1}{\sqrt{1-\zeta_p^2}} \cdot e^{-\zeta_p \omega_{op} t} \cdot \cos\left[\omega_{op} \sqrt{1-\zeta_p^2} \cdot t - \tan^{-1}\left(\frac{\zeta_p}{\sqrt{1-\zeta_p^2}}\right)\right]\right]$$

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Dominant pole approx.

Provided have no DC poles or zeros

$$G(s) = G(0) \cdot \frac{(1 - s/z_1)(1 - s/z_2) \dots (1 - s/z_m)}{(1 - s/p_1)(1 - s/p_2) \dots (1 - s/p_n)}$$

Suppose $|p_1| \ll |p_k|, 2 \leq k \leq n$

$|p_1| \ll |z_k|, 1 \leq k \leq m \quad (\Rightarrow p_1 \in \mathbb{R} \text{ why?})$

$$\text{Then } |G(j\omega)| \cong \frac{G(0)}{\sqrt{1 + \frac{\omega^2}{|p_1|^2}}} \quad \therefore |G(j\omega_{-3dB})| = \frac{G(0)}{\sqrt{2}}$$

$$\Rightarrow \omega_{-3dB} = |p_1| \quad \left(\begin{array}{l} 3dB \\ \text{Bandwidth} \end{array} \right)$$

(New numbering)

Common source amplifiers / freq. response

Aside Course convention

$$I_d = I_D + i_d$$

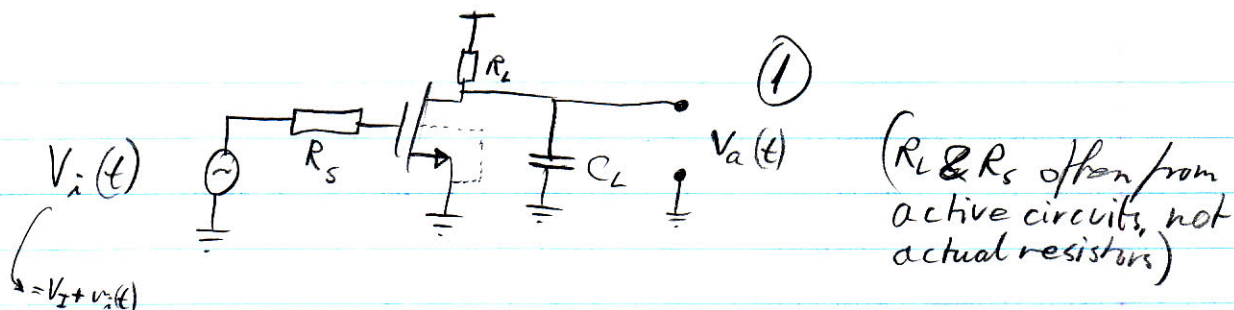
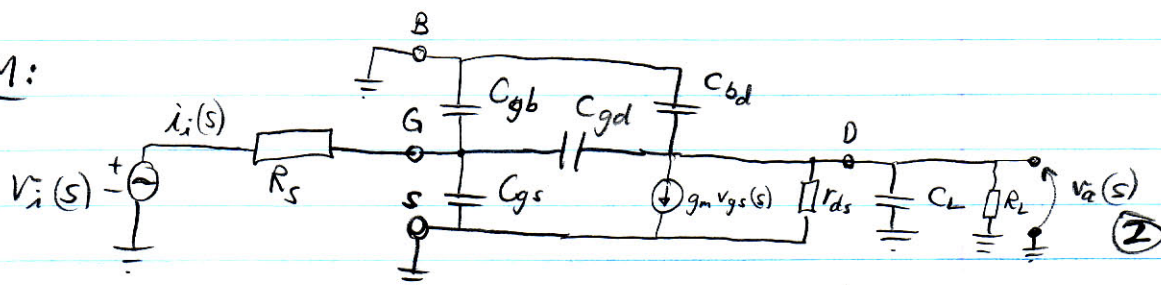
$$V_{gs} = V_{GS} + v_{gs}, \text{ etc...}$$

↑ ↑ ↑
Variations about a constant "bias" value
a constant "bias" level
"total" or "absolute" value

Løse boken bruker
 i_d og v_{gs} for dette.

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SSM:

Let $C_{gs}' = C_{gs} + C_{gb}$ ($\approx C_{gs}$ usually)

$$C_d = C_{bd} + C_L$$

$$R_L' = R_L \parallel r_{ds}$$

eg. for $I_D = 200 \mu A$ $\frac{W}{L} = 50$, $C_{gs}' = 90 \text{ fF}$, $C_{gd} = 20 \text{ fF}$ $\left. \begin{array}{l} C_d = 80 \text{ fF} \\ g_m = 1.7 \times 10^{-3} \text{ S} \\ r_{ds} = 500 \text{ k}\Omega \end{array} \right\}$ ③

Note: ② also SSM of diff pair DM $\frac{1}{2}$ circuit

KCL at G: $\frac{v_i(s) - v_{gs}(s)}{R_S} - v_{gs}(s) \cdot s \cdot C_{gs}' - [v_{gs}(s) - v_o(s)] \cdot s \cdot C_{gd} = 0$

$$\therefore v_{gs}(s) = \left[\frac{1}{\frac{1}{R_S} + s(C_{gs}' + C_{gd})} \right] (v_o(s) \cdot s \cdot C_{gd} + v_i(s) \cdot \frac{1}{R_S}) \quad ④$$

KCL at D: $[v_{gs}(s) - v_o(s)] \cdot s \cdot C_{gd} - g_m v_{gs}(s) - v_o(s) \left[\frac{1}{R_L'} + s C_d \right] = 0$

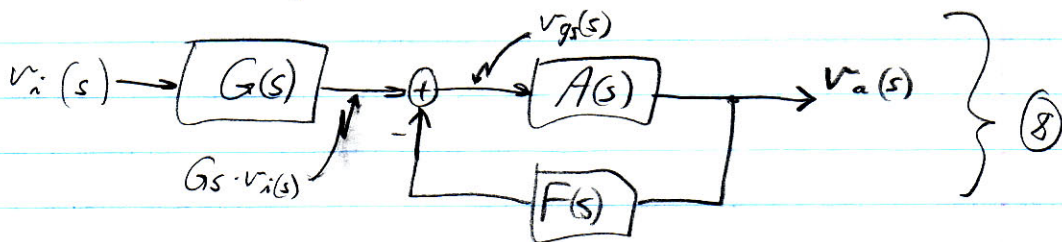
$$\therefore v_o(s) = \left[\frac{1}{\frac{1}{R_L'} + s(C_d + C_{gd})} \right] (s C_{gd} - g_m) \cdot v_{gs}(s) \quad ⑤$$

Let $G(s) = \frac{1}{R_S} \left[\frac{1}{\frac{1}{R_S} + s(C_{gs}' + C_{gd})} \right]$

$$A(s) = \frac{s C_{gd} - g_m}{\frac{1}{R_L'} + s(C_d + C_{gd})}, \quad F(s) = -s C_{gd} \left[\frac{1}{\frac{1}{R_S} + s(C_{gs}' + C_{gd})} \right] \quad ⑥$$

$$\left. \begin{aligned} (4), (6) &\Rightarrow v_{gs}(s) = -F(s) v_a(s) + G(s) v_i(s) \\ (5), (6) &\Rightarrow v_a(s) = A(s) \cdot v_{gs}(s) \end{aligned} \right\} (7)$$

(7) $\Rightarrow$ "Block Diagram" of (2):



(8) using Mason's gain formula

OR

(7) using algebra

$$\left. \begin{aligned} &\Rightarrow A_v(s) = G(s) \frac{A(s)}{1 + A(s)F(s)} \\ &\equiv \left(\frac{v_a(s)}{v_i(s)} \right) \end{aligned} \right\} (9)$$

Block Diagrams

SSM $\Leftrightarrow$ system of equations in s $\Leftrightarrow$ block diagram
(e.g. (2)) (e.g. (4) & (5)) (e.g. (8))

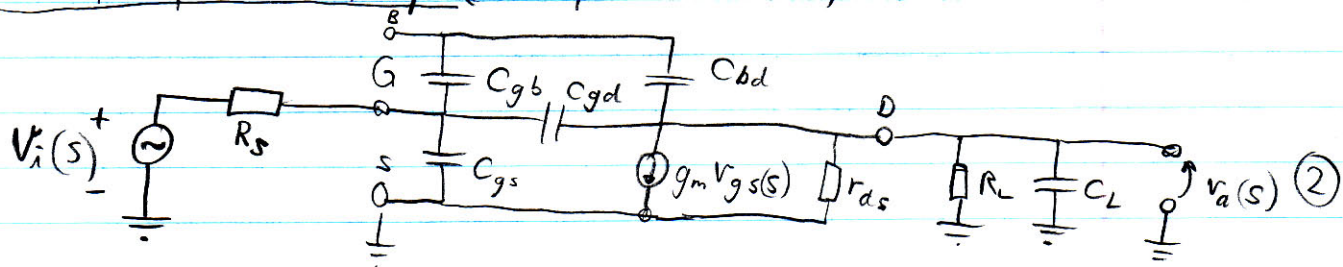
- SSM is symbolic $\Rightarrow$ good for insight
- but components are bidirectional $\Rightarrow$ bad for insight
- can't go directly from SSM to gain & impedances
(need syst. of eqs. first)
- Block diagrams provide insight because components are uni-directional
 $\Rightarrow$ Feedback paths are obvious
- Can go directly from block diagrams to gain & impedances
(using Mason's gain formula)

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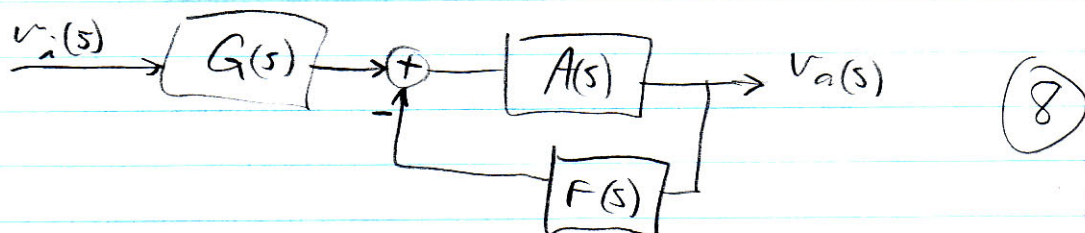
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E.G.

SSM of CS Amp. (recall from last time)

$$\left. \begin{aligned} V_{gs}(s) &= -F(s) \cdot V_o(s) + G(s) V_i(s) \\ V_o(s) &= A(s) \cdot V_{gs}(s) \end{aligned} \right\} \quad (7)$$



$$A_r(s) = G(s) \cdot \frac{A(s)}{1 + A(s)F(s)} \quad (8)$$

Recall Stability $\Leftrightarrow$ no poles in RHP (incl. imag axis?)_{E.G.}

$$(9) \Rightarrow \text{---} \text{---} \Leftrightarrow A(s_0)F(s_0) \neq -1 \text{ for any } s_0 \text{ with } \operatorname{Re}\{s_0\} \geq 0$$

(assumes $F(s) \neq 0$ and no pole of $A(s)$ is a zero of $F(s)$)

First consider $A_{r_0} = A_r(j\omega)|_{\omega=0}$ "DC-gain"

$$A_{r_0} = G(j\omega) \frac{A(j\omega)}{1 + A(j\omega)F(j\omega)} \Big|_{\omega=0}$$

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$$F(j\omega)\big|_{\omega=0} = 0 \quad (\text{makes sense because } F(s) \text{ represents feedback through } C_{gd})$$

$$\therefore A_{v0} = -g_m R_L' \quad (10)$$

Now consider poles & zeros: Let $H(j\omega) = \frac{1}{A_{v0}} \cdot A_v(j\omega)$

$$\therefore A_v(j\omega) = A_{v0} \cdot H(j\omega)$$

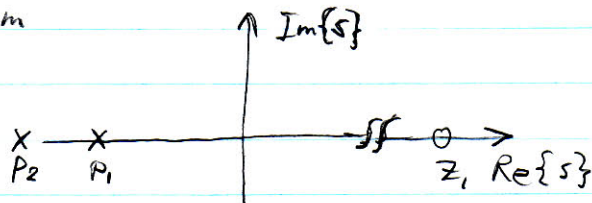
$$(6), (9) \Rightarrow H(s) = \frac{1 - s/z_1}{1 + a_1 s + a_2 s^2}$$

$$\text{where } \begin{cases} z_1 = g_m / C_{gd} \\ a_1 = R_s [C_{gs} + C_{gd} (1 + g_m R_L')] + R_L' (C_{gd} + C_d) \\ a_2 = R_s R_L' [C_d C_{gs} + C_d C_{gd} + C_{gs} C_{gd}] \end{cases}$$

$$\therefore A_v(s) = A_{v0} \left[\frac{1 - s/z_1}{(1 - s/p_1)(1 - s/p_2)} \right] \quad \text{where } \left. \begin{aligned} p_1 &= -\frac{1}{2a_2} (a_1 + \sqrt{a_1^2 - 4a_2}) \\ p_2 &= -\frac{1}{2a_2} (a_1 - \sqrt{a_1^2 - 4a_2}) \end{aligned} \right\} \quad (11)$$

Usually, device parameters $\Rightarrow a_1^2 > 4a_2$ for CS amp.
 $\Rightarrow p_1, p_2$ are real

pole-zero diagram



Ex Using (3) with $R_L = 10 \text{ k}\Omega$, $R_s = 5 \text{ k}\Omega$ gives
 $A_{v0} = -15.5$, $f_{z_1} = \frac{z_1}{2\pi} = 13.5 \text{ GHz}$, $f_{p_1} = \left| \frac{p_1}{2\pi} \right| = 56 \text{ MHz}$

$$f_{p_2} = \left| \frac{p_2}{2\pi} \right| = 860 \text{ MHz}$$

$$\left. \begin{aligned} |p_1| &\ll |p_2| \\ |p_1| &\ll |z_1| \end{aligned} \right\} \Rightarrow p_1 = \text{dominant pole}$$

$$\Rightarrow |A_v(j\omega)| \approx \left| A_{v0} \left(\frac{1}{1 + j\omega/(2\pi \cdot 56 \text{ MHz})} \right) \right| \quad \text{for } \omega \ll f_{p_2} \cdot 2\pi$$

Hand-analysis:
 10~20%
 accuracy

3/5

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$$\Rightarrow 3\text{dB BW} = 56\text{ MHz}$$

Now consider ① with capacitor C_c connected between D and G.

$\Rightarrow C_c$ in parallel with C_{gd} in ②

$\Rightarrow$ All eq. so far hold if C_{gd} is replaced by $C' = C_{gd} + C_c$ } (12)

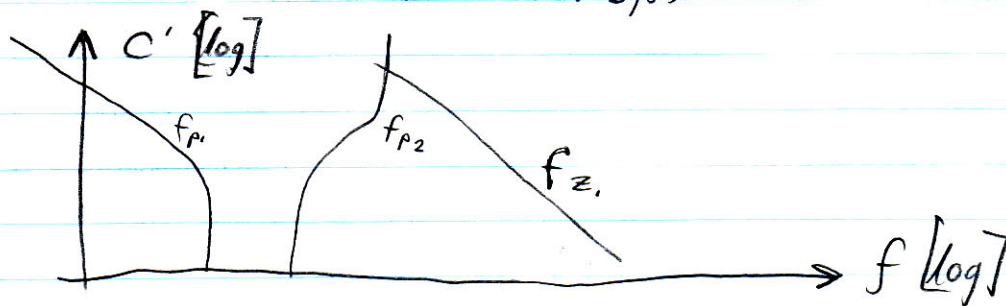
Can show using ⑪ & ⑫ (exercise)

$$p_1 \approx - \frac{1}{(C_d + C')R_L' + (C_{gs}' C')R_s + g_m R_s R_L' C'} \quad (13)$$

$$\approx \frac{-1}{g_m R_s R_L' C'} \quad (\text{for large } R_s \& R_L')$$

$$p_2 \approx \frac{-g_m C'}{C_d C_{gs}' + C'(C_d + C_{gs}')$$

$$z_1 = \frac{g_m}{C'}$$



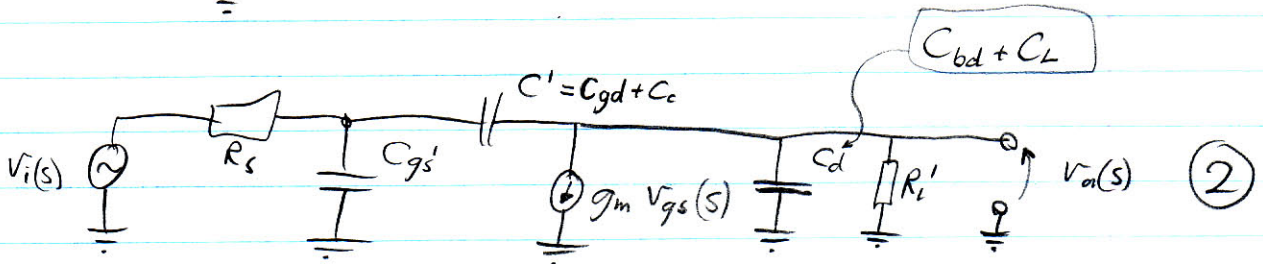
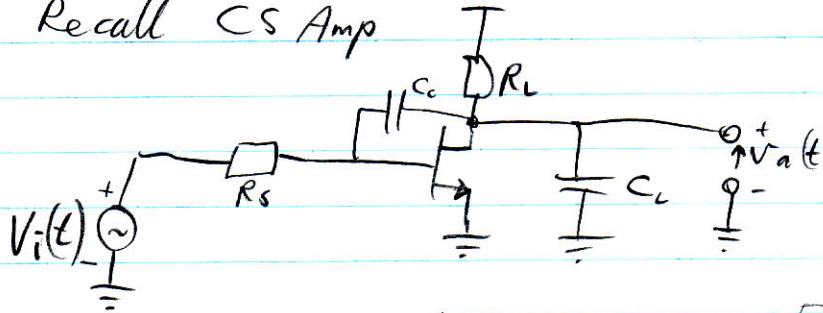
$\therefore$ Increasing C' splits poles but moves zero closer to origin

Reset Numbering system!

(4/5)

Zero-Value Time Constant Analysis

Ex Recall CS Amp



Last time: hard work gave us $p_1 \approx \frac{-1}{(C_d + C')R_L' + (C_{gs}' + C')R_s + g_m R_L' R_s C'}$ (3)

$p_1 = \text{dom. pole}$ so 3dB BW = $\left| \frac{f_1}{2\pi} \right|$

Q If only interested in dom. pole ($\Rightarrow$ 3dB BW) is there an easier way?

A Yes - ZVTC Analysis

ZVTC Theorem

$\beta_1 = \text{negative of } \sum \text{ of all poles}$

Consider circuit containing only sources, resistors and capacitors and poles $p_1, p_2, \dots, p_n$ where $p_k \neq 0$

Let $\beta_1 = -\sum_{k=1}^n \frac{1}{p_k}$

Then $\beta_1 = \sum_{k=1}^n R_k C_k$ (4)

$-\sum(\text{inverse of poles}) = \sum \text{time constants}$

where $C_k = \text{value of } k^{\text{th}} \text{ cap in circuit}$

and $R_k = \text{resistance between nodes of circuit to which}$

C_k is connected (i.e. impedance with all

caps removed)

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Ex (cont...)

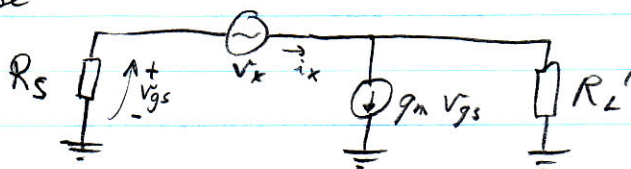
$$|p_1| \ll |p_2| \Rightarrow \beta_1 \approx |1/p_1| \therefore p_1 \approx -\frac{1}{\beta_1}$$

$$\text{Let } C_{gs}' \equiv C_1, \quad C_d \equiv C_2, \quad C' \equiv C_3 \quad (5)$$

$$\text{Inspection of (2)} \Rightarrow R_1 = R_S$$

$$R_2 = R_L'$$

To find R_3 use



$$\left. \begin{aligned} \Rightarrow v_{gs} &= -i_x R_S \\ v_x &= (i_x - g_m v_{gs}) R_L' - v_{gs} \end{aligned} \right\} R_3 = R_L' + R_S + g_m R_L' R_S$$

$$(4) \Rightarrow p_1 \approx \frac{-1}{\underbrace{R_S C_{gs}'}_{T_1} + \underbrace{R_L' C_d}_{T_2} + \underbrace{(R_L' + R_S + g_m R_L' R_S) C'}_{T_3}} = (3) \quad \checkmark$$

Z.V.T.C. Analysis

Vi vet ikke sikkert hvilket RC-ledd som er størst...
Men vi gjør et estimat:

$$\omega_{3dB} \approx |p_{dom}| \approx \frac{1}{\sum_k C_k R_k}$$

$$\approx \sum \left| \frac{1}{p_k} \right|$$

"Zero Value" refererer muligens til at alle "de andre" kondensatorne er nullstilte. På norsk heter det "åpen krets tidskonstantmetoden".

(Nedre grensefrekvens: "Kortslutnings-tidskonstantmetoden", basert på $\sum_k \frac{1}{C_k R_k}$)

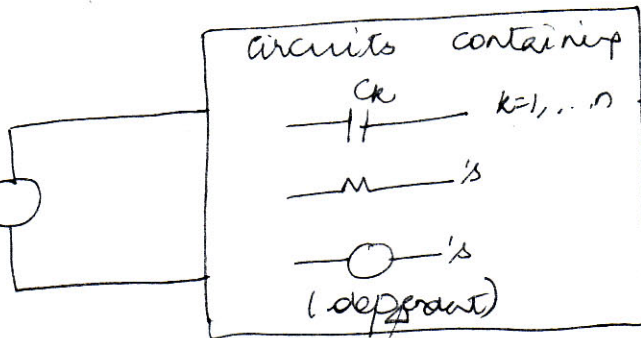
ZVTC Theorem (contd.) (same # system)PROOF:-

Have:

Impulse:

$$v_i(t) = \delta(t)$$

$$(v_i(s) = 1)$$



Impulse Response

$$v_a(t) = g(t)$$

$$(v_a(s) = G(s))$$

(2)

For calculating $P(s)$ & $G(s)$ we need to turn off independent sources

where $G(s) = A_0 \frac{P(s)}{(1-s/p_1)(1-s/p_2) \dots (1-s/p_n)}$

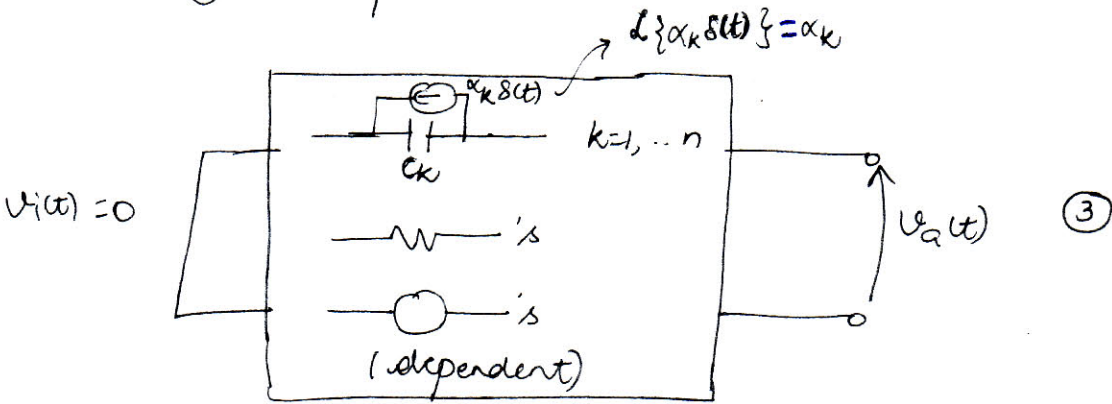
$P(s)$ $\leftarrow$ some polynomial

Note: In (2), $v_i(t) = \delta(t)$ sets initial condition of each capacitor C_k , $k=1, \dots, n$. After $t=0$, $v_i(t)=0$ (Atleast one cap will get charged)

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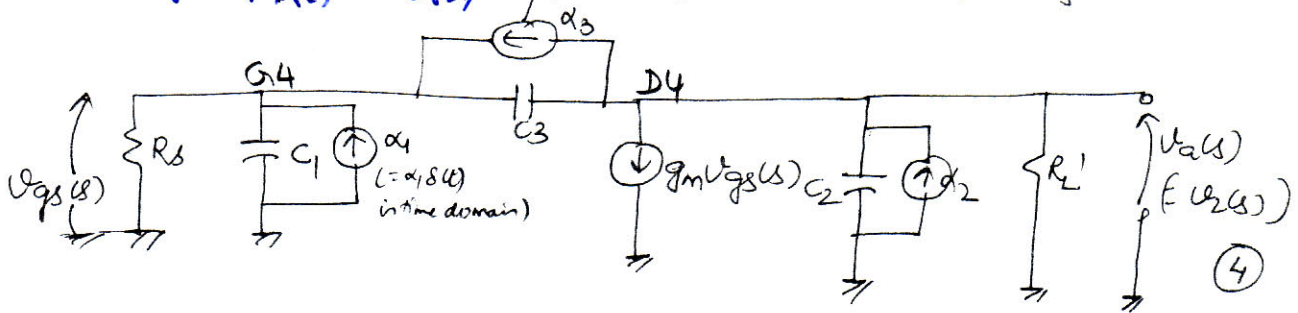
$\Rightarrow g(t) = \text{transient of } \textcircled{2} \text{ if } v_i(t) = 0 \text{ but}$
with appropriate initial conditions on $q_1, q_2, \dots, q_n$.

$\therefore \textcircled{2}$ is equivalent to



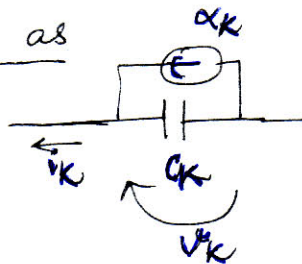
e.g. S.S.M. of G.S. amplifier (last time)

if $v_i(t) = \delta(t)$ produces some $v_o(t)$ as



NOTE: To be specific will use $\textcircled{4}$ in place of $\textcircled{3}$ for today
But all results we find generalize to $\textcircled{3}$.

Define v_k and i_k as



$$i_k(s) = \alpha_k - v_k(s) \cdot s C_k - \textcircled{5}$$

KCL @ any node $\Rightarrow$ equation of the form:

$$d_1 i_1(s) + d_2 i_2(s) + d_3 i_3(s) + q_1 v_1(s) + q_2 v_2(s) + q_3 v_3(s)$$

where $d_k = 1, 0, \text{ or } -1$ (corresponding to current entering, leaving, or node) $= 0$

and $g_k =$ some conductance

e.g. KCL @ gate of ④:

$$-i_1(s) - i_3(s) + \frac{1}{R_S} v_1(s) = 0$$

Can solve all the equations of this form to get:

$$i_1(s) = a_{11} v_1(s) + a_{12} v_2(s) + a_{13} v_3(s)$$

$$i_2(s) = a_{21} v_1(s) + a_{22} v_2(s) + a_{23} v_3(s)$$

$$i_3(s) = a_{31} v_1(s) + a_{32} v_2(s) + a_{33} v_3(s)$$

⑥
where
 $a_{jk} =$
some
conductance

Note: To **force** ⑥ to be linearly independent,
may need to add tiny resistors in series
w/ some of the caps (**add conceptually, not physically**)
("tiny" $\Leftrightarrow$ small enough to have negligible
effect on ckt.)

In matrix notation, ⑥ is $\underline{i}(s) = \underline{A} \cdot \underline{v}(s)$

$$\underline{i}(s) = \begin{bmatrix} i_1(s) \\ i_2(s) \\ i_3(s) \end{bmatrix}_{3 \times 1}, \quad \underline{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3 \times 3} \quad \& \quad \underline{v}(s) = \begin{bmatrix} v_1(s) \\ v_2(s) \\ v_3(s) \end{bmatrix}_{3 \times 1}$$

Ladning på
kondensatorne
i startøjeblikket?

Using ⑤:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} (a_{11} + sC_1) & a_{12} & a_{13} \\ a_{21} & (a_{22} + sC_2) & a_{23} \\ a_{31} & a_{32} & (a_{33} + sC_3) \end{bmatrix} \underline{v}(s) \quad \text{⑦}$$

Call $\underline{\alpha}$ Call $\underline{B}$

(Real life:
c.s. Amp: 3 caps, 2 poles)

Recall "Cramer's Rule"

$$\Rightarrow v_k(s) = \frac{\Delta_k(s)}{\Delta(s)} \quad \text{where } \Delta(s) = \det(\underline{B}) \quad (= \text{polynomial in } s)$$

$$\Delta_k(s) = \det(\underline{B} \text{ with } k\text{th column replaced by } \underline{\alpha})$$

	unit
$\underline{\dot{i}}(s)$	A·sec
$\underline{v}(s)$	V·sec
$\underline{A}, \underline{B}$	$\underline{U} = \frac{A}{V} = \text{Siemens}$
α	$C = \text{A·sec}$

$$\begin{aligned}
 \textcircled{5} \& \textcircled{6} \Rightarrow \alpha_1 &= \dot{i}_1(s) + v_1(s) \cdot s \cdot C_1 \\
 &= a_{11} \cdot v_1(s) + a_{12} \cdot v_2(s) + a_{13} \cdot v_3(s) + v_1(s) \cdot s \cdot C_1 \\
 &= (a_{11} + sC_1) \cdot v_1(s) + a_{12} \cdot v_2(s) + a_{13} \cdot v_3(s) \\
 &\Rightarrow \textcircled{7}
 \end{aligned}$$

Notes:

$$V_2(s) = V_1(s) \text{ in } \textcircled{4}, \text{ so } G(s) = \frac{\Delta_2(s)}{\Delta(s)} \quad \textcircled{8}$$

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9.

(i.e. we can calculate impulse response from Cramer's rule)

We know denominator of $G(s)$ has form: $b_0 + b_1 s + b_2 s^2$

(Because of pole-zero cancellation, we have 2nd order polynomial in real life rather than 3rd order polynomial)
 $= b_0 (1 + \beta_1 s + \beta_2 s^2)$
 $= b_0 (1 - s/p_1) (1 - s/p_2)$

$$\text{Algebra} \Rightarrow \beta_1 = -\sum_{k=1}^n 1/p_k, \quad n=2$$

true for any $n \geq 1$

$$\textcircled{7}, \textcircled{8}, \text{ defn. of } \det(B) \Rightarrow b_0 = \Delta(s) \Big|_{G=C_2=C_3=0} = \Delta(0) \quad \textcircled{9}$$

$$\& \quad b_0 \beta_1 s \equiv b_1 s = h_1 s_1 + h_2 s_2 + h_3 s_3$$

where h_1, h_2, h_3 are real #'s, i.e. $\in \mathbb{R}$

$$\text{Let } \Delta_{ij} = (-1)^{i+j} \det(\underline{B}_{ij}) \text{ where } \underline{B}_{ij} = \underline{B} \text{ with } i\text{th row \& } j\text{th column deleted.}$$

Can expand $\det(\underline{B})$ as

$$\Delta(s) = (a_{11} + s_1) \Delta_{11} + a_{21} \Delta_{21} + a_{31} \Delta_{31}$$

inspection of $\textcircled{7} \Rightarrow C_1$ only occurs in 1st term

$$\therefore h_1 = \Delta_{11} \Big|_{C_2=C_3=0} = \Delta'_{11}(0)$$

Similarly, $h_2 = \Delta_{22}(0)$, $h_3 = \Delta_{33}(0)$

Jan 22, 2008 **10.**

$$\therefore \textcircled{1} \Rightarrow \beta_1 = \frac{\Delta_{11}(0)}{\Delta(0)} \cdot c_1 + \frac{\Delta_{22}(0)}{\Delta(0)} \cdot c_2 + \frac{\Delta_{33}(0)}{\Delta(0)} \cdot c_3$$

Now set $c_1 = c_2 = c_3 = 0$

and $i_2 = i_3 = 0$ to get

$$\begin{bmatrix} i_1 \\ 0 \\ 0 \end{bmatrix} = \underline{A} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Cramer's Rule $\Rightarrow v_1 = \frac{i_1 \det \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}}{\det(\underline{A})}$

$$= i_1 \frac{\Delta_{11}(0)}{\Delta(0)} \quad \text{by defn.}$$

$$\therefore \left. \frac{v_1}{i_1} \right|_{c_1=c_2=c_3=0} = R_1 = \frac{\Delta_{11}(0)}{\Delta(0)}$$

$$i_2 = i_3 = 0$$

Similarly for R_2 and R_3

$$\therefore \beta_1 = R_1 c_1 + R_2 c_2 + R_3 c_3$$

1) One LC should dominate

2) One pole should dominate

1/5

ECE 264A

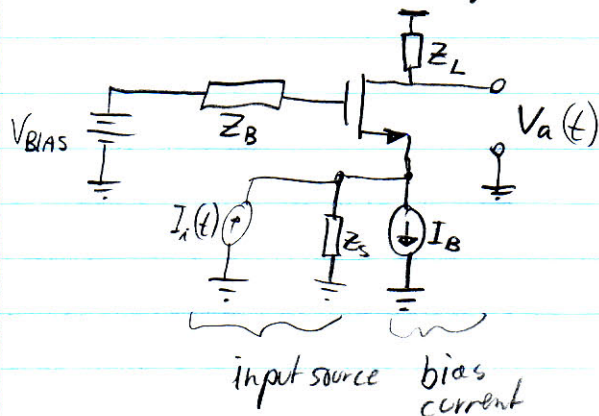
Jan 24, 2008

(o.k. tomorrow with TA 11:00~12:30 in EBU1,3329)

Freq resp of common gate (cascode) stage

Assume
Body
to
ground

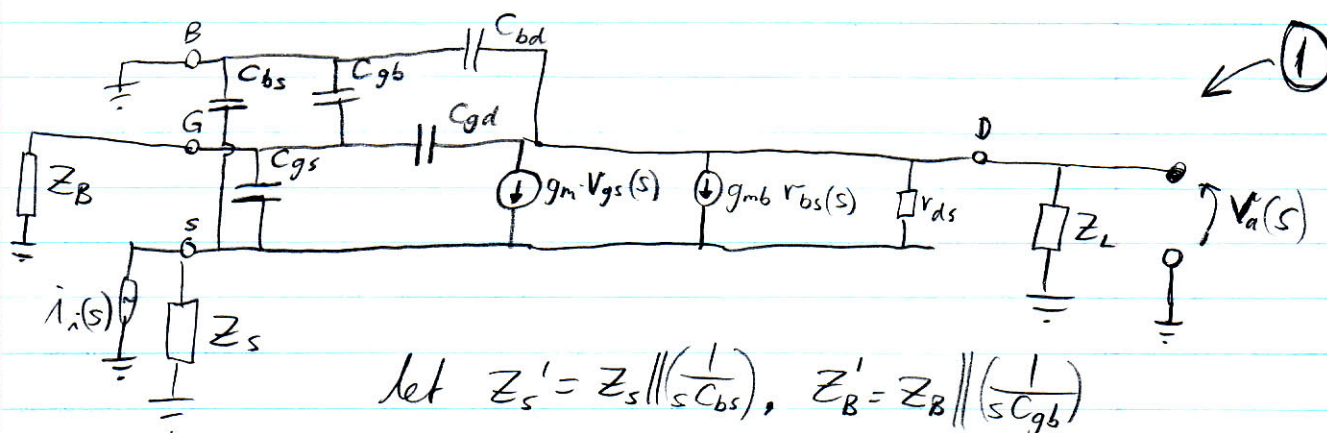
(lowest
potential)



assume

$$Z_B, Z_s, Z_L \equiv \text{[parallel combination of resistor and capacitor]}$$

SSM



$$\text{let } Z_s' = Z_s \parallel \left(\frac{1}{sC_{bs}} \right), \quad Z_B' = Z_B \parallel \left(\frac{1}{sC_{gb}} \right)$$

$$Z_L' = Z_L \parallel \frac{1}{sC_{bd}}$$

$$\text{KCL: (at G)} \quad \frac{V_g}{Z_B'} + (V_g - V_d)sC_{gd} + (V_g - V_s)sC_{gs} = 0$$

$$\therefore V_g = \underbrace{\left[\frac{1}{Z_B'} + s(C_{gd}C_{gs}) \right]^{-1}}_{\text{call it } a(s)} \cdot [sC_{gs}V_s + sC_{gd}V_d] \quad (2)$$

$$\text{KCL: (at S)} \quad \therefore V_s = \underbrace{\left[\frac{1}{Z_s'} + g_m + g_{mb} + \frac{1}{r_{ds}} + sC_{gs} \right]^{-1}}_{\text{call it } b(s)} \left[i_i + (g_m + sC_{gs})V_g + \frac{1}{r_{ds}}V_d \right] \quad (3)$$

(2/5)

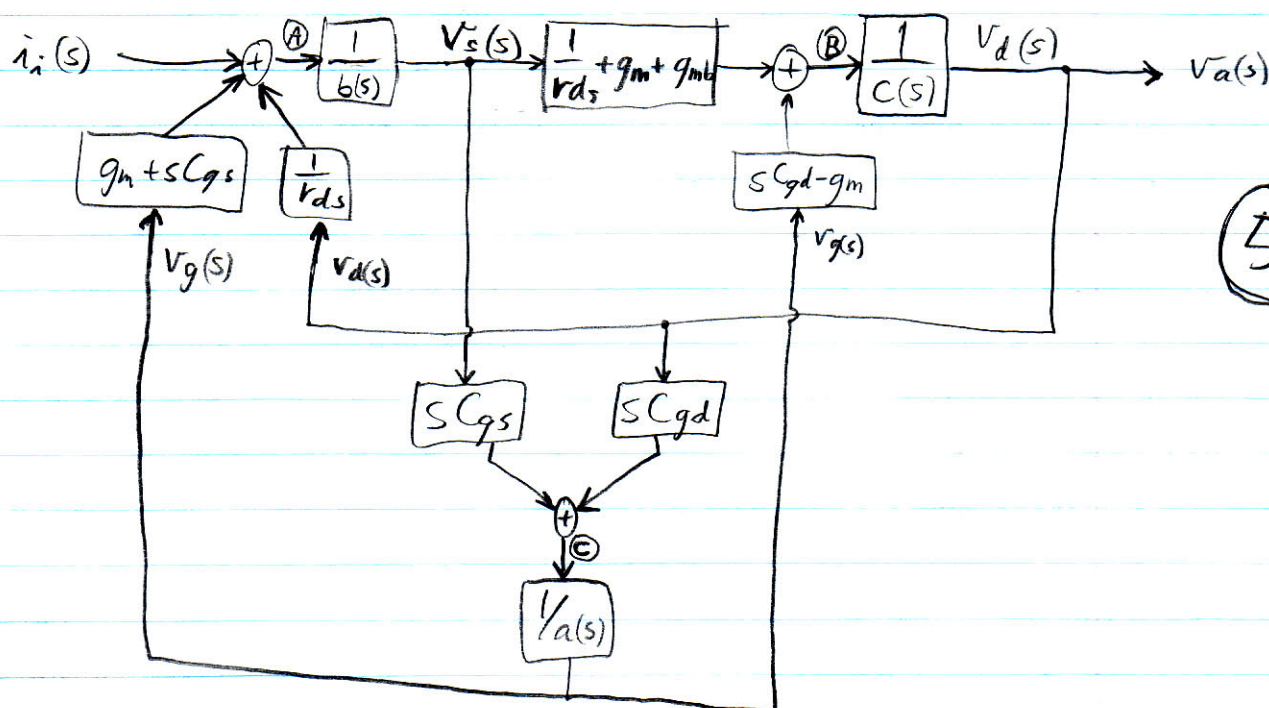
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KCL : ∞

$$\frac{(A+D)}{(A+D)} \quad v_d = \left[\frac{1}{Z'_L} + \frac{1}{r_{ds}} + sC_{gd} \right]^{-1} \left[\left(\frac{1}{r_{ds}} + g_m + g_{mb} \right) v_s + (sC_{gd} - g_m) v_g \right] \quad (4)$$

call it $C(s)$

can generate "block diagram" from (2), (3), (4):



(5)

Observations

- 1) all blocks are unidirectional (in dir. of the arrows)
- 2) all feedback paths arise from parasitic elements
(i.e. if $C_{xy} = 0, r_{ds} = \infty$, then no feedback loops)
with $C_{xy} = 0, r_{ds} = \infty$, (5) reduces to

$$\begin{aligned} i_i(s) &\rightarrow \left[\frac{1}{b(s)} \right] \rightarrow [g_m + g_{mb}] \rightarrow \left[\frac{1}{C(s)} \right] \rightarrow v_a(s) \quad (6) \\ &= \frac{1}{\frac{1}{Z'_L} + g_m + g_{mb}} = Z'_L \end{aligned}$$

$$\therefore \frac{v_a(s)}{i_i(s)} \Big|_{C_{xy}=0, r_{ds}=\infty} = \frac{Z'_L (g_m + g_{mb})}{\frac{1}{Z'_L} + g_m + g_{mb}} = \frac{Z'_L Z'_S (g_m + g_{mb})}{1 + (g_m + g_{mb}) Z'_S} \quad (7)$$

$\rightarrow Z'_L \text{ as } |Z'_S| \rightarrow \infty$

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Inspection of (6) $\Rightarrow$ (7)

Q: Can we find $\left. \frac{v_a(s)}{i_i(s)} \right|_{C_{xy} \neq 0, r_{ds} \neq 0}$ from (5) by inspection?

A: Yes. Using Mason's Gain Formula (M.G.F.)

Application of MGF to (5)

"Loops":

- 1: $A \rightarrow B \rightarrow A$
- 2: $A \rightarrow C \rightarrow A$
- 3: $A \rightarrow C \rightarrow B \rightarrow A$
- 4: $A \rightarrow B \rightarrow C \rightarrow A$
- 5: $B \rightarrow C \rightarrow B$

Note: All loops touch each other

"Forward Paths":

- 1: $i_i \rightarrow A \rightarrow B \rightarrow v_d$
- 2: $i_i \rightarrow A \rightarrow C \rightarrow B \rightarrow v_d$

Note: deleting either path breaks all loops

"Loop gains": $L_1 = \left(\frac{1}{r_{ds}} + g_m + g_{mb} \right) \frac{1}{r_{ds} b(s) c(s)}$

$$L_2 = s C_{gs} (g_m + s C_{gs}) \cdot \frac{1}{a(s) \cdot b(s)}$$

$$L_3 = s C_{gs} (s C_{gd} - g_m) \frac{1}{r_{ds} a(s) b(s) c(s)}$$

$$L_4 = s C_{gd} (g_m + s C_{gs}) \left(\frac{1}{r_{ds}} + g_m + g_{mb} \right) \cdot \frac{1}{a(s) b(s) c(s)}$$

$$L_5 = s C_{gd} (s C_{gd} - g_m) \frac{1}{a(s) c(s)}$$

(8)

"Path gains" $P_1 = \left(\frac{1}{r_{ds}} + g_m + g_{mb} \right) \cdot \frac{1}{b(s) c(s)}$

$$P_2 = s C_{gs} (s C_{gd} - g_m) \cdot \frac{1}{a(s) b(s) c(s)}$$

(9) $\leftarrow$ (units: Ω)

Fact M.G.F. $\Rightarrow \frac{v_a(s)}{i_i(s)} = \frac{P_1 + P_2}{1 - L_1 - L_2 - L_3 - L_4 - L_5}$

(10)

(Both paths touch all the loops)

$\therefore$ Can easily find $G(s) = \frac{v_a(s)}{i_i(s)}$ but it's really messy to look at

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For negligible capacitance in Z_L , Z_B and Z_S then

$$G(s) \cong \frac{\text{second order poly.}}{\text{third order poly.}} \quad \left. \begin{array}{l} (a(s), b(s) \& c(s) \text{ are} \\ \text{1st order polys.}) \end{array} \right\} \begin{array}{l} \text{conductance} \\ \text{(Siemens)} \end{array}$$

(Call full version of $G(s)$ (11))



The good news is (11) = "exact" expr.

The bad news is ——— " ———

(exact expr. is too complicated to give insight)

have third-order system with {2 zeros} {3 poles}

Q So what now?

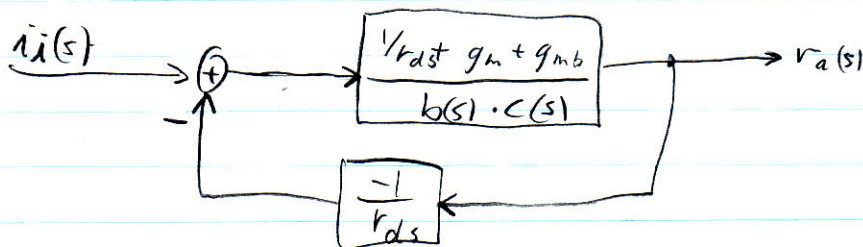
A Plug in numbers (ie freq. and comp. values) into boxes in (5) and eliminate highly attenuated paths.

Ex Z_B usually arises from parasitics

— often $Z_B \neq 0$ causes stability problems in feedback applications

$V_{BIAS} = DC$, so we can use bypass cap to reduce $|Z_B|$

$\therefore$ Suppose $|Z_B| \cong 0$ Then $\left| \frac{1}{a(s)} \right| \approx 0$ and (5) becomes $\approx$



$$\text{Now, MGF gives } G(s) \Big|_{|Z_B|=0} = \frac{\left(\frac{1/r_{ds} + g_m + g_{mb}}{b(s) \cdot c(s)} \right)}{1 - \frac{1}{r_{ds}} \left[\frac{(1/r_{ds} + g_m + g_{mb})}{b(s) c(s)} \right]}$$

let $g_m' = g_m + g_{mb} \quad (\approx 1.2 \cdot g_m)$ assume $g_m \gg \frac{1}{r_{ds}}$

then

$$b(s) = \frac{1}{Z_S'} + g_m' + sC_{gs}, \quad c(s) = \frac{1}{Z_L'} + \frac{1}{r_{ds}} + sC_{gd}$$

(as before)

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$$\therefore G(s) \Big|_{z_B=0} = \frac{g_m'}{\left(\frac{1}{z_s'} + g_m' + sC_{gs}\right)\left(\frac{1}{z_i'} + \frac{1}{r_{ds}} + sC_{gd}\right) - g_m'/r_{ds}} \quad (13)$$

Sanity check: 1) units V_1 ☒

$$2) A(j\omega) \Big|_{r_{ds}, z_s = \infty} = z_i' \quad \checkmark$$

(13) $\Rightarrow$ 2nd order behaviour: 2 poles, no zeros

$\Rightarrow$ always stable denom. of (13) has form $\alpha_0 + \alpha_1 s + \alpha_2 s^2$

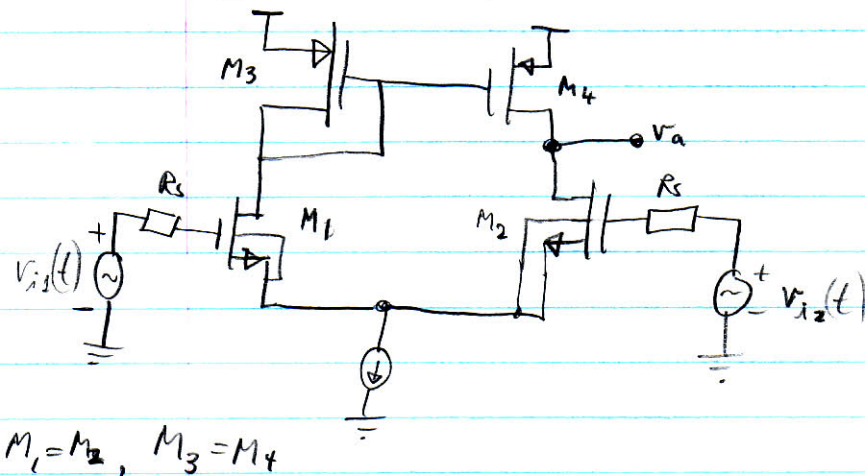
$$\text{with poles} = \frac{-\alpha_1}{2\alpha_2} \pm \frac{1}{2\alpha_2} \underbrace{\sqrt{\alpha_1^2 - 4\alpha_0\alpha_2}}_{< \alpha_1}$$

$$\therefore \text{Re}\{\text{poles}\} < 0$$

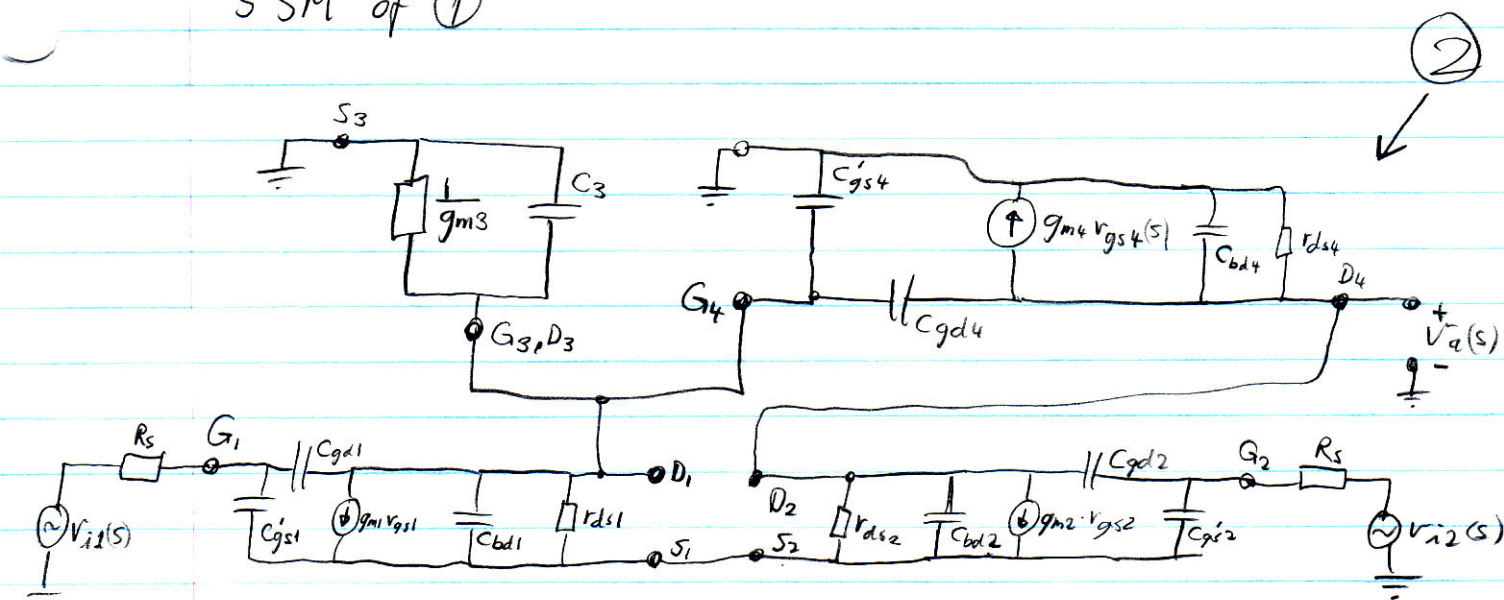
closed book/notes (bring calc.) } mid-term
 ↳ paper + penn (blyant)

Block diagrams & MGF (continued)
 Ex: Diff to single-ended OTA

Operational transconductance amplifier



SSM of ①



where $C_{gsi}' = C_{gsi} + C_{gsi}$, $C_3 = C_{gs3} + C_{bd3}$

Want to analyze differential mode (DM) operation: $v_{i1} = \frac{v_{id}}{2}$, $v_{i2} = -\frac{v_{id}}{2}$

Assume $R_S = \text{negligible for analysis}$

Want BD. contributing nodes: v_{id} , v_{d1} , $v_s \equiv v_{s1} \equiv v_{s2}$, v_o

Input → Drain 1 → Sources → Output

See textbook pp. 140~141

(2/6.)

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KCL at D_1

$$v_{d1} [g_{m3} + s(C_3 + C_{gs4})] + \frac{v_{d1} - v_s}{r_{ds1} \parallel C_{bd1}} + (v_{d1} - v_a) s C_{gd4} + (v_{d1} - \frac{v_{id}}{2}) s C_{gd1} + (\frac{v_{id}}{2} - v_s) g_{m1} = 0$$

Where " $r_x \parallel C_y$ " $\equiv (\frac{1}{r_x} + s C_y)^{-1}$

$$v_{d1} \left[g_{m3} + \frac{1}{r_{ds1}} + s(C_3 + C'_{gs4} + C_{bd1} + C_{gd4} + C_{gd1}) \right] - v_s \underbrace{\left[g_{m1} + \frac{1}{r_{ds1} \parallel C_{bd1}} \right]}_{\text{call it } a(s)} - v_a s C_{gd4} + \frac{1}{2} v_{id} (g_{m1} - s C_{gd1}) = 0$$

$\approx g_{m1} + s C_{bd1}$

$$\therefore v_{d1} = \frac{1}{a(s)} \left[v_s (g_{m1} + s C_{bd1}) + v_a s C_{gd4} - \frac{1}{2} v_{id} (g_{m1} - s C_{gd1}) \right] \quad (3)$$

KCL at D_2

...

$$\therefore v_a \approx \frac{1}{b(s)} \left[v_s (g_{m2} + s C_{bd2}) + v_{d1} (s C_{gd4} - g_{m4}) + \frac{1}{2} v_{id} (g_{m2} - s C_{gd2}) \right] \quad (4)$$

Where $b(s) = \frac{1}{r_{ds4} \parallel r_{ds2}} + s(C_{bd4} + C_{bd2} + C_{gd4} + C_{gd2})$ (used $g_{m2} \gg \frac{1}{r_{ds2}}$)

KCL at S_1, S_2

...

$$\therefore v_s = \frac{1}{c(s)} \left[\frac{v_{d1}}{r_{ds1} \parallel C_{bd1}} + \frac{v_a}{r_{ds2} \parallel C_{bd2}} \right] \quad (5)$$

where $c(s) = g_{m1} + g_{m2} + s(C'_{gs1} + C'_{gs2})$

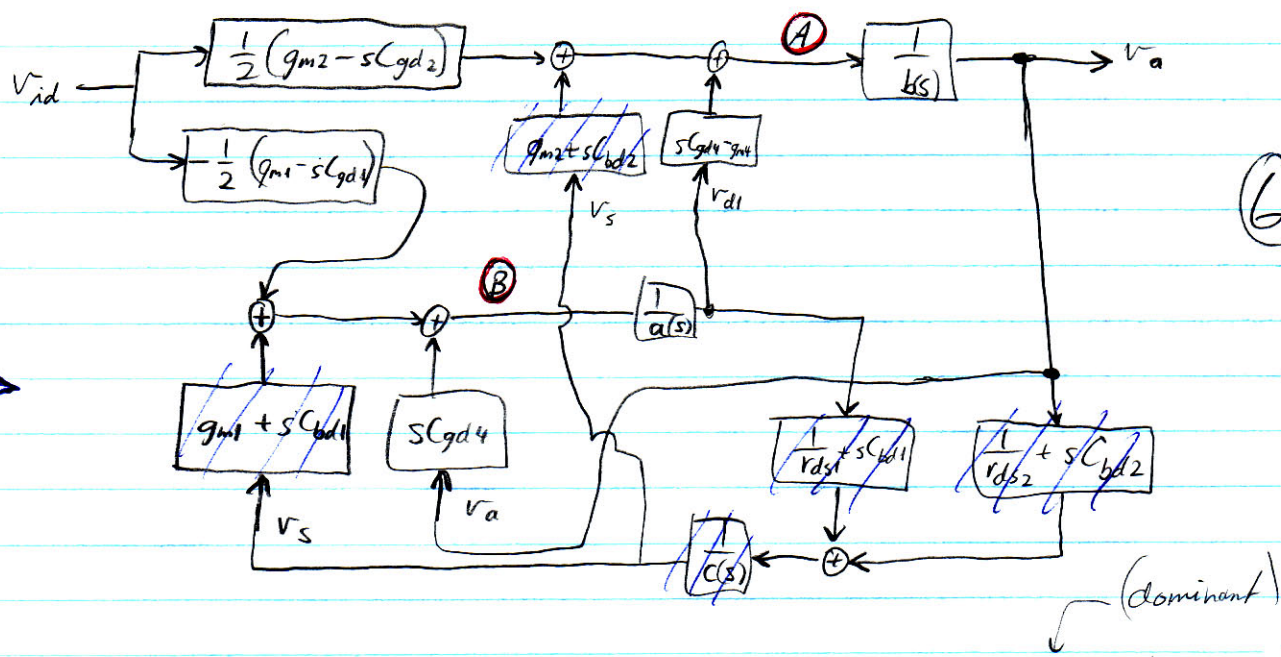
} used $M_1 = M_2$ & $M_3 = M_4$

(3) ~ (5) $\Rightarrow$ block diagram

$$\left[\frac{1}{a(s)} \right] = \left[\frac{1}{b(s)} \right] = \left[\frac{1}{c(s)} \right] = \Omega \quad \leftarrow 1^{st} \text{ order polynomials}$$

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Let $p_1, p_2, \dots$ be the poles $A_v(s) = \frac{v_o(s)}{v_{id}(s)}$ with $|p_1| \leq |p_2| \leq \dots$

Provided $\left| \frac{1}{r_{ds1,2}} + j\omega C_{bd1,2} \right|$ is sufficiently small that

$$|v_s(j\omega)| \ll |v_{d1}(j\omega)|, |v_a(j\omega)| \text{ for } |\omega| \leq |p_2| \quad (7)$$

can eliminate the v_s feedback paths in (6)
(asymmetry $\Rightarrow v_s \neq 0$)

We'll assume (7) holds for now. Can later check the assumption by testing with resulting value of p_2 .

Sanity check: Find $A_{v0} = A_v(j\omega)|_{\omega=0}$ $\omega=0 \Rightarrow s=0$

$$\therefore a(0) = g_{m3} \quad b(0) = \frac{1}{r_{ds2} \parallel r_{ds4}}$$

$$\therefore (6) \Rightarrow v_a = \left(\frac{1}{2} g_{m2} r_{ds2} \parallel r_{ds4} - \frac{1}{2} g_{m1} \cdot \frac{-g_{m4}}{g_{m3}} \cdot r_{ds2} \parallel r_{ds4} \right) v_{id}$$

$$\therefore A_{v0} = g_{m1} \cdot r_{ds2} \parallel r_{ds4} \quad (\checkmark)$$

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Now, apply MGF:Loop: $\textcircled{A} \rightarrow \textcircled{B} \rightarrow \textcircled{A}$ Forward paths 1. $v_{id} \rightarrow \textcircled{A} \rightarrow v_a$ 2. $v_{id} \rightarrow \textcircled{B} \rightarrow \textcircled{A} \rightarrow v_a$

$$\textcircled{6} \Rightarrow P_1 = \frac{g_{m2} - sC_{gd2}}{2 - b(s)}, \quad P_2 = \frac{(g_{m1} - sC_{gd1})(g_{m4} - sC_{gd4})}{2 - a(s) \cdot b(s)}$$

$$L_1 = \frac{sC_{gd4}(sC_{gd4} - g_{m4})}{a(s)b(s)}$$

$$\text{MGF} \Rightarrow A_v(s) = \frac{P_1 + P_2}{1 - L_1}$$

$$= \frac{1}{2} \cdot \frac{a(s)(g_{m2} - sC_{gd2}) + (g_{m1} - sC_{gd1})(g_{m4} - sC_{gd4})}{a(s)b(s) - sC_{gd4}(sC_{gd4} - g_{m4})}$$

 $\textcircled{8}$ $\Rightarrow 2 \text{ zeros, } 2 \text{ poles}$

2nd order $\Rightarrow$ can easily find poles & zeros, but results give little insight

Instead, note: $\left\{ \begin{array}{l} \text{Feedback depends on } sC_{gd4} \text{ (no Miller effect, why?) } \\ \text{Usually } C_{gd4} \ll C_{gsi}, C_{bdi} \\ \therefore \text{Taking } sC_{gd4} \approx 0 \text{ in } \textcircled{8} = \text{reasonable approximation} \end{array} \right.$ $\hookrightarrow Z=0 \text{ (source)}$

$$\text{Now, MGF} \Rightarrow A_v(s) = P_1 + P_2 \quad (L_1 \approx 0) = \frac{2^{\text{nd}} \text{ order poly}}{a(s)b(s)}$$

$$= g_{m1}(r_{ds2} \parallel r_{ds4}) \cdot \frac{(1 - s/z_1)(1 - s/z_2)}{(1 - s/p_1)(1 - s/p_2)}$$

$$\text{where } p_1 = \frac{-1}{(r_{ds2} \parallel r_{ds4})C_2}, \quad p_2 = \frac{-g_{m3}}{C_1}$$

$$\text{where } C_1 = C_{gs3} + C_{bd3} + C_{gs4} + C_{bd1} + C_{gd1}$$

$$C_2 = C_{bd2} + C_{bd4} + C_{gd2}$$

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C_1, C_2 have same order of mag $\left. \begin{array}{l} r_{ds2} \parallel r_{ds4} \gg 1/g_{m3} \text{ (typically)} \end{array} \right\} \Rightarrow \begin{array}{l} p_1 = \text{dom pole} \\ p_2 = \text{non dom. pole (next most dominant)} \end{array}$

Observation

$\frac{1}{g_{m3}}$ = resistance to short-signal $\frac{1}{s}$ at D_1 in ②

$C_1 = \text{cap} \text{ --- } \parallel \text{ ---}$

$r_{ds2} \parallel r_{ds4} = \text{res} \text{ --- } \parallel \text{ --- } D_2 \text{ ---}$

$C_2 = \text{cap} \text{ --- } \parallel \text{ ---}$

$\therefore$ In ② could have found p_1 and p_2 by calculating R_i, C_i
where $R_i = \text{res to gnd. at node } i$

$C_i = \text{cap. --- } \parallel \text{ ---}$

Then $p_1 = \frac{-1}{\text{largest}\{R_i \cdot C_i\}}, p_2 = \frac{-1}{\text{next largest}\{R_i \cdot C_i\}}$

Q : Coincidence?

A : No. This method is reasonable approx in many cases (HW 2)

Slang : Because of this, people often say that the dominant pole occurs at node D_2 and the non-dominant pole occurs at node D_1 .

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Full version of MGF

Block diagram defs.:

- 1) "Path" $\equiv$ route through B.D. (in dir. of arrows) connecting a pair of nodes
- 2) "Path gain" $\equiv$ product of block gains along path
- 3) "Loop" $\equiv$ path which starts and ends at same node with no node along path encountered more than once
- 4) "Loop gain" $\equiv$ path gain of loop
- 5) "Determinant" $\equiv 1 - \sum (\text{all loop gains})$
 $+ \sum (\text{products of loop gains of all loop pairs with no common nodes})$
 $- \sum (\text{triples}) + \sum (\dots)$
...
- 6) "Forward path" $\equiv$ path from input to output containing no full loops

Let $H = \frac{X_{out}}{X_{in}}$ where X_{in} = input of B.D.
 X_{out} = output of B.D.

$$\text{Then } H = \frac{1}{\Delta} \cdot \sum_{k=1}^L P_k \cdot \Delta_k \quad (\text{MGF})$$

where Δ = determinant

L = # of forward paths

P_k = k^{th} forward path

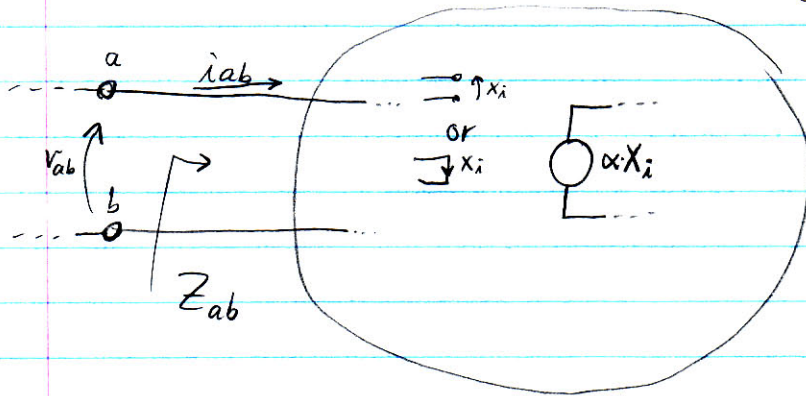
Δ_k = determinant of B.D. that remains after deleting k^{th} path, P_k

"cofactor" = Δ with loops touching the k^{th} path removed.

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Blackman's Impedance Relation

arbitrary SSM circuit
containing ≥ 1 controlled sources



X_i = voltage or current
 αX_i = " " "

(e.g. $X_i = V_{gs}$, $\alpha = g_m$
or $\alpha X_i = \text{current}$)

$$\text{BIR} \quad Z_{ab} = Z_{ab}^0 \cdot \frac{1 + T_{sc}}{1 + T_{oc}} \quad (1)$$

$$\left(\equiv \frac{V_{ab}(s)}{i_{ab}(s)} \right)$$

nullstilt!

where $Z_{ab}^0 \equiv$ Impedance between a and b with controlled source removed
 $T_{sc} \equiv -\alpha \cdot \frac{X_i}{X_x}$ when $V_{ab} = 0$ (a, b shorted) ($\alpha = 0$)

and αX_i is replaced by an independent "test source", X_x

$T_{oc} \equiv$ Same as T_{sc} except with $i_{ab} = 0$ (a, b open circuited)

Note: if have more than one controlled sources, pick any one of them (call it the reference source).

T_{enc} or "loop gain"

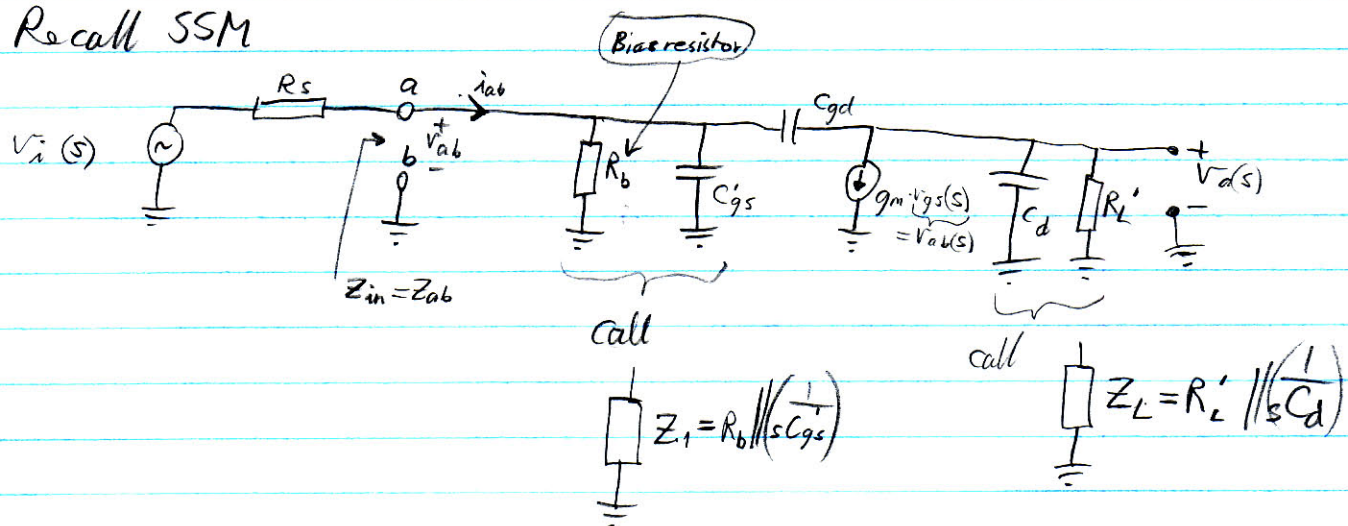
See note
of March 18, 2008

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Ex 1 C.S. Amp

Recall SSM



BIR: $\alpha = g_m$, $X_i = V_{ab}$, $x_x = i_x$

$$Z_{ab}^o = Z_i \parallel \left(\frac{1}{sC_{gd}} + Z_L \right) = \frac{Z_i \cdot (1 + Z_L \cdot sC_{gd})}{1 + (Z_i + Z_L) \cdot sC_{gd}}$$

$$T_{sc} = 0 \quad (\text{because } X_i = V_{ab} = 0)$$

$$T_{oc} = -g_m \cdot \frac{1}{i_x} \left[-i_x \left(Z_L \parallel \left(Z_i + \frac{1}{sC_{gd}} \right) \right) \cdot \frac{Z_L}{Z_i + \frac{1}{sC_{gd}}} \right]$$

$$= \frac{g_m Z_i Z_L}{Z_L + Z_i + \frac{1}{sC_{gd}}} \quad (\text{note } \rightarrow 0 \text{ as } C_{gd} \rightarrow 0)$$

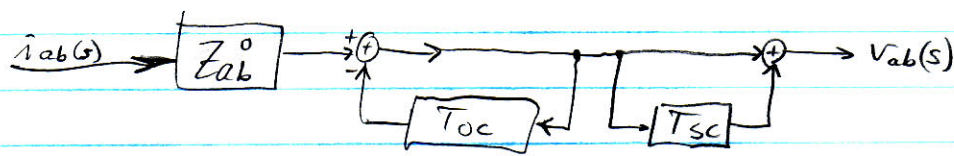
$$\therefore \textcircled{1} \Rightarrow Z_{in} = Z_{ab}^o \cdot \frac{1}{1 + T_{oc}} = Z_i \cdot \frac{1 + sC_{gd} \cdot Z_L}{1 + [Z_L + Z_i(1 + g_m Z_L)] \cdot sC_{gd}}$$

Miller effect

②

Observations

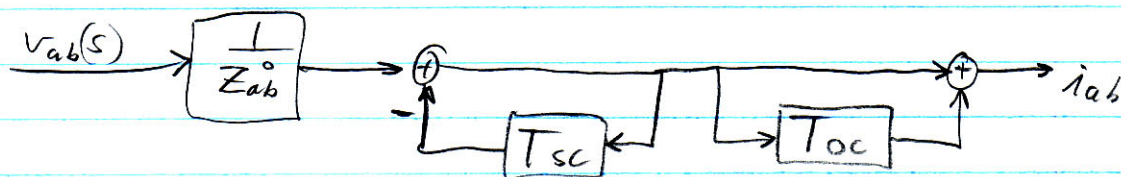
- 1) T_{oc} represents a feedback path around the reference source
Why? def $\Rightarrow$ If output of ref. source has no connection to its controlling voltage (or current) then $T_{oc} = 0$
Also, MGF and ① give:



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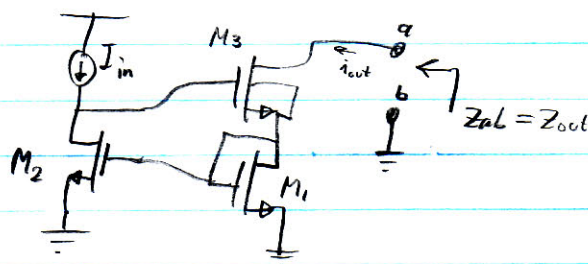
Or.



$\therefore T_{sc}$ also arises from a feedback path within the circuit

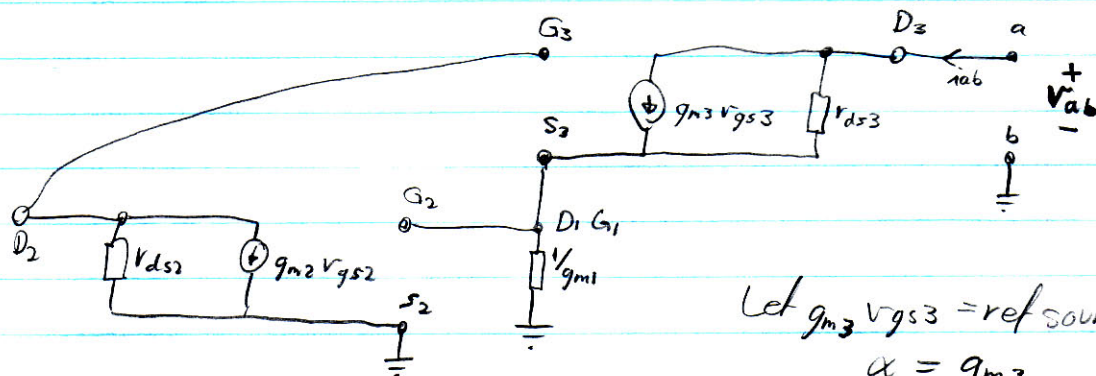
2) Feedback can either increase or decrease Z_{ab}

Ex 2 Wilson current mirror (low freq analysis)



Low freq. SSM (Using $\square \approx \square \parallel \frac{1}{g_{m1}}$)

(Set $i_{in} = 0$ to find Z_{out})



Let $g_{m3} v_{gs3} = \text{ref source} \therefore$

$$\alpha = g_{m3}$$

$$X_i = v_{gs3}$$

Inspection of ③ $\Rightarrow Z_{ab}^o = r_{ds3} + \frac{1}{g_{m1}} \approx r_{ds3}$

③

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T_{oc} : $i_{ab} = 0 \Rightarrow v_{gs2} = 0 \Rightarrow v_{gs3} = 0 \Rightarrow x_i = 0 \therefore T_{oc} = 0$

T_{sc} : with $g_{m3} v_{gs3}$ replaced by i_x and $v_{ab} = 0$ have

$$\left. \begin{aligned} v_{gs2} &= i_x \left(r_{ds3} \parallel \left(\frac{1}{g_{m1}} \right) \right) \approx \frac{i_x}{g_{m1}} \\ \text{KVL} \Rightarrow v_{gs2} + v_{gs3} &= -g_{m2} \cdot v_{gs2} \cdot r_{ds2} \end{aligned} \right\} \Rightarrow v_{gs3} = -\frac{i_x}{g_{m1}} (1 + g_{m2} r_{ds2})$$

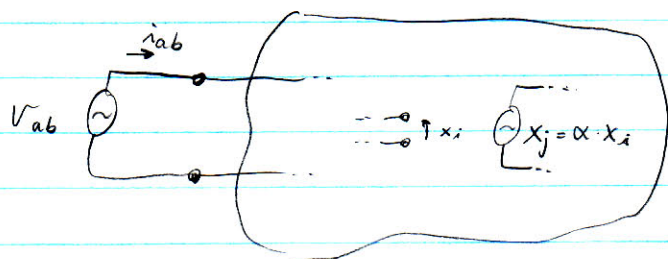
$$\therefore T_{sc} = \frac{g_{m3}}{g_{m1}} (g_{m2} \cdot r_{ds2} + 1) = g_{m3} r_{ds2} + 1$$

(for $M_1 = M_2 = M_3$)

$\therefore \textcircled{1} \Rightarrow Z_{ab} = r_{ds3} (2 + g_{m2} \cdot r_{ds2}) \therefore \text{Feedback increased } Z_{ab}$

Proof of BIR:

Linearity $\Rightarrow$ can write



SSM circuit as before (all indep. sources set to zero)

$$\left. \begin{aligned} v_{ab} &= A \cdot i_{ab} + B \cdot x_j \\ x_i &= C \cdot i_{ab} + D \cdot x_j \end{aligned} \right\} \textcircled{4} \quad \begin{aligned} &\text{(two-port representation)} \\ &A(s), B(s), C(s), D(s) \equiv \text{transfer functions} \end{aligned}$$

Solving $\textcircled{4}$ for $Z_{ab} = \frac{v_{ab}}{i_{ab}}$ gives $Z_{ab} = A \cdot \frac{1 - \frac{\alpha(AD-BC)}{A}}{1 - \alpha D} \quad \textcircled{5}$

$\textcircled{4} \Rightarrow A = \left. \frac{v_{ab}}{i_{ab}} \right|_{x_j=0} = \text{def of } Z_{ab}^0 \quad \textcircled{6}$

Now suppose $v_{ab} = 0$ and $x_j = x_x$ (indep. of x_i)

Then $\textcircled{4} \Rightarrow \begin{aligned} i_{ab} &= -\frac{B}{A} \cdot x_x \\ x_i &= C \cdot i_{ab} + D \cdot x_x \end{aligned}$

$$\Rightarrow -\alpha \frac{x_i}{x_x} = -\frac{\alpha(AD-BC)}{A} \quad \textcircled{7}$$

$\equiv T_{sc}$

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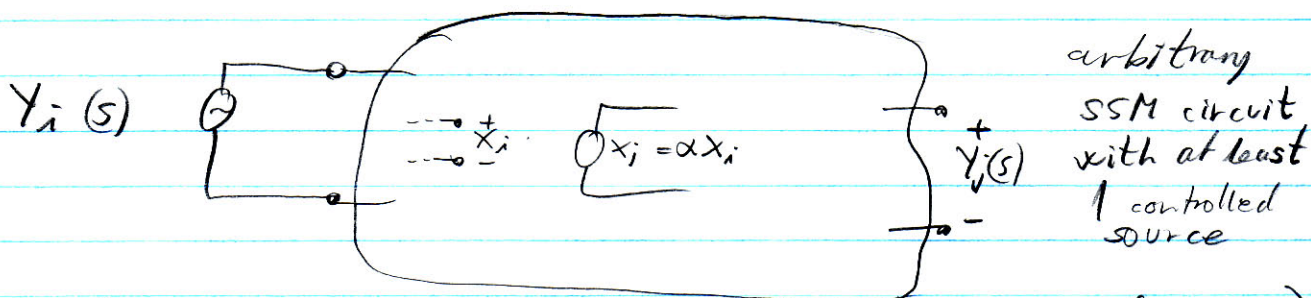
Now suppose $i_{ab} = 0$ and $x_j = x_x$ (indep. of x_i)
 Then ④ similarly $\Rightarrow T_{oc} = -\alpha D$ ⑧

$$\therefore \textcircled{5} \sim \textcircled{8} \Rightarrow Z_{ab} = Z_{ab}^o \cdot \frac{1+T_{sc}}{1+T_{oc}} \quad \square$$

□

Asymptotic Gain Relation

⑨



$x_i \neq Y_i$ required

any controlled source in circuit ($\equiv$ "ref. source")

$\left. \begin{matrix} x_i, x_j \\ Y_i, Y_j \end{matrix} \right\}$ voltages or currents

$$AGR: A(s) = A_\infty \cdot \frac{T}{1+T} + A_o \cdot \frac{1}{1+T} \quad \textcircled{10}$$

$$\left(= \frac{Y_j(s)}{Y_i(s)} \right)$$

where $T \equiv -\alpha \cdot \frac{x_i}{x_x}$ where $Y_i = 0$ and x_j replaced by an independent "test source", x_x
 "loop gain"

$$A_\infty(s) \equiv \left. \frac{Y_j(s)}{Y_i(s)} \right|_{\alpha \rightarrow \infty} \equiv \text{"asymptotic gain"}$$

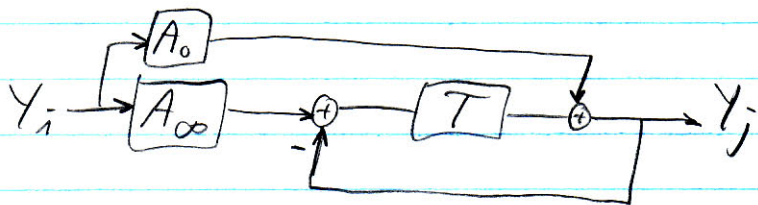
$$A_o(s) \equiv \left. \frac{Y_j(s)}{Y_i(s)} \right|_{\alpha=0} \equiv \text{"direct transmission term"}$$

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Observations

MGF of (10) $\Rightarrow$ can redraw (9) as



$\Rightarrow$ large $T \Leftrightarrow$ feedback dominates behavior $\Rightarrow A(s) \cong A_\infty(s)$

$A_0(s)$ arises from forward path through feedback network

"Reference source" in BIR & AGF:

See note of March 18, 2008

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ECE 264A

Feb. 12, 2008

E.G.

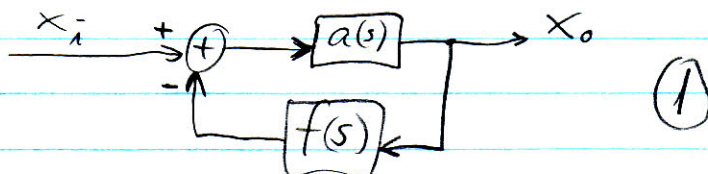
Avg.: 68
Med.: 70 } Mid-term

Feedback

Let $x_i(s)$ = input signal (V or I)

$x_o(s)$ = output ——— " ———

Then a circuit incorporates feedback if its B.D. can be reduced to:



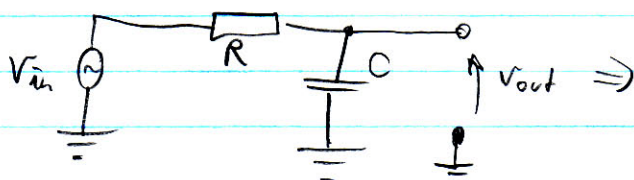
MGF + (1) $\Rightarrow$

$$A(s) = \frac{a(s)}{1 + a(s)f(s)}$$

(2)

$$\text{where } A(s) \equiv \frac{x_o(s)}{x_i(s)}$$

Ex 1 "Passive feedback"



$$\therefore a(s) = 1 \\ f(s) = sRC$$

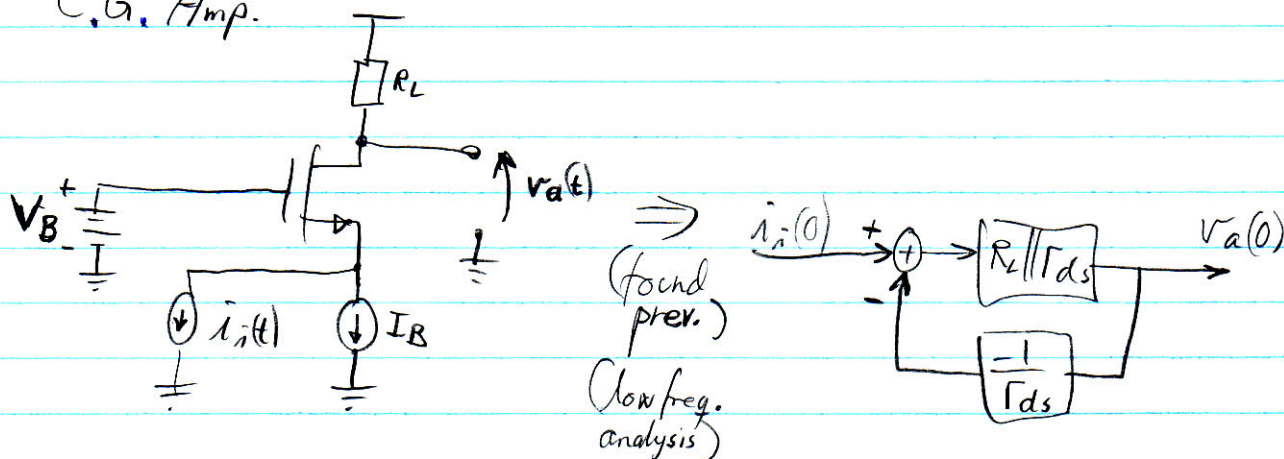
(more generally, poles $\Rightarrow$ feedback)

$$\frac{v_{out}}{v_{in}} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} = \frac{1}{1 + sRC} \Rightarrow v_{out} = v_{in} - v_{out} \cdot s \cdot RC$$

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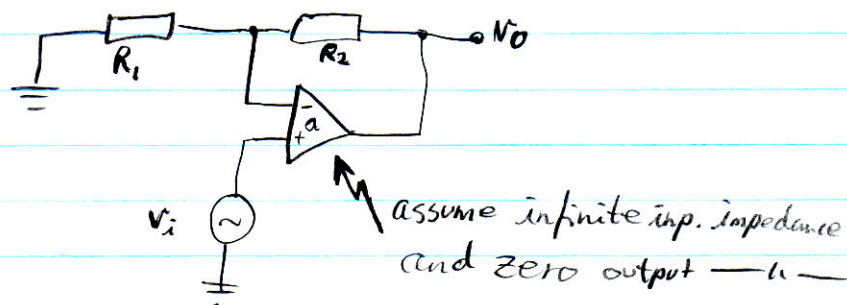
Ex 2 "Parasitic feedback"

C.G. Amp.



Note: In Ex1 feedback reduces the magnitude of $X_o \equiv$ "neg. feedback"
In Ex2 $\sim$ increases $\sim$ $\equiv$ "pos. feedback"

Ex 3 "Intentional active feedback"



$$v_o = a \left[v_i - \frac{R_1}{R_1 + R_2} v_o \right]$$



$$A = \frac{a}{1 + a \left(\frac{R_1}{R_1 + R_2} \right)} \rightarrow 1 + \frac{R_2}{R_1} \text{ as } a \rightarrow \infty$$

neg. feedback

$\therefore$ large $a \Rightarrow A$ depends only of the ratio of R 's

(3)

Easy in IC's

accurate in IC's

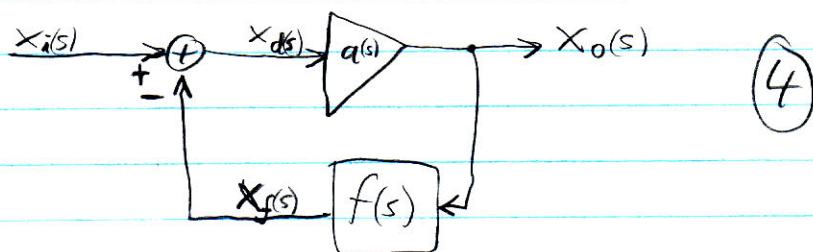
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Intentional active feedback $\Rightarrow$ feedback around high gain amp
 $\Rightarrow$ many benefits (e.g. ③)

Negative Intentional Active Feedback

Def.: In the following feedback system



$a(s) \equiv$ "open loop gain", $f(s) \equiv$ "feedback factor"

$T(s) \equiv a(s) \cdot f(s) \equiv$ "loop gain", $A(s) \equiv$ "closed loop gain"
(recall $A(s) = \frac{a(s)}{1 + a(s) \cdot f(s)}$)

Let $A_{ideal} = \lim_{a \rightarrow \infty} A = \frac{1}{f}$ (e.g. $A_{ideal} = 1 + \frac{R_2}{R_1}$ in Ex 3)

Asymptotic
gain-relation:
jan. 31

$$A = A_{ideal} \cdot \frac{1}{1+T}$$

$\therefore$ ② $\Rightarrow A = A_{ideal} \left(1 - \frac{1}{1+T} \right)$ fractional deviation of A from A_{ideal}

$A = \frac{1}{f} \left(1 - \frac{1}{1+a \cdot f} \right) = \frac{1}{f} \cdot \frac{a \cdot f}{1+a \cdot f} = \frac{a}{1+a \cdot f}$

Also, MGF & ④ $\Rightarrow$

$$X_d = \frac{1}{1+T} \cdot X_i \rightarrow 0 \text{ as } T \rightarrow \infty \quad (a \rightarrow \infty)$$

$$X_f = \frac{1}{1+1/T} \cdot X_i \rightarrow X_i \text{ as } T \rightarrow \infty \quad (a \rightarrow \infty)$$

$\therefore X_f$ "tracks" X_i with "error signal" X_d ($d = \text{difference}$)

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Benefits:

- (i) gain desensitivity
- (ii) non-linear distortion reduction
- (iii) in-loop noise suppression
- (iv) broadbanding
- (v) input/output impedance adjustment

(Potential) Drawbacks

- (i) $A < a$
- (ii) excessive phase shift can cause instability

$$\left(\frac{V}{V}\right)' = \frac{V'V - VV'}{V^2}$$

(i) Gain desensitivity

$$(2) \Rightarrow \frac{dA}{da} = \frac{1}{(1+af)^2} \Rightarrow \frac{\Delta A}{\Delta a} \approx \frac{1}{(1+af)^2} \quad \text{small } \Delta a$$

$$\therefore \frac{\Delta A}{A} \approx \left(\frac{1}{1+T}\right) \cdot \frac{\Delta a}{a} \Rightarrow \text{large variation in } a \Rightarrow \text{small } \frac{\Delta A}{A}$$

$\Rightarrow A$ is "stable" w.r.t variations in a

$$(2) \Rightarrow \frac{dA}{df} = -A^2$$

$$\therefore \frac{\Delta A}{\Delta f} \approx -A \cdot \frac{1}{f} \cdot \frac{T}{1+T}$$

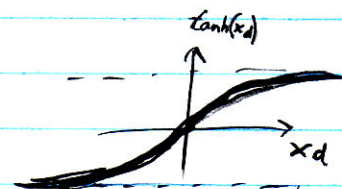
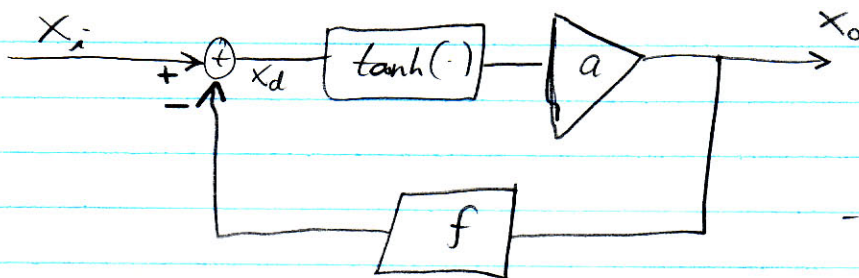
$$\therefore \frac{\Delta A}{A} \approx -\left(\frac{T}{1+T}\right) \frac{\Delta f}{f} \Rightarrow A \text{ is not "stable" w.r.t variations}$$

≈ 1 for $|T| \gg 1$

$\Rightarrow$ want high gain amplifier (not nec. well defined) and precise feedback factor.

$\frac{\Delta A}{A} = \frac{-T}{1+T} \cdot \frac{\Delta f}{f}$	$\frac{\Delta A}{A} = \frac{1}{1+T} \cdot \frac{\Delta a}{a}$
--	---

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(ii) Non-linear distortion reductionEx 4

$$x_o = a \cdot \tanh(x_d)$$

$$\therefore \tanh^{-1}\left(\frac{x_o}{a}\right) = x_d = x_i - f \cdot x_o$$

$$\therefore \frac{x_o}{a} + \frac{x_o^3}{3a^3} + \frac{x_o^5}{5a^5} + \dots = x_i - f x_o$$

$$\therefore x_o + \frac{x_o^3}{3a^2} + \frac{x_o^5}{5a^4} + \dots = a(x_i - f x_o)$$

$\rightarrow 0 \text{ as } a \rightarrow \infty$

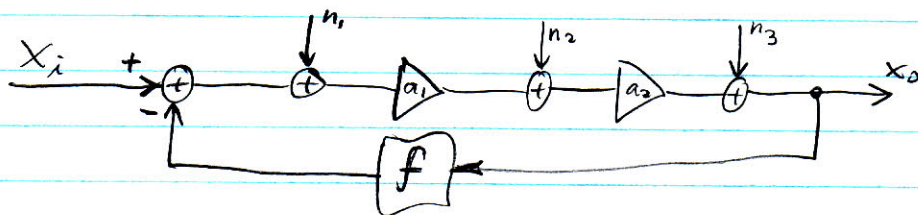
$$\therefore x_o \approx A \cdot x_i \text{ for large } a \text{ and } x_o \rightarrow \underbrace{\frac{1}{f} \cdot x_i}_{\text{no distortion}} \text{ as } a \rightarrow \infty$$

- Heuristics:
- 1) Follows from gain desensitivity
(distortion prior to $a \Leftrightarrow$ input dependent gain variation)
 - 2) Large $a \Rightarrow$ small $x_d \Rightarrow$ small portion of non-linear curve is traversed $\Rightarrow \approx$ linear

6./6

Feb. 12, 2008

(iii) In-loop noise suppression



open loop gain:
 $a = a_1 a_2$

$$MGF \Rightarrow x_o = \frac{a}{1+af} \left(x_i + n_1 + \frac{n_2}{a_1} + \frac{n_3}{a_1 a_2} \right)$$

not suppressed

somewhat suppressed

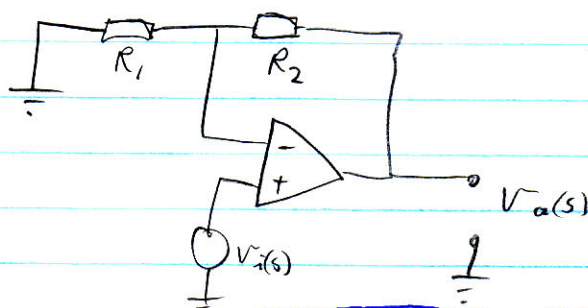
most suppressed

(iv) Broadbanding

Ex 5 suppose $a \approx \frac{a_0}{1-j\omega/\omega_{p0}}$ (dom. pole approx.)

$F \approx \text{indep. of } \omega$

eg.



$$f = \frac{R_1}{R_1 + R_2}$$

Then (2) $\Rightarrow$

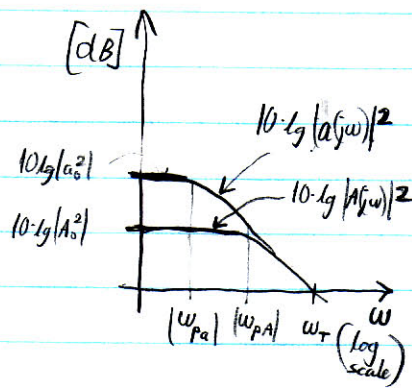
$$A = \frac{a}{1+af} = \frac{1}{f} \cdot \frac{af}{1+af} = \frac{1}{f} \cdot \frac{1}{1+\frac{1}{f} \cdot \frac{1}{a}} = A_{ideal} \cdot \frac{1}{1+1/f}$$

$$A(j\omega) = \underbrace{\left(1 + \frac{R_2}{R_1}\right)}_{1/f} \cdot \frac{1}{1 + \left(1 + \frac{R_2}{R_1}\right)(1-j\omega/\omega_{p0})/a_0}$$

$$a(j\omega) \Rightarrow A(j\omega) = \frac{A_0}{1-j\omega/\omega_{pA}}$$

$$\text{where } A_0 = \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{1}{1 + \left(1 + \frac{R_2}{R_1}\right)/a_0} \quad (5)$$

$$\text{and } \omega_{pA} = \omega_{p0} \left(1 + a_0 \cdot \frac{R_1}{R_1 + R_2}\right) \quad (6)$$

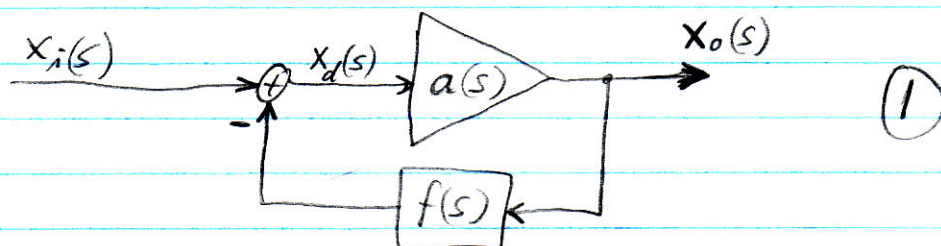


(1/6)

Feb. 14, 2008

ECE 264A

E.G.

Negative intentional active feedback (cont. from pp. $\frac{3 \sim 6}{6}$ last time)Recall: Can always write the block diagram of a circuit with feedback as: $a(s) \equiv$ "open loop gain" $f(s) \equiv$ "feedback factor" $T(s) \equiv a(s) \cdot f(s) \equiv$ loop gain← (i M.G.F.: $T = a \cdot f$ unlike definitions!) (1) $A(s) = \frac{x_o(s)}{x_i(s)} \equiv$ "closed loop gain"

$$\text{MGF} \Rightarrow A(s) = \frac{a(s)}{1 + a(s) \cdot f(s)} \quad (2)$$

Broadbanding (cont.)

$$\text{Suppose: } a(j\omega) = \underbrace{a_0}_{\text{DC-gain}} \cdot \frac{1}{1 - j\omega/\omega_{pa}} \quad (3)$$

→ dominant pole approx.

$$f(j\omega) = \text{const.}$$

$$(2), (3) \Rightarrow A(j\omega) = A_0 \cdot \frac{1}{1 - j\omega/\omega_{pa}} \quad \text{where}$$

$$A_0 = \frac{a_0}{1 + a_0 \cdot f} \quad (\text{DC closed loop gain})$$

$$\& \quad \omega_{pa} = (1 + a_0 \cdot f) \cdot \omega_{pa}$$

(4)

2/6

Feb. 14, 2008

$|A_o \cdot \omega_{pa}| \equiv$ "closed loop gain bandwidth product", $GBW_{\text{closed loop}}$

$|a_o \cdot \omega_{pa}| \equiv$ "open loop gain bandwidth product", $GBW_{\text{open loop}}$

$$(4) \Rightarrow A_o \cdot \omega_{pa} = a_o \cdot \omega_{pa}$$

$\therefore GBW_{\text{open loop}} = GBW_{\text{closed loop}}$ in this case.

$\therefore$ Increasing f (between 0 and 1) decreases A_o , but increases the 3dB BW.

"Unity Gain Frequency"

$\equiv \omega_t$ such that $a(j\omega_t) = 1$, $\omega_t \in \mathbb{R}$

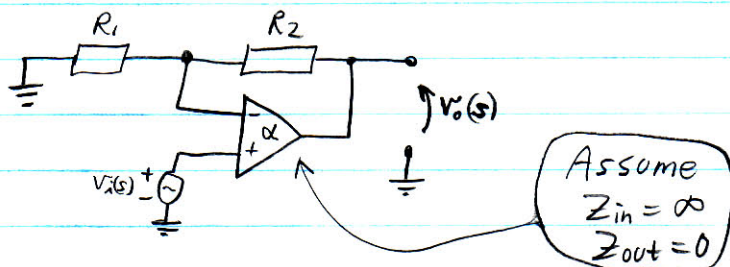
$$\therefore (3) \Rightarrow \left| a_o \cdot \frac{1}{1 - j\omega_t/\omega_{pa}} \right| = 1$$

$$\Rightarrow (a_o^2 - 1) \cdot \omega_{pa}^2 = \omega_t^2$$

$$\Rightarrow \omega_t \approx a_o \cdot |\omega_{pa}| \quad (a_o \gg 1)$$

$\therefore GBW = \omega_t$ in this case (not always in other cases)

Ex 1 (from last time)

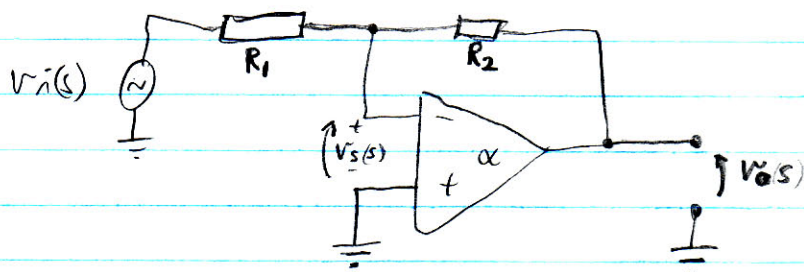


$$\Rightarrow (1) \text{ if } a = \alpha, \quad f = \frac{R_1}{R_1 + R_2}$$

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Feb 14, 2008

Ex2 (Same except different input location)



(6)

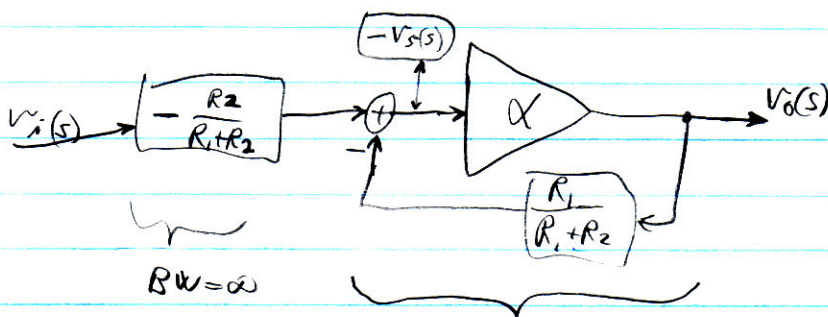
$$v_o(s) = \alpha(s) \cdot [-v_s(s)]$$

$$\therefore v_s(s) = v_i(s) - R_1 \cdot \left[\frac{v_i(s) - v_o(s)}{R_1 + R_2} \right]$$

$$\therefore -v_s(s) = -v_i(s) \left[1 - \frac{R_1}{R_1 + R_2} \right] - v_o(s) \cdot \left[\frac{R_1}{R_1 + R_2} \right]$$

$$\rightarrow = \frac{R_2}{R_1 + R_2}$$

$\therefore$ BD of (6) is:



$$\equiv (1) \text{ with } a = \alpha \text{ \& } f = \frac{R_1}{R_1 + R_2}$$

(Same as Ex. 1.)

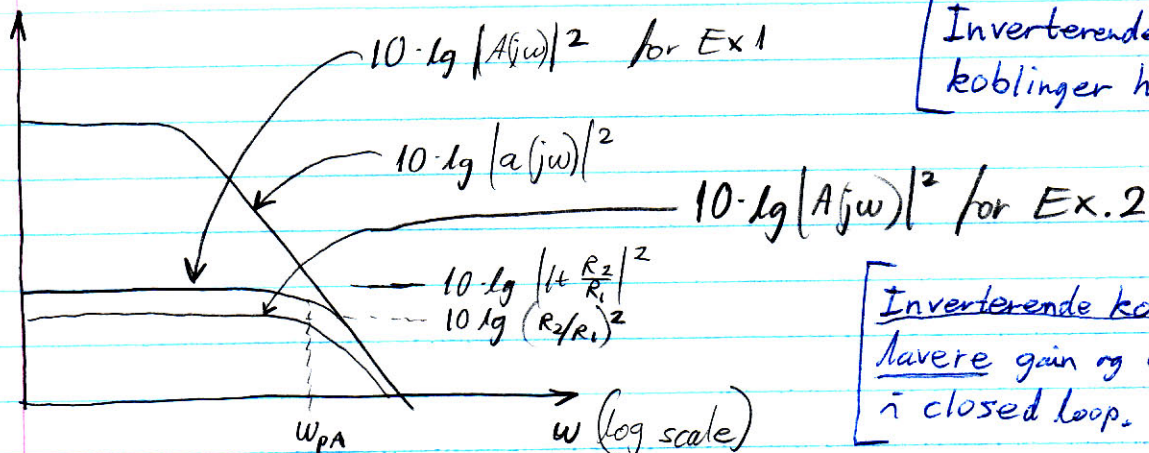
$\Rightarrow$ BW = same as Ex1

But GBW of (6) less than that of Ex1 by $\frac{R_2}{R_1 + R_2}$

$$\text{Also, } GBW_{\text{closed loop}} = \frac{R_2}{R_2 + R_1} \cdot \omega_t = \frac{R_2}{R_1 + R_2} \cdot a_0(\omega_{pa})$$

GBW_{open loop}

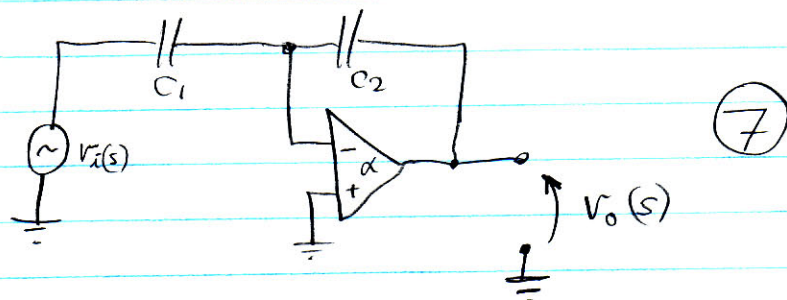
Feb 14, 2008



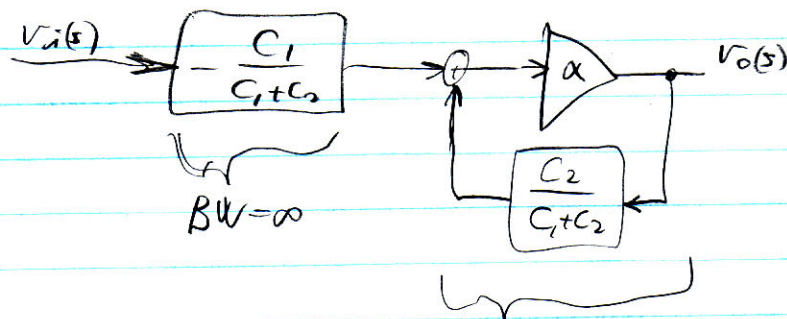
Inverterende og ikke-inv. koblinger har samme B.W.

Inverterende kobling gir lavere gain og GBW produkt i closed loop.

Ex 3 Same as Ex 2 with R's replaced by C's (important later in switched cap. circuits)



Exercise show BD of ⑦ is



$\equiv$ ① with $a = \alpha$
 $f = \frac{C_2}{C_1+C_2} \Rightarrow f = \text{const} (\in \mathbb{R})$

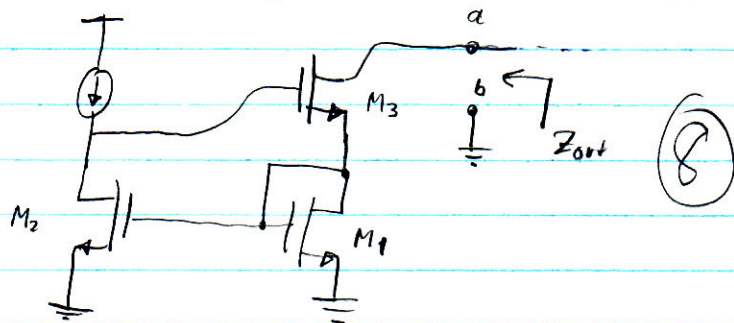
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$$\therefore \textcircled{4} \Rightarrow BW = \left(1 + a_o \frac{C_2}{C_2 + C_1}\right) \cdot |\omega_{pa}|$$
$$\text{Also } GBW_{\text{closed loop}} = \frac{C_1}{C_1 + C_2} \cdot \omega_t = \frac{C_1}{C_1 + C_2} \cdot a_o \cdot |\omega_{pa}|$$

(V) Impedance transformation

Ex. 4 Wilson current mirror

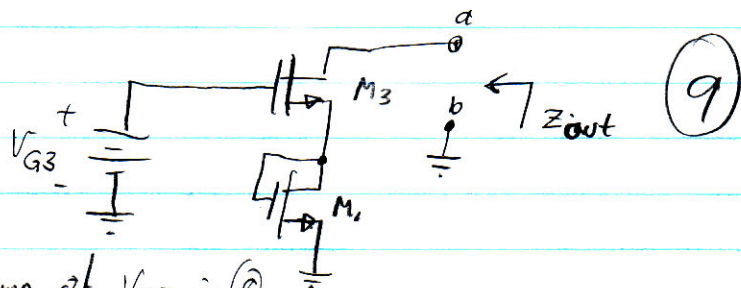


previously found:

$$Z_{out} \approx r_{ds3} \cdot g_{m2} \cdot r_{ds2}$$

(low freq. analysis, neglecting body effect)

Now consider



Where V_{G3} = DC comp. of V_{G3} in $\textcircled{8}$

$$\text{Can show: } Z_{out} \text{ of } \textcircled{9} \approx r_{ds3} \left(2 + \frac{1}{g_{m1} \cdot r_{ds3}}\right)$$
$$\approx 2 \cdot r_{ds3}$$

Note: $\textcircled{9}$ is equiv. to $\textcircled{8}$ with feedback disabled

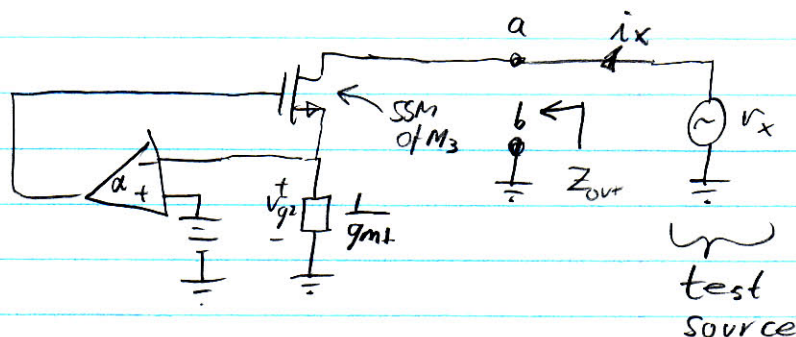
$\therefore$ feedback increases Z_{out} by $\left(\frac{1}{2} g_{m2} \cdot r_{ds2}\right)$ factor

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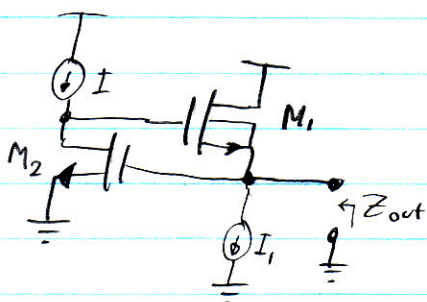
Heuristics:

(8) $\equiv$



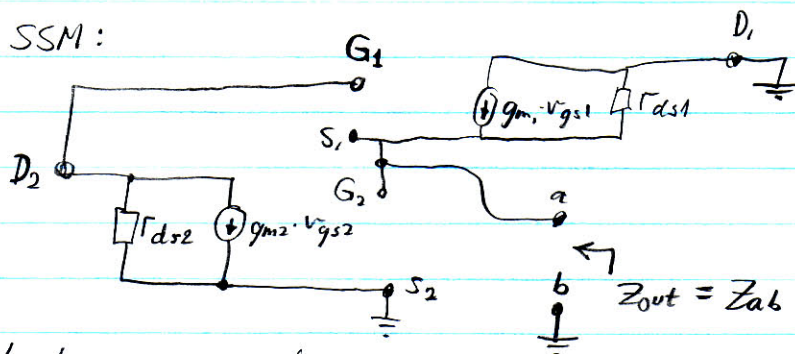
If v_x increases, i_x increases $\Rightarrow v_{g2}$ increases
 $\Rightarrow v_{g3}$ decreases
 $\Rightarrow$ feedback acts to reduce i_x

Ex 5 Voltage source



(10)

Low freq SSM:



(11)

BIR: Let $g_{m1} \cdot v_{g1} \equiv \text{ref. source}$
 $\therefore v_{gs1} = x_i, \alpha = g_{m1}, Z_{ab}^0 = r_{ds1}$

Exercise: Show $T_{sc} = 0, T_{oc} = g_{m1} \cdot g_{m2} \cdot r_{ds1} \cdot r_{ds2}$

$$\therefore Z_{ab} = r_{ds1} \cdot \frac{1}{1 + g_{m1} g_{m2} \cdot r_{ds1} \cdot r_{ds2}} \approx \frac{1}{g_{m1} g_{m2} r_{ds2}} = \text{small!}$$

Feedback reduced output imp. by a factor of $g_{m2} \cdot r_{ds2}$

Stability

Let $A(s) = \frac{a'_0 + a'_1 s + a'_2 s^2 + \dots + a'_m s^m}{b'_0 + b'_1 s + b'_2 s^2 + \dots + b'_n s^n}$ (1)

→ If $A(0) \neq 0$ or ∞ , can write (1) as

$A(s) = A_0 \cdot \frac{1 + a_1 s + \dots + a_m s^m}{1 + b_1 s + b_2 s^2 + \dots + b_n s^n}$ (2)

(other choices of $T(s)$ are possible if we allow $a(s)$ has poles)

... then:

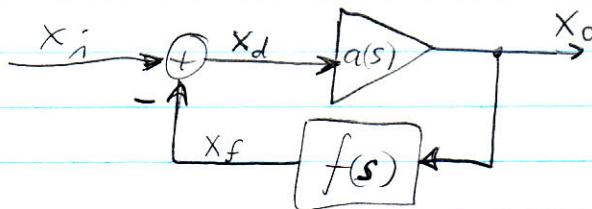
$\therefore A(s) = \frac{a(s)}{1 + a(s) \cdot f(s)}$

where

$a(s) \cdot f(s) = T(s)$

$a(s) = A_0 (1 + a_1 s + \dots + a_m s^m)$

$\therefore$ (2) can be impl. as:



Def.: A system is stable if (and only if) it satisfies the following property:

For each bounded input signal, $x_i(t)$, the output signal, $x_o(t)$, must also be bounded

($x(t)$ is bounded means $\exists B \in \mathbb{R}$ st. $|x(t)| < B \forall t$)

$\therefore$ "Stability" $\equiv$ "bounded input, bounded output, stability"

"BIBO"

"Such that"

April 18, 2007

ECE 171A

Bounded input: $R(s) = \frac{A_R(s)}{B_R(s)}$

... hvor $B_R(s)$ sine røtter har negativ realdel eller, om de er på den vertikale akse, multiplisitet maks 1 (ellers får vi terms med rampe-form)

$\Rightarrow$ Stabilt sys. må ha neg. realdel på polene for å garantere bounded output.

2/4

Feb. 19, 2008

Claim 1 Let $h(t)$ be the impulse response of an LTI system and let $H(s)$ be its transfer function. Then the system is stable iff either of the following hold:

- i) All poles of $H(s)$ are in the LHP (not including imag. axis) (ie. if s_0 is a pole then $\text{Re}\{s_0\} < 0$)
- ii) $\int_{-\infty}^{\infty} h(\tau) d\tau < \infty$

iff $\equiv$
"if and only if"

proof Exercise (review material)

Claim 2 If an LTI has a transfer function given by (2), then it is stable iff

$$T(s_0) = -1 \Rightarrow \text{Re}\{s_0\} < 0$$

proof = Exercise.

Def. "Nyquist plot" $\equiv$ plot of $\text{Im}\{T(j\omega)\}$ vs. $\text{Re}\{T(j\omega)\}$ as ω increases from $-\infty$ to ∞

Nyquist Criterion:

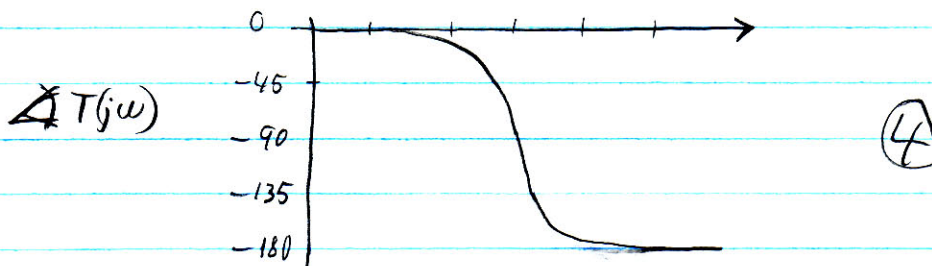
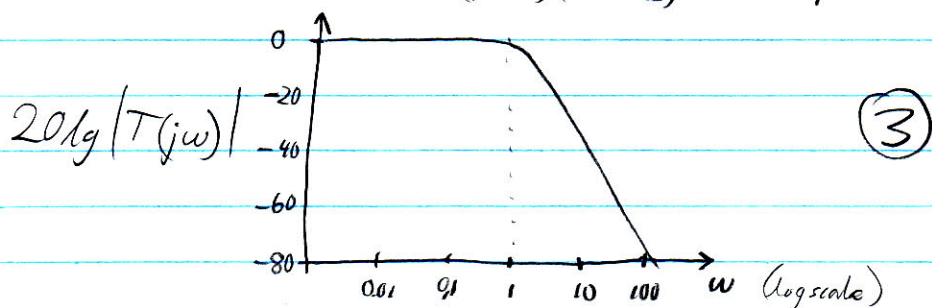
Provided there are no RHP pole-zero cancellations in $T(s) = a(s) \cdot f(s)$ an LTI system is stable iff the net no. of CCW encirclements of the point $(-1, 0)$ by the Nyquist plot equals the no. of RHP poles in $T(s)$

$\therefore$ Nyquist crit. $\Rightarrow$ stability test $\longleftrightarrow$ {not so useful these days (because of computers)}
will soon see: Nyquist crit. $\Rightarrow$ insight $\Rightarrow$ concepts of phase & gain margin } very useful

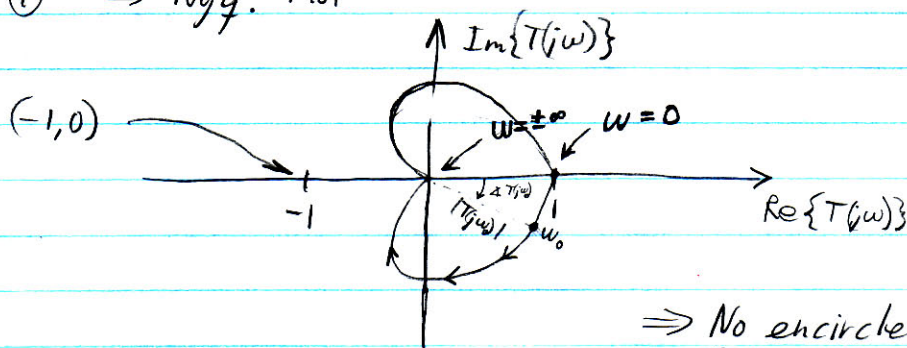
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Feb. 19, 2008

Ex 1 Suppose $T(s) = \frac{1}{(1+s)(1+s/2)}$ poles: $-1, -2$



③, ④ $\Rightarrow$ Nyq. Plot



$\Rightarrow$ No encirclements of $(-1, 0)$
($T(s)$ has no RHP poles by inspection)

Nyq. crit $\Rightarrow$ system with t.f. $A(s) = \frac{a(s)}{1+T(s)}$
is stable (provided there are no pole-zero cancellations in $a(s)f(s)$) (why?)

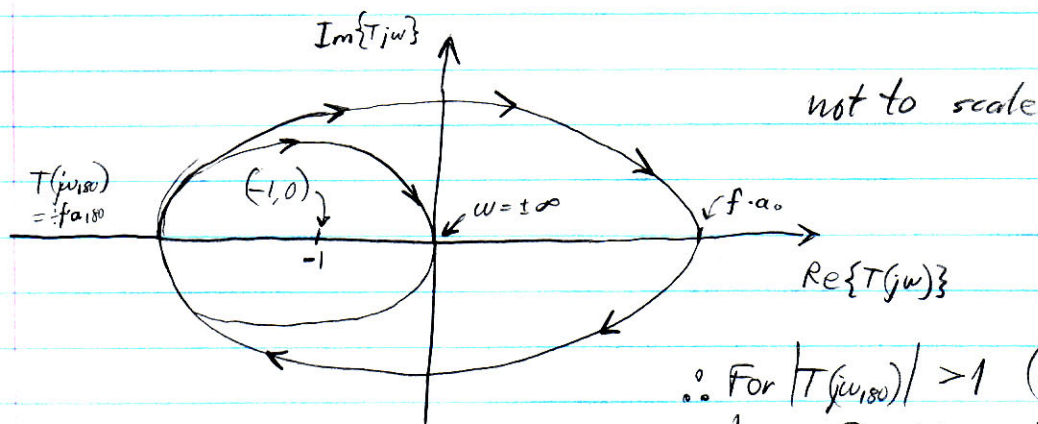
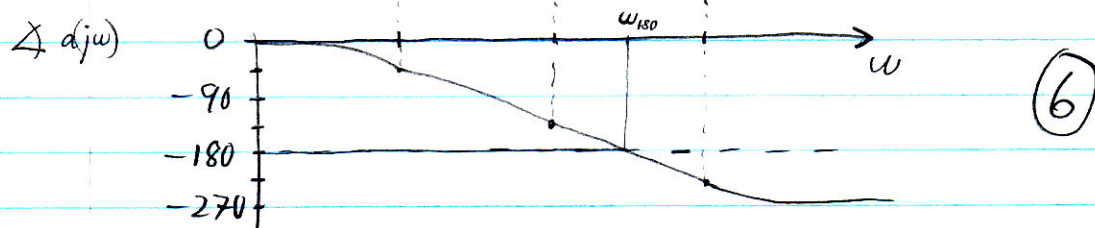
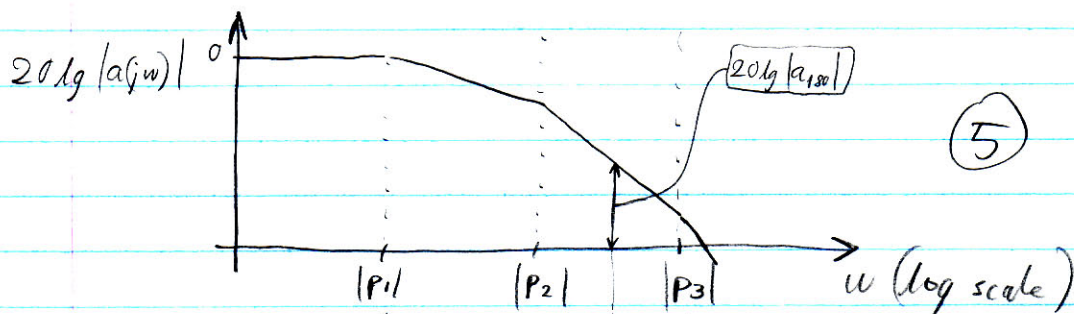
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Feb. 19, 2008

Ex 2 Suppose $a(s) = \frac{a_0}{(1-s/p_1)(1-s/p_2)(1-s/p_3)}$ and $f = \text{const.}$

$$\therefore T(j\omega) = f \cdot a(j\omega)$$

For $|p_1| \approx \frac{1}{10} \cdot |p_2| \approx \frac{1}{100} |p_3|$ and $\text{Re}\{p_i\} < 0, i=1,2,3$

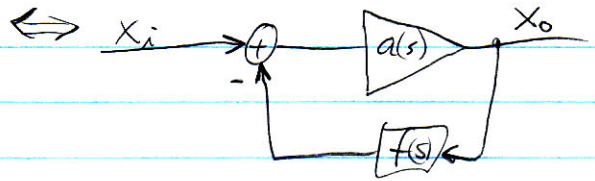


$\therefore$ For $|T(j\omega_{180})| > 1$ (the case shown) have 2 c.w. encirclements, but $T(s)$ has no RHP poles

$\therefore$ Nyquist crit $\Rightarrow$ unstable if $|T(j\omega_{180})| > 1$
 $\Rightarrow$ stable if $|T(j\omega_{180})| < 1$

Nyquist Criterion (cont.)

Let $A(s) = \frac{a(s)}{1 + a(s)f(s)}$



$T(s) = a(s)f(s)$

Assume: No zero of $f(s)$ is a pole of $a(s)$

Then $T(s_0) = -1 \Leftrightarrow s_0$ is a pole of $A(s)$

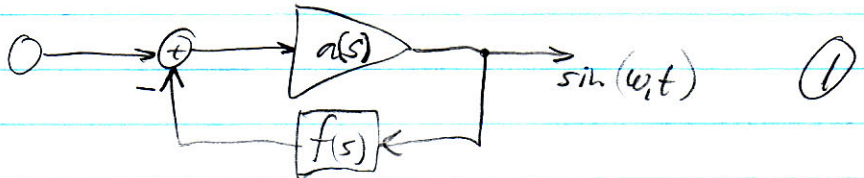
Ex. 1 Suppose $T(j\omega_1) = -1$ for some $\omega_1 \in \mathbb{R}$

$\Rightarrow A(s)$ has pole on imag. axis $\Rightarrow$ oscillation

$\Rightarrow$ unstable

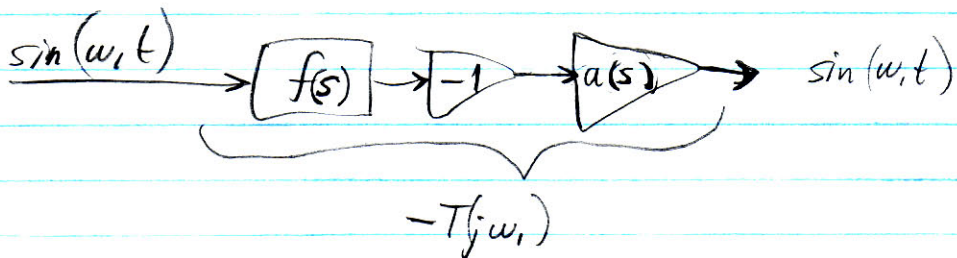
$\therefore$ can choose initial cond. s.t.

s.t. =
"such that"



occurs.

This happens because ① is equiv. to



$T(j\omega_1) = -1 \Rightarrow |T(j\omega_1)| = 1$

and

$\angle T(j\omega_1) = \pi$

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Feb. 21, 2008

E.G.

Ex 2 Suppose $T(s_0) = -1$ for $s_0 = \sigma_0 + j\omega$, $\sigma_0 \geq 0$
 $\Rightarrow$ have ① with $\sin(\omega, t)$ replaced by $\underbrace{e^{\sigma_0 t}}_{\rightarrow \infty \text{ as } t \rightarrow \infty} \cdot e^{j\omega t}$
 $\Rightarrow$ ① input $\Rightarrow$ unbounded output

Ex 3 Suppose $A(s)$ is stable, but for some $\omega_1 \in \mathbb{R}$,
slang

$$|T(j\omega_1)| = 1 - |\varepsilon| \text{ where } |\varepsilon| = \text{small \#}$$

and $\angle T(j\omega_1) = \pi$

Then have ringing for any input step
e.g.: $x_i = \text{step}$ $x_o = \text{ringing}$

In many systems, ringing is undesirable because it slows the settling time.

$\Rightarrow$ We need to design with "gain margin" s.t. $|T(j\omega)| \neq 1$
when $\angle T(j\omega) = \pi$ with sufficient "margin" to minimize ringing.

Nyg. crit. = method of evaluating "marginal stability"
using simulated or calculated freq. response plots

Recall Nyq. Plot $\equiv$ Plot of $\text{Im}\{T(j\omega)\}$ vs. $\text{Re}\{T(j\omega)\}$ as ω
increases from $-\infty$ to ∞

Nyg. Crit $\equiv$ An LTI system is stable iff the net
number of CCW encirclements of $(-1, 0)$
by the Nyq. plot equals the number of
RHP poles of $T(s)$.

3/5

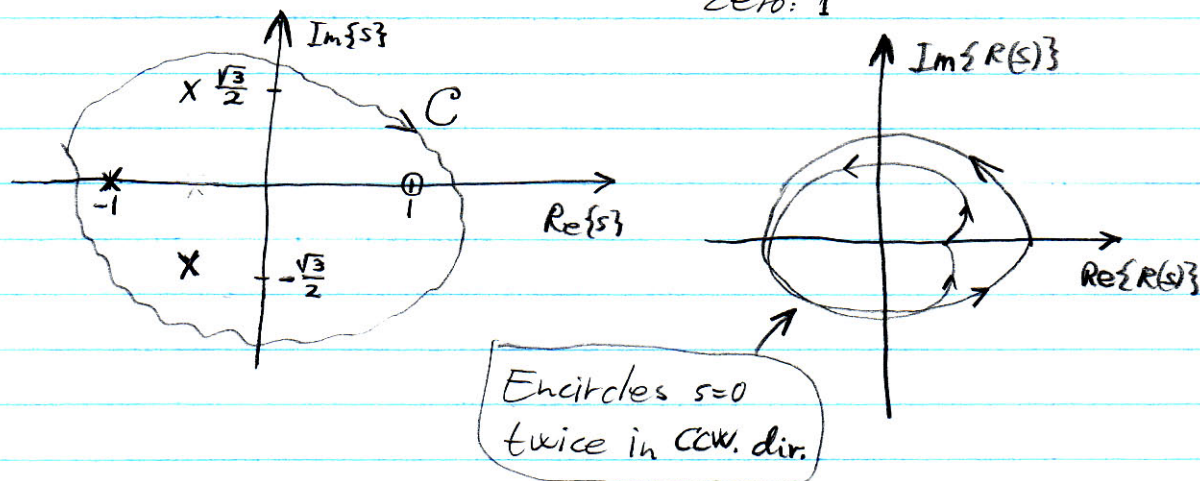
Feb. 21, 2008

E.G.

The Nyquist crit. is based on The Encirclement Property (E.P.).

E.P.: Let $R(s)$ be any rational fn. The plot of $R(s)$ along a closed CW path, C , in the complex s -plane encircles the point $s=0$ in a clockwise direction a net no. of times equal to the no. of zeros $\div$ no. of poles within the contour.

Ex. 4 Suppose $R(s) = \frac{s-1}{(s+1)(s^2+s+1)}$ poles: $-1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j$
Zero: 1



Why does this work?

Ex. 5 Suppose $R(s)$ has only 2 (RHP) zeros and no poles

$(V_1, V_2 \in \mathbb{R}, > 0)$
 $\therefore R(s) = V_1 V_2 e^{j(\theta_1 + \theta_2)}$

As C is traversed: $\begin{cases} \text{net change in } \theta_1 \text{ is } 0 \\ \text{net change in } \theta_2 \text{ is } -2\pi \end{cases}$

$\therefore \angle R(s) = \theta_1 + \theta_2$

$\therefore$ As C is traversed, the net change in $\angle R(s)$ is -2π
 $\Rightarrow R(s)$ circles origin once (in cw dir.)

Feb. 21, 2008

E.G.

Proof of Nyq. crit.

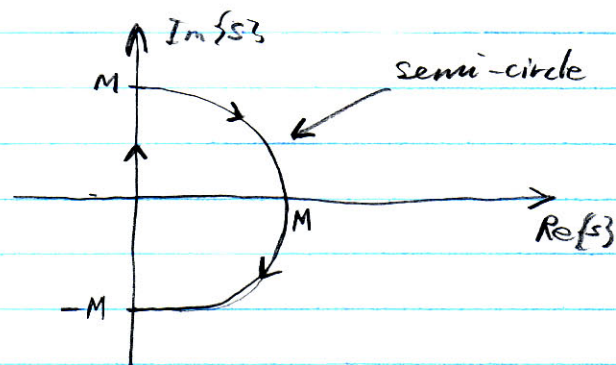
Restrictions of proof:

- 1) Rational $T(s)$
- 2) $|T(j\omega)| < \infty \quad \forall \omega \in \mathbb{R}$
- 3) $T(s)$ has $(\# \text{poles}) \geq (\# \text{of zeros})$

(But Nyq. crit also applies without restrictions 1) & 3) and can be modified to handle poles on the imag axis.)

Consider contour C as follows:

As $M \rightarrow \infty$, C contains all RHP poles of $A(s)$



$$1), 3) \Rightarrow T(s) = \frac{c_0 + c_1 s + \dots + c_{N_1} s^{N_1}}{d_0 + d_1 s + \dots + d_{N_2} s^{N_2}}$$

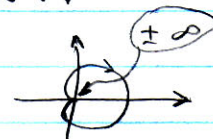
Where $N_1 \leq N_2$

$$\therefore \lim_{|s| \rightarrow \infty} T(s) = \begin{cases} \frac{c_{N_1}}{d_{N_1}} & \text{if } N_1 = N_2 \\ 0 & \text{if } N_1 < N_2 \end{cases} = \text{const} (\in \mathbb{R})$$

$\Rightarrow$ The plot of $T(s)$ along the semi-circular part of C approaches a single point on real axis as $M \rightarrow \infty$

$\Rightarrow$ Nyq. plot is equiv. to plot of $T(s)$ along C as $M \rightarrow \infty$

e.g., recall: Nyq. plot of $T(s) = \frac{1}{(1+s)(1+s/2)}$ is



$\Rightarrow$ Can apply encirclement property to Nyq. plot

Let $R(s) = 1 + T(s)$. Then plot of $R(s)$ along C encircles the origin exactly as many times as that of $T(s)$ encircles $(-1, 0)$.

$\therefore$ E.P. $\Rightarrow$ net no. of cw. encirclements of $(-1, 0)$
 $= \# \text{ of zeros of } (1+T(s)) \text{ minus } \# \text{ of poles of } 1+T(s) \text{ inside } C.$

5/5

Feb. 21, 2008 E.G.

But:

- zeros of $1+T(s) \equiv$ poles of $A(s)$
- poles of $1+T(s) \equiv$ poles of $T(s)$
- inside $C =$ all RHP as $M \rightarrow \infty$

$\Rightarrow$ Nyq. crit.



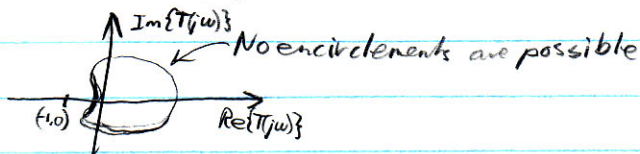
Important special case of Nyquist criterion:

Suppose $\angle T(0) = 0$ and $T(s)$ has no RHP poles

If $\angle T(j\omega) = \pi$ for only one pos. value, $\omega = \omega_\pi$,

Then $A(s)$ is stable iff $|T(j\omega_\pi)| < 1$

eg. picture:



Note:

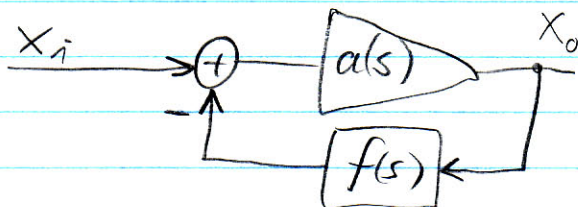
In general, $|T(j\omega_\pi)| > 1 \Rightarrow$ instability(!)

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ECE 264A

Feb. 26, 2008

E.G.

OH:
1~2³⁰ this
Friday (2/29)Gain & Phase Margin

$$A(s) = \frac{a(s)}{1 + \frac{a(s) \cdot f(s)}{T(s)}}$$

Assume no pole of $a(s)$ is a zero of $f(s)$.

Nyg. crit $\Rightarrow$ 2 special cases:

i) Suppose $\angle T(0) = 0$, $T(s)$ has no RHP poles and $|\angle T(j\omega)| = \pi$ has no more than one positive solution, $\omega = \omega_\pi$

Then $A(s)$ is stable iff $|T(j\omega_\pi)| < 1$

ii) Suppose $\angle T(0) = 0$, $|T(0)| > 1$, $T(s)$ has no RHP poles, and $|T(j\omega)| = 1$ has no more than one positive solution, $\omega = \omega_u$

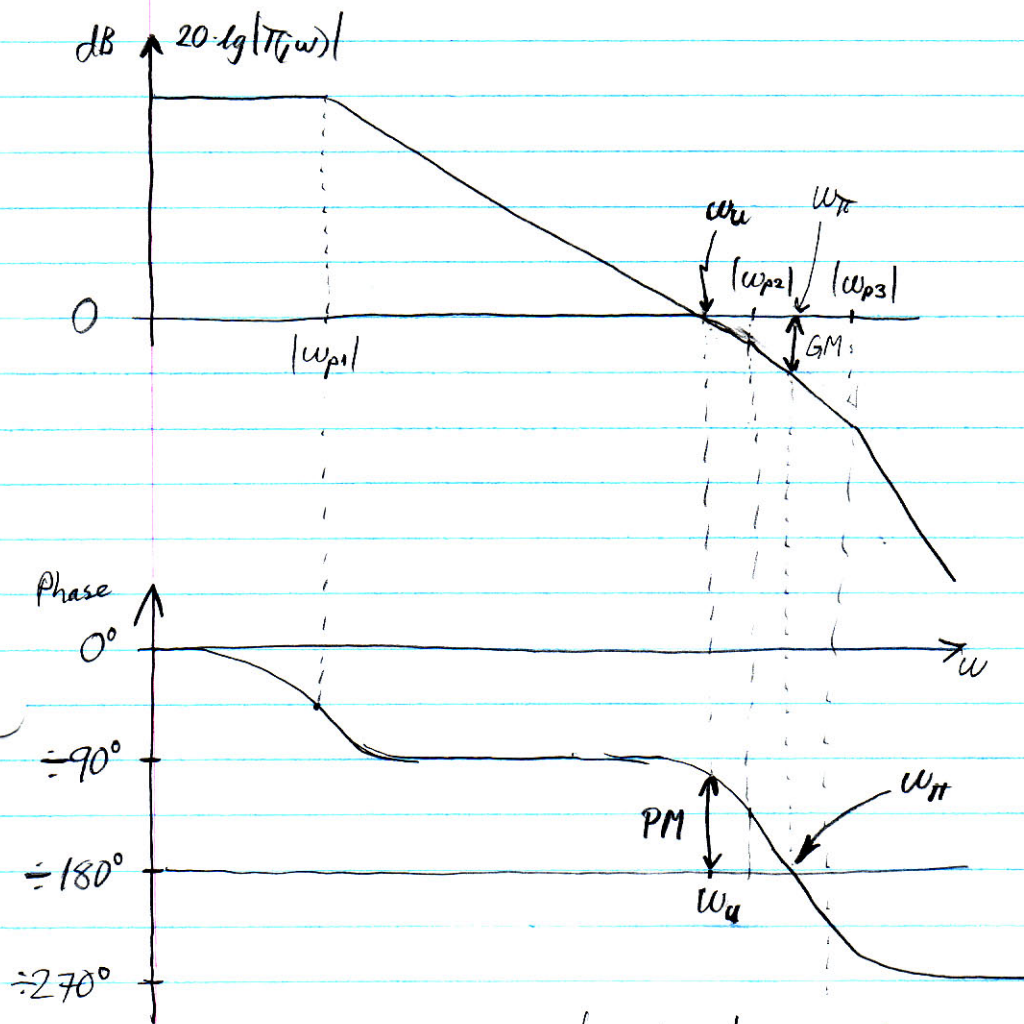
Then $A(s)$ is stable iff $-\pi < \angle T(j\omega_u) < \pi$

i) $\Rightarrow$ Basis of "Gain margin" (GM) definition (soon)
ii) $\Rightarrow$ Basis of "Phase margin" (PM) definition (soon)

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Def. of PM & GM (via 3-pole T(s) example)



ie. $GM \equiv 20 \lg \left| \frac{1}{T(j\omega_u)} \right|$ (pos. as shown)

$PM \equiv 180^\circ - |\angle T(j\omega_u)|$ (pos. as shown)

3/7

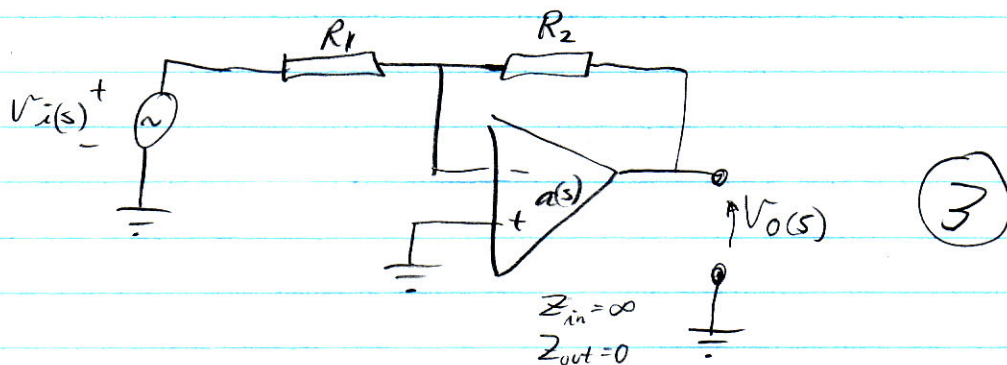
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∴ ①, $GM > 0 \Leftrightarrow$ stable (closed loop)

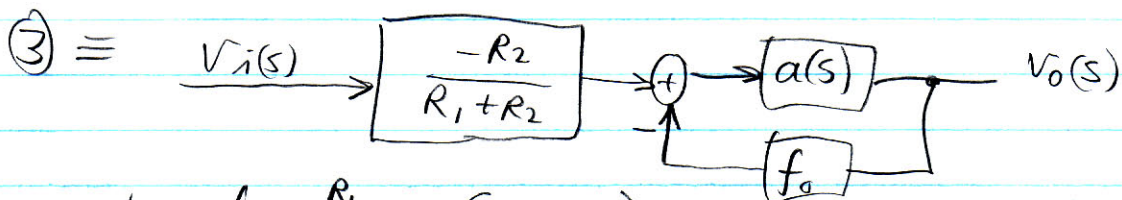
②, $PM > 0 \Leftrightarrow$ stable (closed loop)

Also, larger GM (PM) $\Leftrightarrow$ better marginal stability
(i.e. step response has less ringing; can tolerate larger component errors without instab.)

Consider:



Previously found:



where $f_o = \frac{R_1}{R_1 + R_2}$ ($0 < f_o < 1$)

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Ex 1 (3) with $a(s) = \frac{a_0}{1 - s/\omega_{pa}}$, $\omega_{pa} < 0$ ($\in \mathbb{R}$)
 $a_0 > 0$ ($\in \mathbb{R}$)

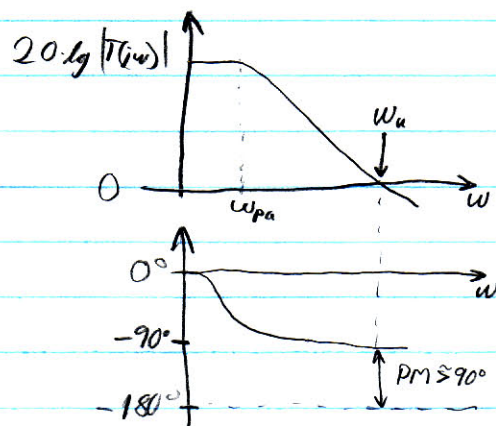
$$\therefore T(j\omega) = \frac{a_0 \cdot f_0}{1 - j\omega/\omega_{pa}}$$

Depending on $a_0 f_0$

$$90^\circ < PM < 180^\circ$$

$$\text{e.g. } \lim_{a_0 f_0 \rightarrow 1} PM = 180^\circ$$

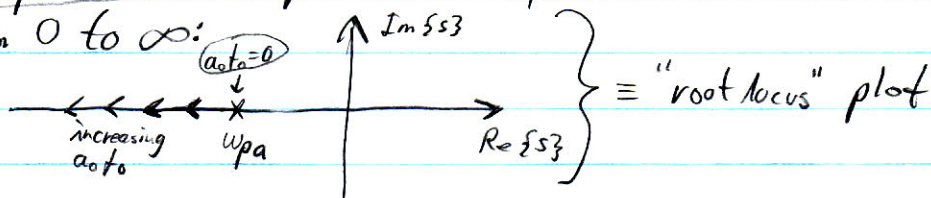
$$\lim_{a_0 f_0 \rightarrow \infty} PM = 90^\circ$$



Feb. 14

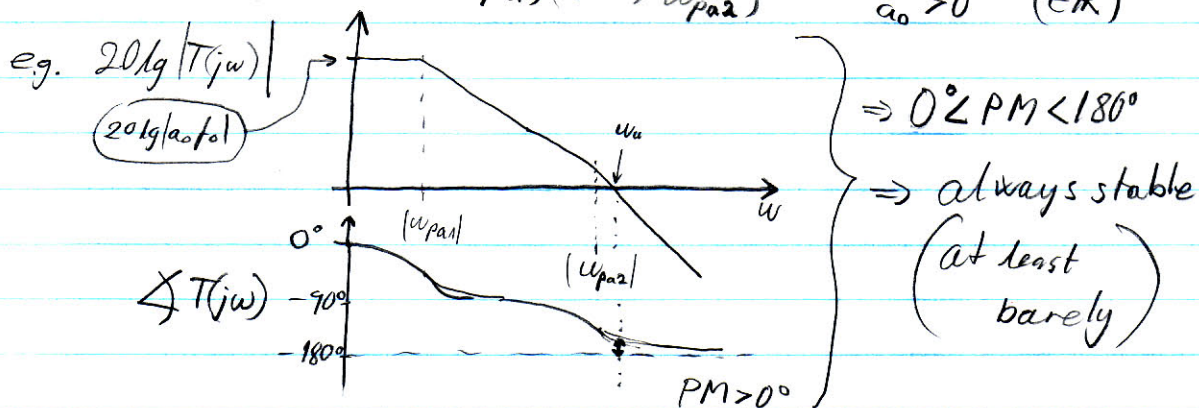
Recall: $A(s) \equiv \frac{V_o(s)}{V_i(s)} = A_0 \cdot \frac{1}{1 - s/\omega_{pa}}$ where $A_0 \in \mathbb{R}$
 $\omega_{pa} = (1 + a_0 f_0) \cdot \omega_{pa}$

Plot of ω_{pa} position in s -plane as $a_0 f_0$ increases from 0 to ∞ :



$\therefore$ as expected, it's stable for any choice of $a_0 f_0 > 0$

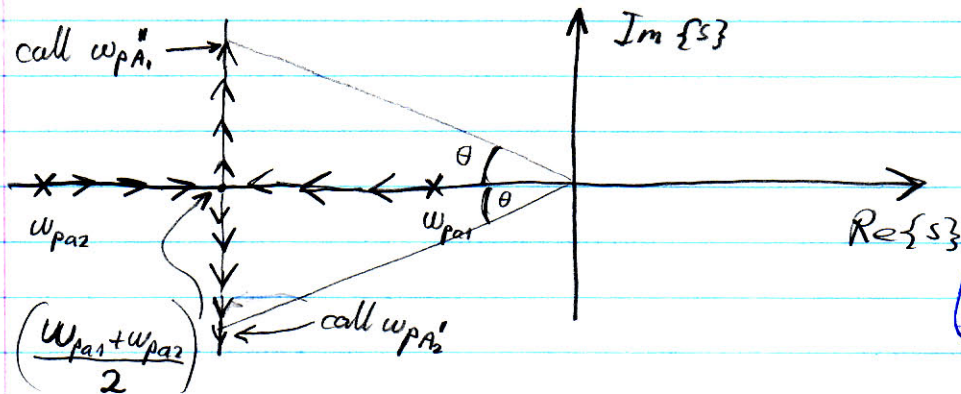
Ex 2 (3) with $a(s) = \frac{a_0}{(1 - s/\omega_{pa1})(1 - s/\omega_{pa2})}$, $\omega_{pa1} < 0$ ($\in \mathbb{R}$) $i=1,2$
 $a_0 > 0$ ($\in \mathbb{R}$)



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Root Locus plot (verify) (pos. of ω_{PA_i})



$$\frac{a(s)}{1+f \cdot a(s)} = \frac{a_0 \cdot \omega_{pa1} \cdot \omega_{pa2}}{s^2 - (\omega_{pa1} + \omega_{pa2}) \cdot s + (1+a_0 \cdot f) \cdot \omega_{pa1} \omega_{pa2}}$$

Recall $\theta = \cos^{-1} \zeta$ where ζ = damping ratio

$\therefore \zeta = \cos \theta$, so

$\zeta \rightarrow 0$ as $\theta \rightarrow 90^\circ$

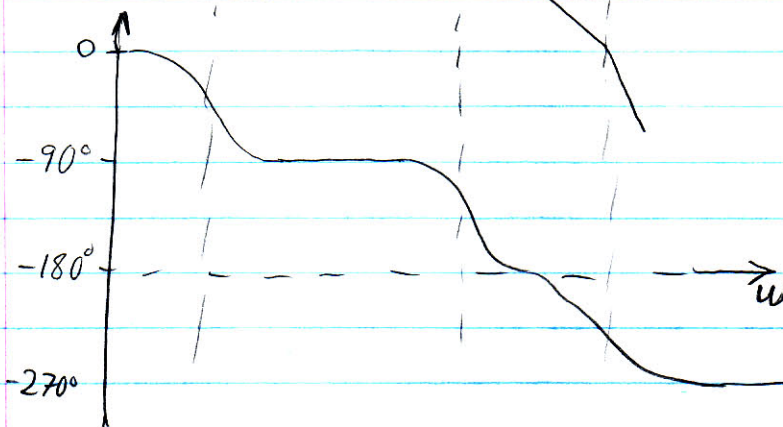
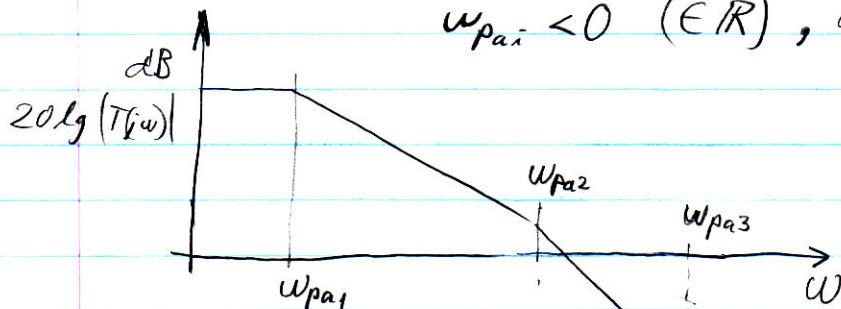
(i.e. as $a_0 f \rightarrow \infty$)

(i.e. as $PM \rightarrow 0$)

$\therefore$ Smaller $PM \Rightarrow$ more ringing.

Ex. 3 ③ with $a(s) = \frac{a_0}{(1-s/\omega_{pa1})(1-s/\omega_{pa2})(1-s/\omega_{pa3})}$

$\omega_{pa_i} < 0$ ($\in \mathbb{R}$), $a_0 > 0$ ($\in \mathbb{R}$)



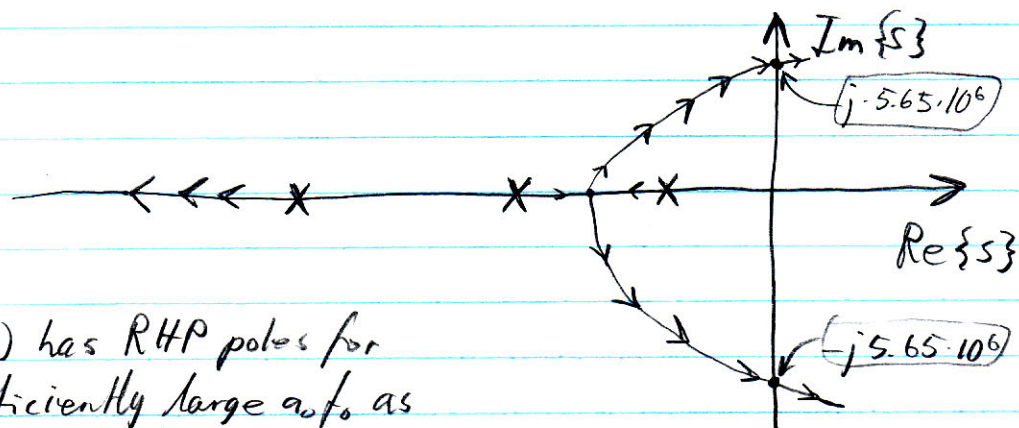
$\Rightarrow -90^\circ < PM < 180^\circ$

$\Rightarrow$ can be unstable

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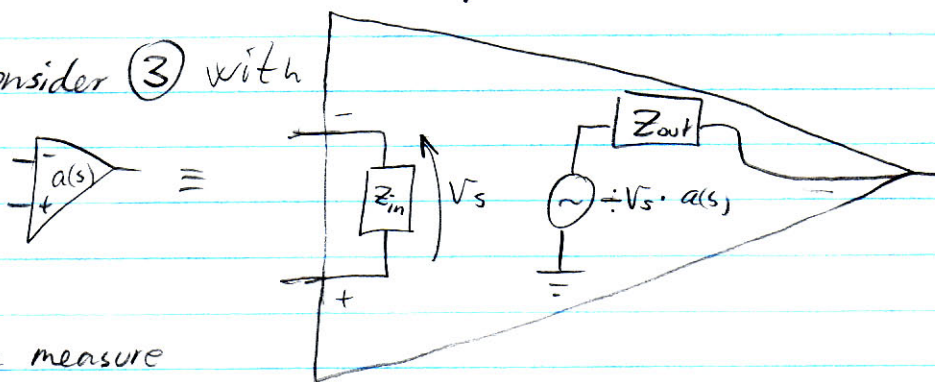
e.g. Suppose $\omega_{p1} = -10^6 \frac{\text{rad}}{\text{sec}}$, $\omega_{p2} = -2 \cdot 10^6 \frac{\text{rad}}{\text{sec}}$, $\omega_{p3} = -10^7 \frac{\text{rad}}{\text{sec}}$
Then the root locus plot is as follows:



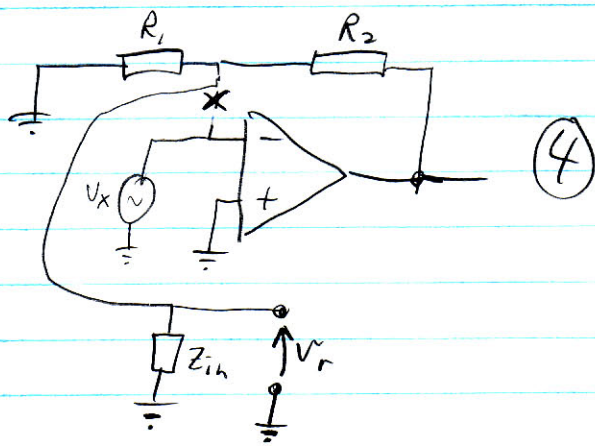
$\therefore A(s)$ has RHP poles for sufficiently large ω_0 as expected.

Using SPICE to estimate $T(j\omega)$

Ex4 Consider (3) with



Suppose we measure $T(j\omega)$ using:



have:

- 1) Broken feedback loop
- 2) Applied test source after break
- 3) Loaded node prior to break as if it were not broken

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First suppose $Z_{in} = \infty$, $Z_{out} = 0$

$$\text{Inspection} \Rightarrow \frac{v_r}{v_x} = -a(s) \cdot \frac{R_1}{R_1 + R_2} = -a(s) \cdot f_o = -T(s)$$

$$\therefore T(s) = -\frac{v_r(s)}{v_x(s)}$$

$\therefore$ can use this approach to "measure" $T(j\omega)$
Often works, but must be careful in general.

Suppose $Z_{in} \neq \infty$, $Z_{out} \neq 0$

$$\text{AGR} \Rightarrow A(s) = A_{\infty} \cdot \frac{T}{1+T} + A_0 \cdot \frac{1}{1+T}$$

Before $A_0 = 0$ because $Z_{out} = 0$
Now, $A_0 \neq 0$

We still get $T(j\omega)$ using (4) but analysis will miss poles contributed by $A_0(s)$

$$N_{CW, (0,0)}^{(1+T)} = Z_{(1+T)} - P_{(1+T)} =$$

$$= P_{cl.l.} - P_T = N_{CW, (-1,0)}^T$$

$$N_{CCW, (-1,0)}^T = P_T - P_{cl.l.}$$

RHP poles & RHP

Zeros in general

(map from s to
either $T(s)$ or
to $(1+T(s))$).

SEC:



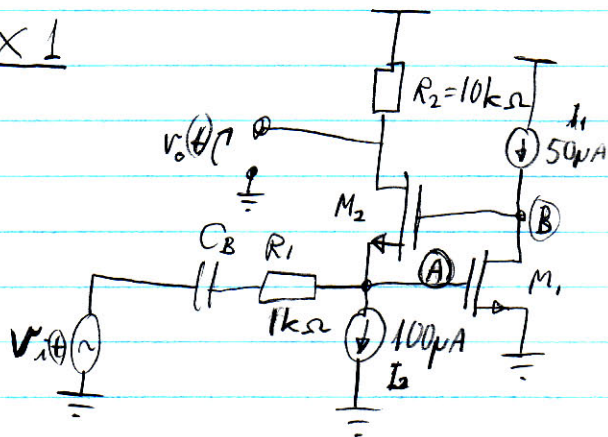
For stability, $P_{cl.l.} = 0$

$$\Rightarrow N_{CCW, (-1,0)}^T = P_T$$

$$N_{CCW, (-1,0)}^T = \text{"\# RHP poles, open loop"} \div \text{"\# RHP poles, closed loop"}$$

Using SPICE to estimate $T(j\omega)$

Ex 1



(1)

$M_1: \frac{W}{L} = 15 \mu\text{m}/1 \mu\text{m}$
 $M_2: \frac{W}{L} = 20 \mu\text{m}/1 \mu\text{m}$
 $C_B = \text{large DC-blocking cap (assume } C_B \approx \infty)$

Observations

- (i) (A) is a low impedance node:
 - inside the loop bandwidth $\Rightarrow$ (A) = almost virtual ground (H.W)
 - even with feedback disabled, impedance of (A) $\approx 1k\Omega \parallel \left(\frac{1}{g_{m2}}\right)$
- (ii) Similar reasoning $\Rightarrow$ driving point resistance between nodes (A) and (B) is relatively small ($\approx 1/g_{m1}$)

(ii) $\Rightarrow$ Reasonable to neglect C_{gd1} , C_{gs2} (2)

Apply A.G.R. with $g_{m1} \cdot v_{gs1} \equiv \text{ref. source}$

$$\therefore A(s) = A_{\infty}(s) \cdot \frac{T(s)}{1+T(s)} + \underbrace{A_0(s)}_{\text{gain of C.G. amplifier formed by } M_2 \text{ (but with high impedance gate connection)}} \cdot \frac{1}{1+T(s)}$$

$\frac{1}{g_{m2}} \approx 1k\Omega$, so $A_0(0) \approx 5$

The point is: $T(s)$ does not give the whole stability "picture" because $A_0(s)$ might have marginal stability.

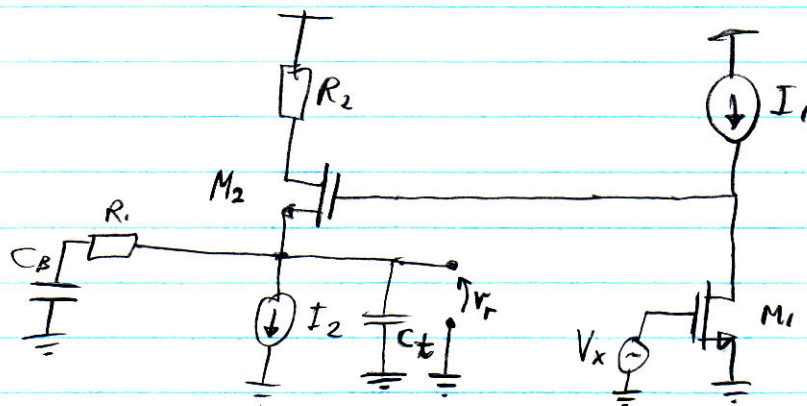
(2/5)

Feb. 28, 2008

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- Good approach
- 1) Measure $T(s)$ to estimate the degree of stab.
 - 2) If simulations show more ringing than expected, consider $A_o(s)$

② $\Rightarrow$ Can "measure" $T(j\omega)$ using:



③

where
 $C_t \equiv C_{gs1}$ (termination impedance)
DC-value of V_x
= DC-value of node (A) in ①

$\Rightarrow$

have "broken" feedback loop and applied test signal

$$\textcircled{3} \Rightarrow T(j\omega) = - \frac{v_r(j\omega)}{v_x(j\omega)}$$

$\rightarrow$ (same $T(j\omega)$ as in A.G.R. if ② holds - verify)

$\Rightarrow$

simple test to find PM, GM, or Nyquist plot
simulations $\Rightarrow$ PM $\approx 90^\circ$

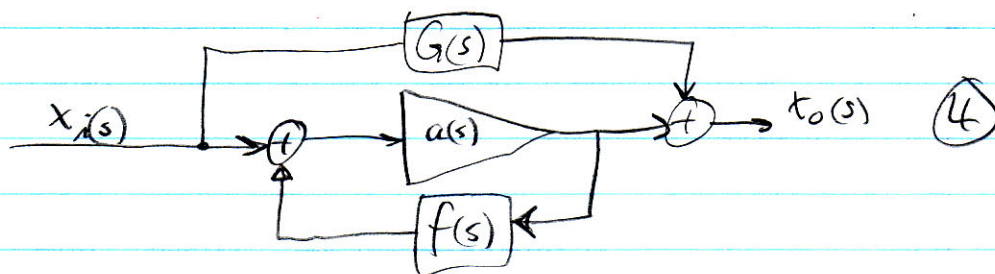
In general, this always works, but must:

- 1) Bias input node following break point as in the closed loop circuit.
- 2) Must terminate the node (prior to breakpoint) as in the closed loop circuit.

HW 6 $\Rightarrow$ Method of avoiding need to terminate output node.

(3/5)

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Theory behind breaking feedback loopsEx

$$H(s) \equiv \frac{x_o(s)}{x_i(s)} = \underbrace{\frac{a(s)}{1+a(s)f(s)}}_{\text{call } A(s)} + G(s)$$

Both $G(s)$ and $A(s)$ } • have poles (in general)
 } • affect stability

In circuits, often } • $A(s) \Rightarrow$ desired behavior (dominant)
 } • $G(s) \Rightarrow$ parasitic signal-path (non-dominant)

$\therefore$ Feedback loop in (4) $\Rightarrow A(s)$

Knowing $A(s) \nRightarrow$ insight into how $a(s)$ or $f(s)$ can be modified to improve performance.

But

Knowing $T(j\omega) \Rightarrow$ " ——— || ——— ...
 ... ——— || ——— "

(via Nyq. crit., etc..)

Can "break" any point of feedback loop in (4) and "measure" $T(j\omega)$ via simulation.

The problem: Usually have a circuit, not a B.D. to analyze. Previously have used A.G.R. for specific circuits to justify measuring $T(j\omega)$ directly from circuit. (i.e. without first conv. to a B.D.)

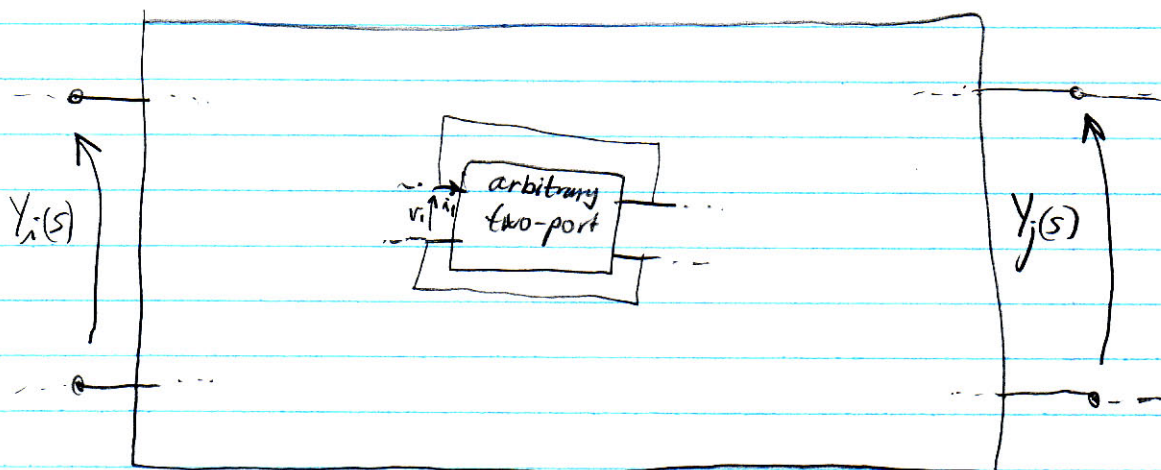
Does this work in general?

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Arbitrary LTI circuit containing a feedback loop



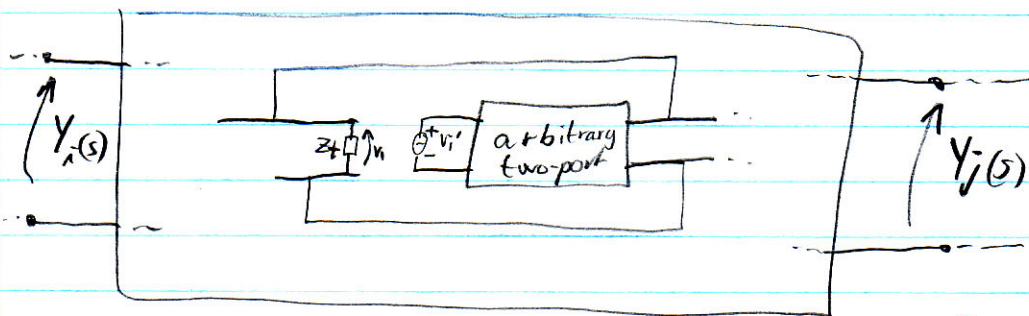
(5)

(Y is voltage, but could be current!)

$$\text{Let } H(s) = \frac{Y_j(s)}{Y_i(s)}$$

Note: Can draw (5) s.t. $Y_i(s)$ = current and/or $Y_j(s)$ = current

Can redraw (5) as:



(6)

where $v_i' = \alpha \cdot v_i$, with $\alpha \equiv 1$, $Z_f \equiv \frac{v_i(s)}{i_i(s)}$

(input imp. of two-port depends on loading of $Y_i(s)$ & $Y_j(s)$ ports)

Now, suppose $v_i'(s)$ replaced by independent source, v_x
LTI-system $\Rightarrow v_i = C(s) \cdot Y_i(s) + D(s) \cdot v_x(s)$

Feedback loop "enabled" $\Leftrightarrow D(s) \neq 0$ (e.g. $v_i \equiv y_i \Rightarrow \textcircled{7}$ violated)

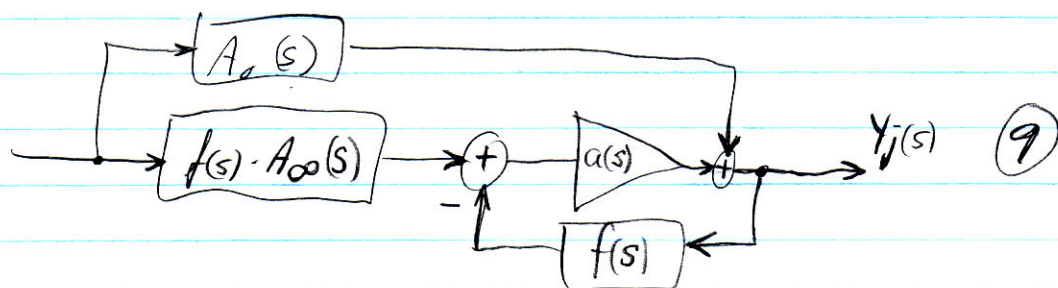
Provided ⑦ holds, can apply A.G.R.

Recall A.G.R. $\Rightarrow H(s) = A_{\infty}(s) \cdot \frac{T(s)}{1+T(s)} + A_0(s) \cdot \frac{1}{1+T(s)}$ (8)

where $T(s) = -\frac{v_i(s)}{v_x(s)}$ for $y_i(s) = 0$ and $v_i'(s)$ replaced by $v_x(s)$.

$$A_0(s) = y_i(s)/y_{i0}(s) \text{ with } x=0, A_\infty = \lim_{x \rightarrow \infty} \frac{y_i(s)}{y_{i0}(s)}$$

∴ Block Diagram of ⑥:



Where $a(s)$, $f(s)$ are any functions s.t. $a(s) \cdot f(s) = T(s)$

Observations

- 1) $T(j\omega)$ in ⑧ same as that found by breaking feedback loop in ⑤, term. the break with $Z_t(s)$, and injecting v_x (or i_x) at input of broken loop
 $\Rightarrow$ Method always works provided ⑦ holds
- 2) Can use Nyq. crit. (or P.M., G.M.) to assess relative stability of F.B. loop in ⑧ (If F.B. loop has marginal stab. then ⑨ (ie ⑤) "—————" "—————")

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Note: For the version of the Nyquist Plot and Criterion presented previously, must have at least as many poles as zeros.

(More ^(than poles) zeros $\Rightarrow$ inf. B.W. !)

Compensation

The problem: Given an amplifier and feedback network, how do we adjust or add components to achieve a desired stability margin?

Most often $T(s)$ satisfies:

- (i) $|\angle T(0)| < \pi$ (neg. feedback at DC)
- (ii) $T(s)$ has no RHP (incl. $j\omega$ axis) poles
- (iii) $|T(j\omega)| = 1$ has one pos. solution, $\omega = \omega_u$

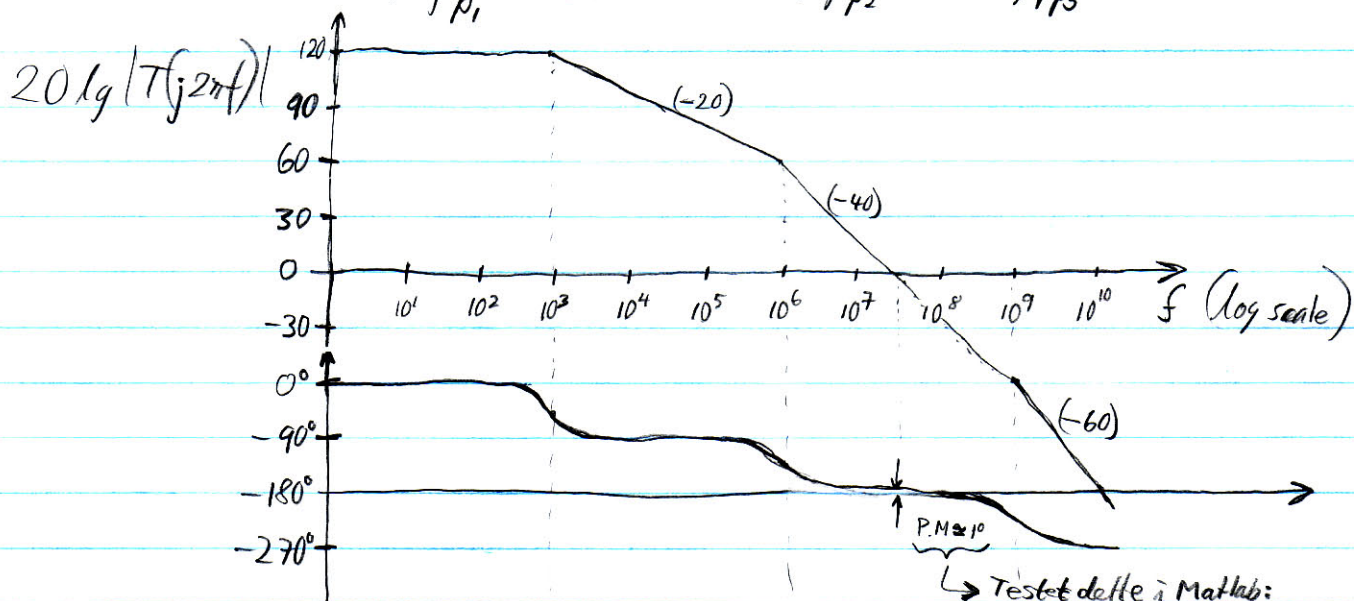
①

① $\Rightarrow$ PM, GM valid indicators of stab. margin.

Ex 1: $T(s) = T_0 \cdot \frac{1}{(1-s/\omega_{p1})(1-s/\omega_{p2})(1-s/\omega_{p3})}$ ②

with

$$T_0 = 10^6, f_{p1} = \frac{\omega_{p1}}{2\pi} = 10^3 \text{ Hz}, f_{p2} = 10^6 \text{ Hz}, f_{p3} = 10^9 \text{ Hz}$$



Testet dette i Matlab:
Fikk 0.004°

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Observations

- i) Both $\angle T(j\omega)$ and $|T(j\omega)|$ decrease monotonically
- ii) $PM \approx 45^\circ$ requires $|f_{p2}| > f_u$

$\therefore$ Only 3 ways (compensation methods) to increase P.M.
given $T(s)$ has the form of (2)

- 1) Reduce $T_0 \Rightarrow \omega_u$ decreases, but $\angle T(j\omega)$ unchanged
 $\Rightarrow$ P.M. increases

But: Trades C.L. accuracy for P.M.

(feedback benefits requires large T_0)

- 2) Reduce $|f_{p1}|$

$\Rightarrow \omega_u$ decreases, but $\angle T(j\omega)$ almost

unchanged for $\approx 4 \cdot |f_{p1}|$ (2 octaves = 0.6 decades)

$\Rightarrow$ P.M. increases

But: Trades C.L. BW for P.M.

- 3) Increase $|f_{p2}|$

$\Rightarrow \omega_u$ increases until $|f_{p2}| > f_u$, then remains unchanged
but $\angle T(j\omega_u)$ increases

$\Rightarrow$ P.M. increases (up to 90°)

But: Usually not possible without simultaneously
reducing $|f_{p1}|$.

Fargon:

1) $\equiv$ "gain compensation"

2) $\equiv$ "dom. pole compensation"

2)+3) $\equiv$ "pole splitting compensation" or "miller compensation"

See (3/5)
Jan. 17

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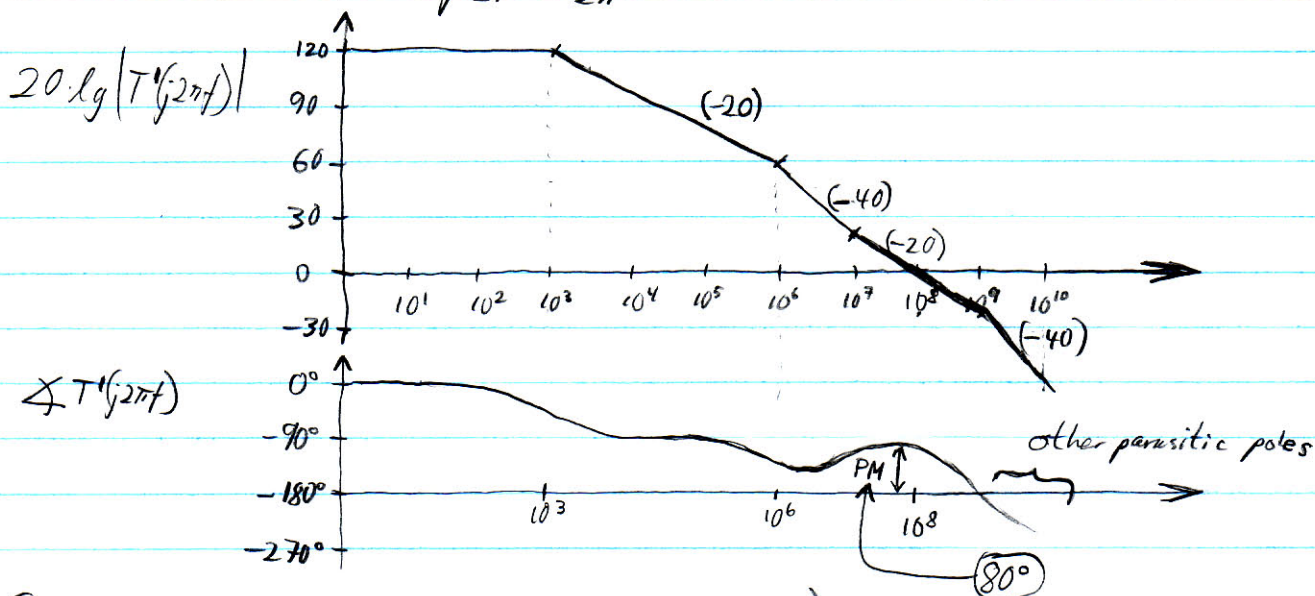
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Ex 2 Suppose we can add LHP zero to $T(s)$ given by ②

e.g. let $T'(s) = \underbrace{T(s)}_{\text{same as in Ex 1.}} \cdot (1 - s/\omega_{z1})$

with $f_{z1} = \frac{\omega_{z1}}{2\pi} = 10^7 \text{ Hz}$



(zero approx at $1.2 \cdot \omega_u \leftarrow$ rule of thumb)

Observations

- i) LHP zero added pos. phase shift without significantly changing f_u (f_u gikk fra $10^{7.5}$ til 10^8 Hz)
 - $\Rightarrow$ Increased PM (fra $\sim 0^\circ$ til 80°)
 - $\Rightarrow$ Adding LHP zero $\equiv$ compensation strategy

- ii) A RHP zero would have decreased PM $\rightarrow$ to $-89.4^\circ \Rightarrow$ unstable!
called "lead compensation"

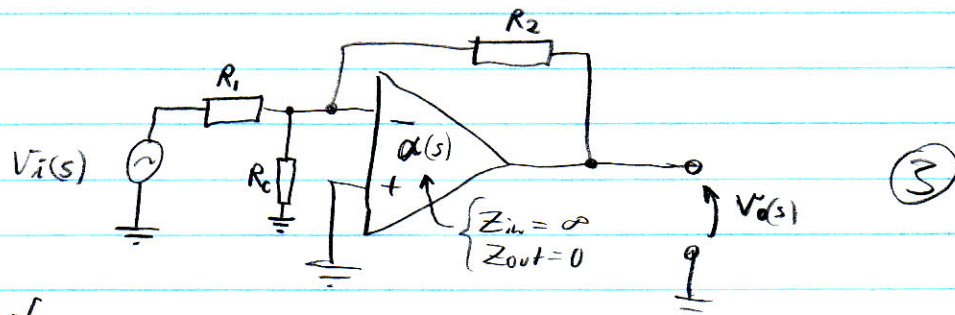
Lead compensation is used in 2-stage CMOS op-amps (soon).

$\rightarrow$ Vil nesten ikke endre ω_u men likevel påvirke fasen til en viss grad. Denne tommelfinger-regelen er ikke brukt her. (Her er $|z| < \omega_u$.)

4/4

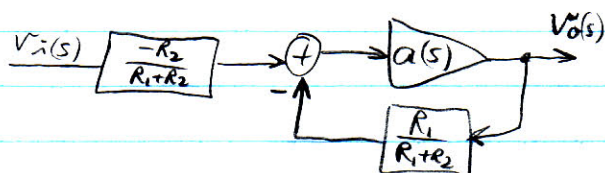
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Ex3 Gain compensation



Can verify:

③ ⇒



Where

$$a(s) = \frac{R_1 \parallel R_2 \parallel R_C}{R_1 \parallel R_2} \cdot \alpha(s)$$

e.g. Suppose $\alpha(s) = 3$ -pole, no-zero transfer fcn.

⇒ $T(s)$ given by ②

⇒ decreasing $R_C \Rightarrow$ smaller T_0 but same $\angle T(j\omega)$ as before

e.g. suppose with $R_C = \infty$, $T(s)$ = same as Ex 1

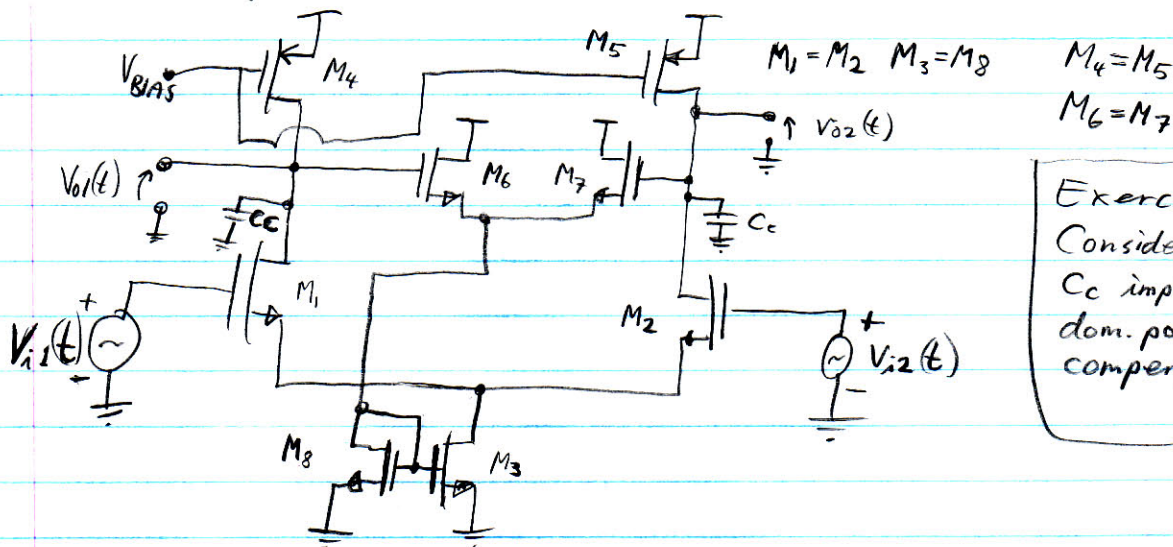
Then $PM \approx 4^\circ$ (very poor relative stability) { Jeg fikk 0.004° PM i Matlab...

e.g. suppose $R_1 = R_2$ and $R_C = \frac{R_1}{1000}$

Then $PM \approx 35^\circ$

But went from $T_0 = 120\text{dB}$ to $T_0 = 66\text{dB}$

Ex4 Dom. pole compensation in CMFB



Exercise:
Consider how C_c implements dom. pole compensation!

1/5

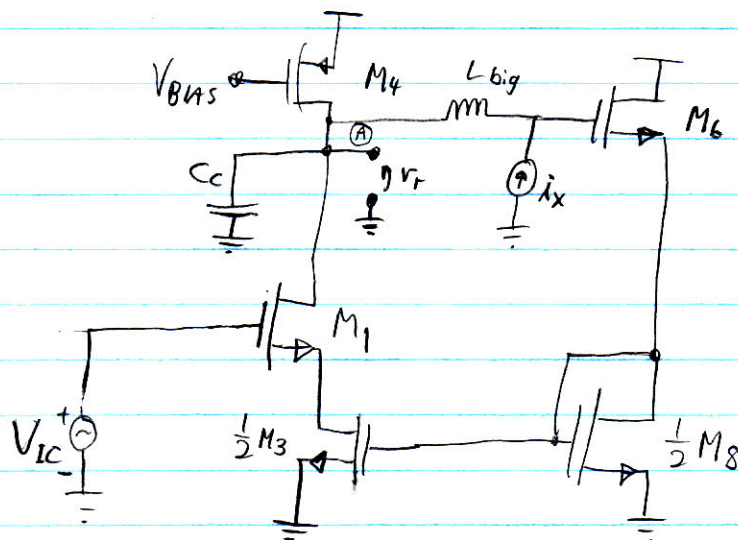
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Ex 4 from last time

(recall diff pair with CMFB diagram)

CM $\frac{1}{2}$ -circuit & T_{Gw} measurement config



($L_{big} \Rightarrow$ zero-freq voltages at two terminals of L_{big} are same, but L_{big} blocks non-zero frequency components)

valid if $C_{gs6} \ll C_c$

Note: trans impedance from node A to all other nodes is small *
 $\Rightarrow \frac{1}{C_c \cdot R_A} = \text{pole of } T(s)$ (C_c has little effect on other poles of $T(s)$)

where $R_A = \text{small-sig. res. from node A to small-sig ground}$

$\Rightarrow A = \text{dom. pole node}$

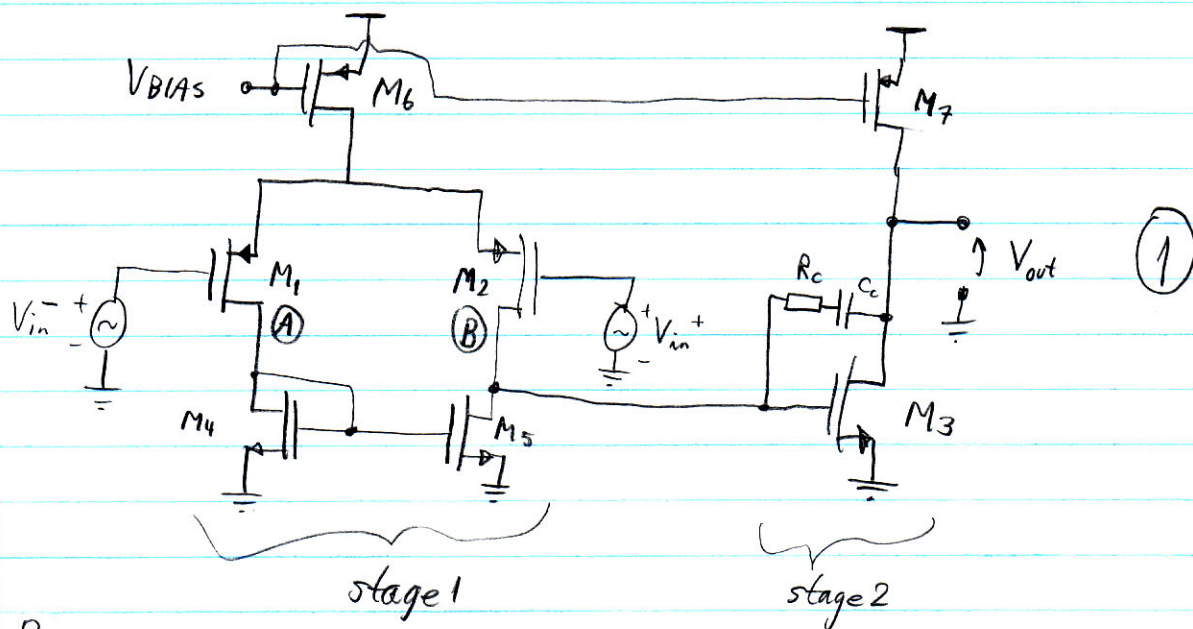
$\Rightarrow \text{can reduce } |f_{p1}| \text{ (only) by increasing } C_c$

Strøm injisert inn i node A vil i liten grad påvirke spenningene på andre noder i kretsen. $\frac{V_{andre}}{I_A} = \text{liten} = Z_{trans.}$

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Ex Two-stage op-amp with lead compensation



Remarks:

1) Typ. designed s.t. pole assoc. with (A)

i.e. $p_A = \frac{-g_{m4}}{C_A}$ ← cap from (A) to small-sig. ground

is s.t. $|p_A| \gg \omega_u$ ← unity-gain freq.

2) A source-follower buffer stage is sometimes added to (1)

3) Could replace pMOSs by nMOSs and vice versa, but pMOSs have better $1/f$ noise performance so (1) as shown has better $1/f$ noise (why?).

(3/5)

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First consider with $R_c = 0$

Using prev. results and Remark 1):

(1) has 2 significant poles, p_1 & p_2 and 1 significant zero, z_1 given by:

$$p_1 \approx -\frac{1}{g_{m3} R_1 R_2 C_c} \quad (2) \quad \text{where} \quad R_1 \equiv r_{ds2} \parallel r_{ds5}$$

$$R_2 \equiv r_{ds3} \parallel r_{ds7}$$

and $C_c \gg C_{gd}$ is assumed

$$p_2 \approx -\frac{g_{m3} C_c}{C_{d3} C_{gs3}' + C_c (C_{d3} + C_{gs3}')} \quad (3)$$

Where C_{d3} = total cap. from output to ground from M_3, M_7 , and any load.

$$\approx -\frac{g_{m3}}{C_{d3}} \quad (\text{often})$$

$$z_1 \approx \frac{g_{m3}}{C_c} \quad (4)$$

 p_1 = dom. pole (sets open loop BW) p_2 = non-dom pole (with p_1 sets open-loop phase shift at unity gain freq.) z_1 = RHP-zero (adds neg. phase shift - just like a LHP pole)

$$\text{e.g. } g_{m3} = 1 \cdot 10^{-3} \Omega^{-1}, R_1 = R_2 = 100 \text{ k}\Omega, A_{V_{o1}} = 70 \quad (5)$$

$$C_{d3} = C_{gs3}' = 350 \text{ fF}, C_c = 1 \text{ pF}$$

↓
DC-gain of stage 1

$$(2) - (5) \Rightarrow \begin{aligned} p_1 &= -100 \cdot 10^3 \frac{\text{rad}}{\text{sec}} & (-16 \text{ kHz}) \\ p_2 &= -1.2 \cdot 10^9 \frac{\text{rad}}{\text{sec}} & (-191 \text{ MHz}) \\ z_1 &= 10^9 \frac{\text{rad}}{\text{sec}} & (159 \text{ MHz}) \end{aligned}$$

$$A_{V_{o2}} (\equiv \text{DC-gain of stage 2}) = -g_{m3} \cdot R_2 = -100$$

$$\therefore A_v(s) \approx 7000 \cdot \frac{(1 - s/10^9)}{(1 + s/10^5)(1 + s/1.2 \cdot 10^9)}$$

$$\therefore \omega_u \approx 7.1 \cdot 10^8 \frac{\text{rad}}{\text{sec}} \quad (113 \text{ MHz}) \quad (\text{verify})$$

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$$\angle A_r(j\omega_u) = \tan^{-1}\left(-\frac{7.1 \cdot 10^8}{10^9}\right) - \tan^{-1}\left(\frac{7.1 \cdot 10^8}{10^5}\right) - \tan^{-1}\left(\frac{7.1 \cdot 10^8}{1.2 \cdot 10^9}\right)$$

$$\cong \underbrace{-35.4^\circ}_{\text{zero}} - \underbrace{90^\circ}_{\text{dom. pole}} - \underbrace{30.6^\circ}_{\text{non-dom. pole}} = -156^\circ$$

If $f(s)=1$, then $T(s) = A_r(s)$ so

$$\text{P.M.} = 180^\circ - 156^\circ = \underline{24^\circ} \text{ (low P.M.)}$$

- Note:
- 1) Increasing C_c reduces neg. phase shift from p_2 at ω_u but increases $\angle z_1$ at ω_u (because ω_u decreases but so does z_1)
 - 2) Situation is actually worse than calculated because of pole at (A)

Heuristics: As ω increases, feed fwd. path through C_c begins to dominate the gain path through M_3 . Gain path through M_3 has negative polarity, but feed-fwd. path through C_c has phase $\rightarrow 0$ as $\omega \rightarrow \infty \Rightarrow$ As $\omega \rightarrow \infty$, have pos. feedback

Now consider with $R_c > 0$

Can show, $R_c \neq 0 \Rightarrow$ have $z_1 \cong \frac{1}{C_c(1/g_{n3} - R_c)}$ (6)
have a 3rd pole, p_3

Usually:

- 1) $|p_3| \gg |p_1|, |p_2|, |z_1|$, so can ignore p_3
 - 2) p_1, p_2 are hardly affected by R_c
- } (7)

(6) $\Rightarrow R_c > \frac{1}{g_{n3}} \Rightarrow z_1$ "moves" to LHP
 $\Rightarrow z_1$ contributes pos phase shift

Fact: (7) breaks down for very large R_c .

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Typical rule of thumb: Choose R_c s.t. $|z_1| \cong 1.2 \cdot \omega_u$ (8)

$$(6), (8) \Rightarrow 1.2 \cdot \omega_u \cong \left| \frac{1}{C_c (1/g_{m3} - R_c)} \right|$$

$$R_c = \frac{1}{g_{m3}} + \frac{1}{1.2 \cdot \omega_u \cdot C_c} \quad (9) \quad (\text{for } R_c > \frac{1}{g_{m3}})$$

e.g. (using (5)) (9) $\Rightarrow R_c = 2.17 \text{ k}\Omega$

$$\text{Now } \angle A_v(j\omega) = \tan^{-1}\left(\frac{\omega_u}{1.2 \cdot \omega_u}\right) - 90^\circ - 30.6^\circ \cong -80.8^\circ$$

$$\therefore \text{new P.M. for } f(s)=1: \text{P.M.} \cong 99.2^\circ$$

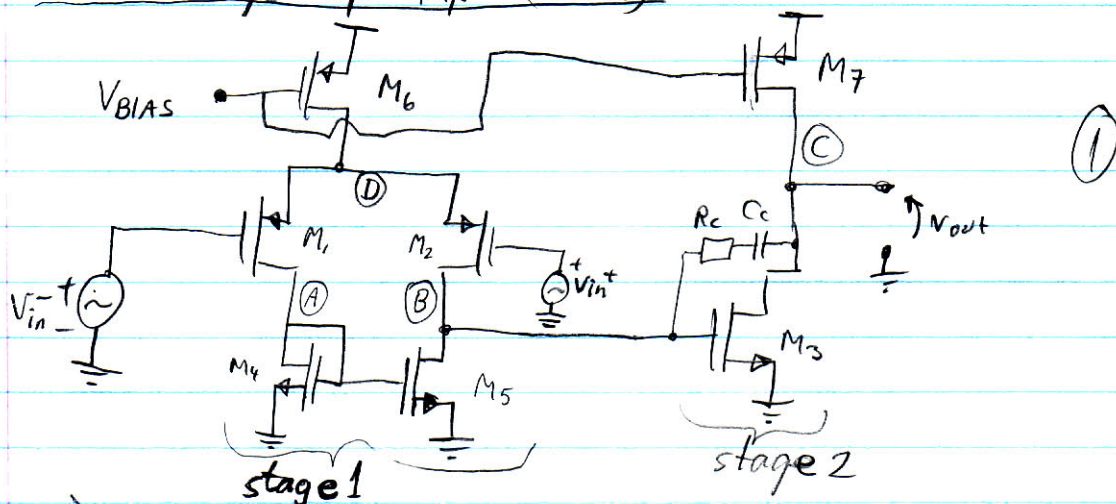
$\Rightarrow$ Can afford to reduce C_c ($\Rightarrow$ increase BW) to reduce P.M. (typ. want P.M. $> 65^\circ$)

Closed loop system:

- critically damped if P.M. = 76°
- gain peaking if P.M. $< 66^\circ$

Final exam: WLH 2204 (Thurs, March 20)
Extra Office Hours on Wed. of next week

Two-stage Op-Amp (cont.)



1) Common mode rejection of stage 1 $\Rightarrow$ usually can neglect pole associated with node D when considering diff. mode of op-amp.

2) Usually designed s.t. pole associated with A } (2)
 i.e. $p_A \approx -\frac{g_{m4}}{C_A}$ is s.t. $|p_A| \gg \omega_u$
 Then $M_2, M_5, M_3, M_7, R_C, C_C$ determine P.M. when 1 is used in feedback system.

3) Prev. found:

$$A_v(j\omega) \equiv \frac{V_{out}(j\omega)}{V_{in}^+(j\omega) - V_{in}^-(j\omega)} \approx \underbrace{g_{m1} R_1}_{\text{DC-gain of stage 1}} \cdot \underbrace{g_{m3} R_2}_{\text{DC-gain of stage 2}} \frac{(1-j\omega/Z_1)}{(1-j\omega/p_1)(1-j\omega/p_2)} \quad (3)$$

where $R_1 \equiv r_{ds2} \parallel r_{ds5}$, $R_2 \equiv r_{ds3} \parallel r_{ds7}$

$$p_1 \approx -\frac{1}{g_{m3} R_1 R_2 C_C} \quad (\text{dom. pole}) \quad (4)$$

$$p_2 \approx \frac{-g_{m3}}{C_{d3} + C_{g3}} \quad (\text{first non-dom. pole}) \quad (5)$$

$$\text{and } Z_1 \approx \frac{1}{C_C (g_{m3} - R_C)} \quad (\text{dom. zero}) \quad (6)$$

Note:
 ④-⑥ hdd for typ. design parameters

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General:
Zero is
placed
near
 ω_u

4) $p_1 \Rightarrow 3\text{dB BW}$ and introduces -90° phase at ω_u
 $p_2, z_1 \Rightarrow \text{PM}$ (i.e. P.M. varies with p_2, z_1 ; not with p_1)
assumes $|p_2|, |z_1| \approx 10 \cdot |p_1|$

5) R_c chosen s.t. z_1 in LHP $\Rightarrow$ introduces pos. phase shift at ω_u

Fargoh: PM of an open-loop op-amp $\approx 180^\circ + \angle A_v(j\omega_u)$
why? When connected as voltage follower, i.e. $f=1$, (hardest
non-attenuating config to compensate) loop gain $\equiv T(j\omega) \equiv A_v(j\omega)$

Usually, ① designed s.t. $\underbrace{|p_2|, |z_1|}_{\text{same order of mag.}} > \omega_u$

$$\therefore \textcircled{3} \Rightarrow |A_v(j\omega_u)| \approx \left| g_{m1} g_{m3} R_1 R_2 \cdot \frac{1}{1 - j\omega_u/p_1} \right|$$
$$\approx g_{m1} g_{m3} R_1 R_2 \cdot \left| \frac{p_1}{j\omega_u} \right|$$

$$\therefore \textcircled{4} \Rightarrow \approx \left| \frac{g_{m1}}{C_c \cdot j \cdot \omega_u} \right|$$

$$\therefore |A_v(j\omega_u)| = 1 \Rightarrow \omega_u \approx \frac{g_{m1}}{C_c} \quad \textcircled{7}$$

$$\text{P.M.} = 180^\circ - \tan^{-1} \frac{\omega_u}{z_1} + \tan^{-1} \frac{\omega_u}{p_2} - \underbrace{90^\circ}_{\text{contrib. by } p_1} \quad \textcircled{8}$$

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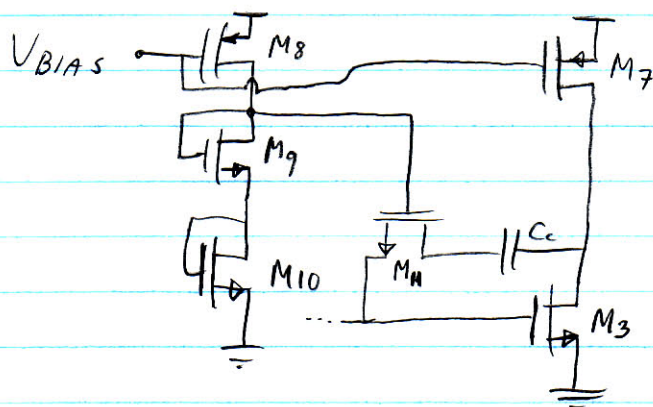
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Problem: How to maintain constant P.M. across temp./process/supply voltage variations?

(5) - (8) $\Rightarrow$ P.M. depends on $g_{m1}, g_{m3}, C_{d3}, C_{gs3}, C_c, R_c$
 These parameters vary and don't track each other

A Solution: Replace stage 2 of (1) by:



with $\frac{W_7/L_7}{W_3/L_3} = \frac{W_8/L_8}{W_{10}/L_{10}}$ (10)

(9)

M_{11} biased by $M_8 - M_{10} \equiv R_c$

No DC-current $\Rightarrow M_{11}$ in triode

$$\Rightarrow R_c \approx \frac{1}{\mu_n C_{ox} (W_{11}/L_{11}) (V_{GS11} - V_{Tn})}$$

call V_{eff11}

$$g_{m3} = \mu_n C_{ox} (W_3/L_3) \cdot V_{eff3}$$

$$\therefore R_c g_{m3} = \frac{(W_3/L_3) V_{eff3}}{(W_{11}/L_{11}) V_{eff11}} \quad (11)$$

$$\therefore (6) \Rightarrow Z_1 = \frac{g_{m3}}{C_c \cdot \left[1 - \frac{(W_3/L_3) V_{eff3}}{(W_{11}/L_{11}) V_{eff11}} \right]} \quad (12)$$

$$(7), (12) \Rightarrow \frac{W_{11}}{Z_1} = \frac{g_{m1}}{g_{m3}} \cdot \left[1 - \frac{(W_3/L_3) V_{eff3}}{(W_{11}/L_{11}) V_{eff11}} \right]$$

$$(5), (7) \Rightarrow \left| \frac{W_u}{p_2} \right| = \frac{g_{m1}}{g_{m3}} \cdot \frac{C_{d3} + C_{gs3}}{C_c}$$

$$\frac{g_{m1}}{g_{m3}} = \frac{\sqrt{2 \mu_p C_{ox} (W_1/L_1) I_{D1}}}{\sqrt{2 \mu_n C_{ox} (W_3/L_3) I_{D3}}}$$

$$\mu_0 C_{ox} \approx K'$$

Facts:

- 1) $\mu_p/\mu_n \approx \text{const. for a given process}$
 - 2) $I_{D1}/I_{D3} \approx \text{const. because derived from a common bias network}$
 - 3) $\frac{C_{d3} + C_{gs3}}{C_c} \neq \text{const.}, \text{ but does not vary much over process \& temp. because dominated by oxide capacitor}$
- $\Rightarrow \frac{g_{m1}}{g_{m3}} \approx \text{const.}$

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$\therefore P.M. \cong \text{const. provided } V_{eff3}/V_{eff11} \cong \text{const.}$

$$\textcircled{10} \Rightarrow V_{eff10} = V_{eff3}$$

$$V_{eff9} = V_{eff11}$$

$$\therefore \frac{V_{eff3}}{V_{eff11}} = \frac{V_{eff10}}{V_{eff9}} = \frac{\sqrt{\frac{2 \cdot I_{D10}}{\mu_n \cdot C_{ox} (W_{10}/L_{10})}}}{\sqrt{\frac{2 I_{D9}}{\mu_n \cdot C_{ox} (W_9/L_9)}}} = \sqrt{\frac{W_{10}/L_{10}}{W_9/L_9}} = \left\{ \begin{array}{l} \text{ratio of} \\ \text{like quantities} \end{array} \right.$$

$\therefore P.M. \cong \text{indep. of process \& temperature}$

The "reference source" in Blackman's Impedance Relation (BIR) and the Asymptotic Gain Formula (AGF):

The values of the variables in BIR (Z_{ab}^o, T_{sc}, T_{oc}) or in AGF (A_∞, A_o, T) depend on which reference source we use.

Of course, the resulting impedance (Z_{ab}) or gain (A) is the same, independent of our choice of reference source.

BIR & AGF was covered on Jan. 31 in class.