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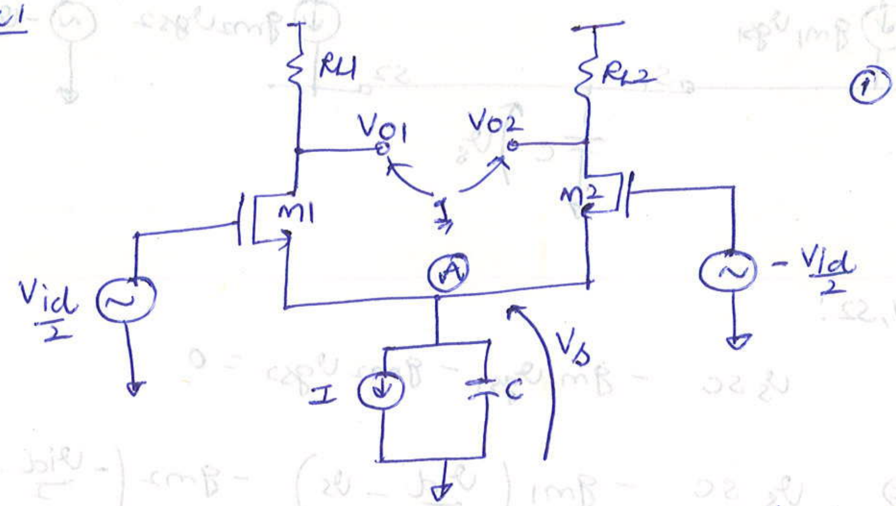
REC #1 APRIL 1st '08

Zeros from Differential circuit Paths:

(Loads need not be resistive)

Ex1



Advantages of using current mirror load?

For demonstration simplicity $r_{dsi} = \infty$, $g_{mbi} = 0$, $i = 1, 2$

C = Only significant cap in the circuit ②

Paradox:

ECE 264A material $\Rightarrow$ 1) have pole, $p_A = \dfrac{-1}{R_{(A)} C_{(A)}}$

where R_A = small signal resistance to gnd

C_A = " " capacitance to gnd

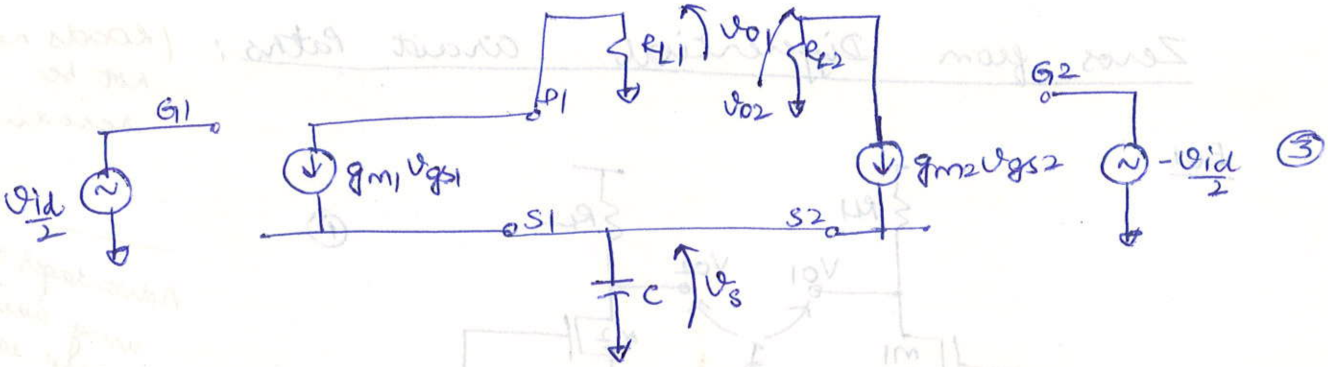
$\Rightarrow$ $p_{(A)} = -\dfrac{g_{m1} + g_{m2}}{C}$

ECE 164 material $\Rightarrow$ 2) if $g_{m1} = g_{m2}$, $v_s = 0$ (symmetry)

So $p_{(A)}$ has no effect on ckt.

To resolve this paradox, must analyze ①

Let $R_{L1} = R_{L2} = R_L$



③

KCL @ S1, S2:

$$v_s sC - g_{m1} v_{gs1} - g_{m2} v_{gs2} = 0$$

$$\Rightarrow v_s sC - g_{m1}\left(\frac{v_{id}}{2} - v_s\right) - g_{m2}\left(-\frac{v_{id}}{2} - v_s\right) = 0$$

$$\Rightarrow v_s = \frac{1}{2}\,\frac{g_{m1} - g_{m2}}{g_{m1} + g_{m2} + sC}\, v_{id} \quad ④$$

③ $\Rightarrow$

$$v_{o1} = -g_{m1} v_{gs1} R_{L1}$$

$$= -g_{m1}\left(\frac{v_{id}}{2} - v_s\right) R_L$$

$$= -g_{m1} R_L \frac{v_{id}}{2}\left(1 - \frac{g_{m1} - g_{m2}}{g_{m1} + g_{m2} + sC}\right)$$

③ $\Rightarrow v_{o1} = -g_{m1} R_L \left(\frac{v_{id}}{2} - v_s\right), \quad v_{o2} = -g_{m2} R_L \left(-\frac{v_{id}}{2} - v_s\right)$

$$\therefore v_{od} \equiv v_{o1} - v_{o2} = -\frac{1}{2}(g_{m1} + g_{m2}) R_L v_{id} + \frac{1}{2}(g_{m1} - g_{m2}) R_L (2 v_s)$$

④ $\Rightarrow v_{od} = -\frac{1}{2} R_L v_{id} \left[g_{m1} + g_{m2} - \frac{(g_{m1} - g_{m2})^2}{\ldots} \right]$

$$= -\frac{1}{2} R_L v_{id} \left[\frac{4 \; g_{m1} g_{m2} \pm (g_{m1} + g_{m2}) sC}{g_{m1} + g_{m2} + sC} \right]$$

$$\therefore \; A_{dm} = \frac{v_{od}}{v_{id}} = -\frac{1}{2} R_L \left[\frac{\frac{4 \, g_{m1} g_{m2}}{g_{m1}+g_{m2}} \pm sC}{1 - \frac{s}{\underbrace{-(g_{m1}+g_{m2})/C}_{= p_A}}} \right]$$

$$= -g_m R_L \left(\frac{1 - s/z_1}{1 - s/p_A} \right)$$

$$g_m = \frac{2 \, g_{m1} g_{m2}}{g_{m1} + g_{m2}}, \quad z_1 = \frac{2 g_m}{C}$$

$\therefore$ As $g_{m1} \to g_{m2}$, $z_1 \to p_A$ $\Rightarrow$ pole-zero cancellation

Resolution of paradox: We really do have a pole @ $s = p_A$, but in a perfectly matched version of (1), we also have a zero that cancels the pole.

Q So what?

A We never have perfect matching, so differential pair amplifiers have at least one pole-zero doublet

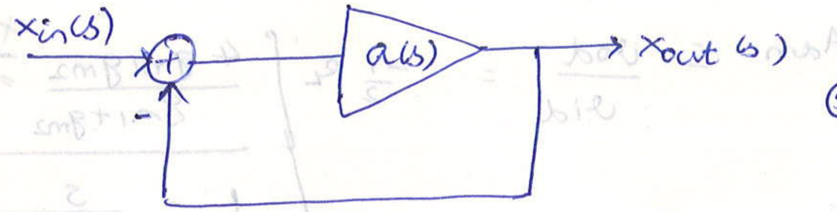
Jargon: Pole-zero doublet = closely spaced pole & zero

Q So what?

A Pole-zero doublet $\Rightarrow$ slow settling component in closed-loop amplifier step response.

Paul Grey's paper

Ex2 Unity gain feedback system w/ pole-zero doublet



(5)

where $\quad a(s) = \dfrac{a_0 \,(1 - s/z_x)}{(1 - s/p_d)(1 - s/p_x)}$

and $\quad p_x = a_p p_d \qquad$ ($p_d \equiv$ dominant pole

$\quad z_x = a_z p_d \qquad p_x, z_x \equiv$ pole-zero doublet)

$\quad a_0 > a_p \;, a_z > 0$

Assume $\quad$ 1) $a_0 >> 1 \quad$ 2) $|p_d| < |p_x|, |z_x| < \dfrac{a_0 |p_d|}{10}$

3) $|p_d| << |p_x|, |z_x|$

Can show (HW)

$$A(s) \equiv \frac{x_{out}(s)}{x_{in}(s)} = \frac{1 - s/z_x}{(1 - s/p_1)(1 - s/p_2)} \quad (6)$$

where $\quad p_2 \cong - \dfrac{GBW \, z_x}{z_x \cancel{-} GBW}$ (7) $\quad$ ($GBW \equiv a_0 |p_d|$)

$+$

$$p_1 \cong - \frac{GBW \, p_x}{z_x} \quad (8)$$

Recall (6) $\Rightarrow \quad A(s) = \dfrac{A_1}{1 - \frac{s}{p_1}} + \dfrac{A_2}{1 - \frac{s}{p_2}}$

where $\quad A_1 = A(s)\left(1 - \dfrac{s}{p_1}\right)\Big| \quad , A_2 = A(s)\left(1 - \dfrac{s}{p_2}\right)\Big|$

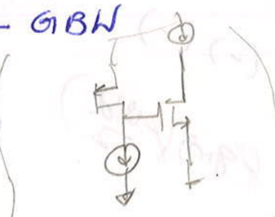
Let $x_{step}(t) = x_{out}(t)\Big|_{x_{in}(t) = u(t)}$ where $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

HW 2 $\Rightarrow$ $x_{step}(t) = \underbrace{A_1 (1 - e^{P_1 t}) u(t)}_{\cong\, x_{step}(t) \text{ w/o doublet}} + \underbrace{A_2 (1 - e^{P_2 t}) u(t)}_{\text{slow settling component.}}$

(Recall, ⑤ w/ $P_x = Z_x$ (i.e. no doublet) has

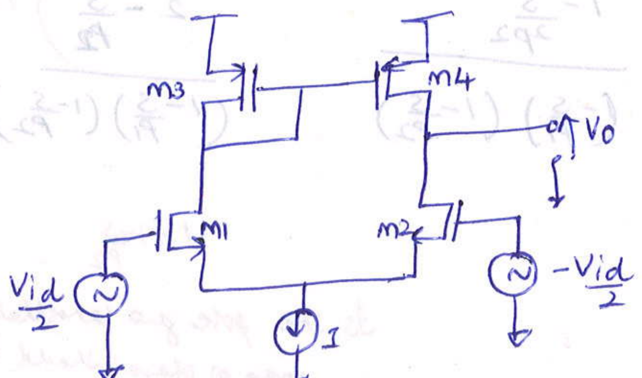
$$A(s) = \frac{1}{1 - \frac{S}{P_1}} \quad \text{w/} \quad P_1 \cong -GBW$$

P. Z doublet: due to inherent NL or due to mismatch?



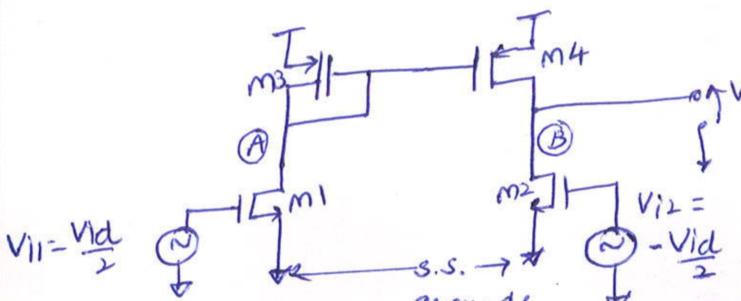
zero can be far away

Ex 3 Pole zero doublet in diff to single-ended amp:



$\cong$

SSM reduces to that of (assumes large r_{ds1} & r_{ds2}) & low C_{d1} & C_{d2})



⑨

Linearity of SSM

$\Rightarrow v_0 = v_{01} + v_{02}$

where $v_{01} = v_0 \Big|_{\substack{v_{i1} = v_{id}/2 \\ v_{i2} = 0}}$

and $v_{02} = v_0 \Big|_{\substack{v_{i2} = -v_{id}/2 \\ v_{i1} = 0}}$

⑦ $\Rightarrow v_{01} = \left(\frac{1}{2} v_{id}\, g_{mn} \frac{1}{g_{mp}} \cdot g_{mp} (r_{dsn} \| r_{dsp})\right)$

$$\left(\frac{1}{(1 - s/P_1)(1 - s/P_2)}\right)$$

where $P_1 = \frac{-1}{r_{dsn} \| r_{dsp}\, C_{B}}$

$P_2 = -\frac{g_{mp}}{C_{A}}$

Recall C_{x} = small signal cap. from node x to small signal ground

$v_{o2} \to$ 2 inversions

Similarly, (9) $\Rightarrow v_{o2} = \frac{1}{2} v_{id}\, g_{mn} (r_{dsn} \| r_{dsp}) \frac{1}{(1 - s/P_1)}$

(−)(−)

$(g_m)(+\frac{v_{id}}{2})$

$$v_o = v_{o1} + v_{o2} = \frac{1}{2} v_{id}\, g_{mn} (r_{dsn} \| r_{dsp}) \left[\frac{1}{(1-\frac{s}{P_1})(1-\frac{s}{P_2})} + \frac{1}{1-\frac{s}{P_1}} \right]$$

$$\therefore a(s) = \underbrace{g_{mn} (r_{dsn} \| r_{dsp})}_{\equiv a_0} \frac{1 - \frac{s}{2P_2}}{(1-\frac{s}{P_1})(1-\frac{s}{P_2})} \cong \frac{2 - \frac{s}{P_2}}{(1-\frac{s}{P_1})(1-\frac{s}{P_2})}$$

$1 - \frac{s}{P_2}$

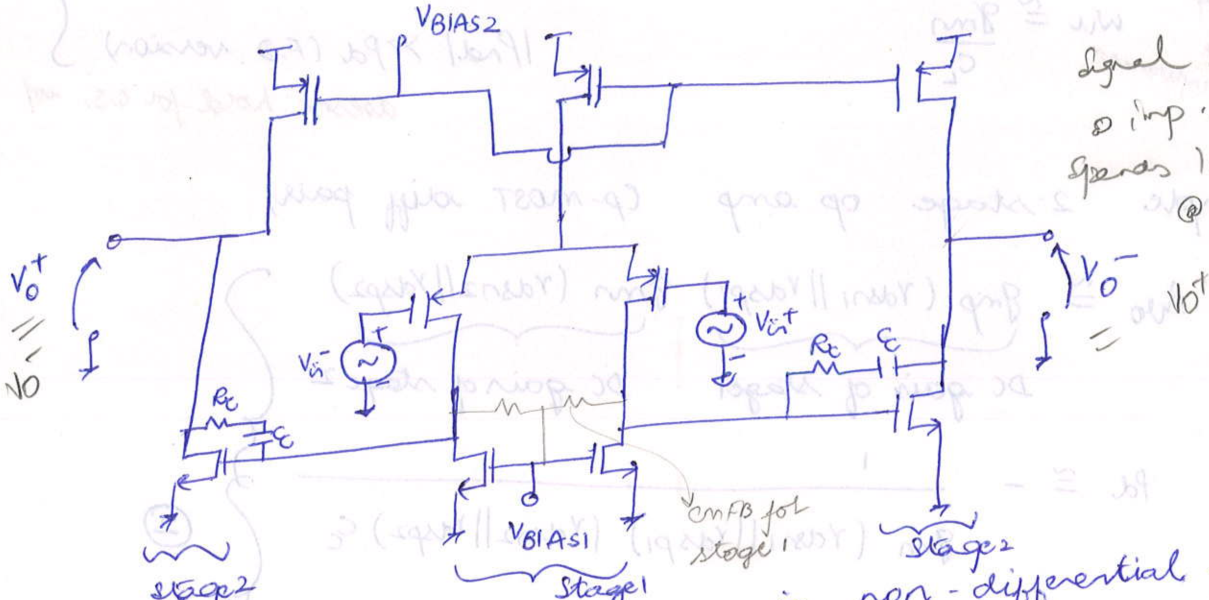
$\equiv \frac{v_o}{v_{id}}$

$1 + 1 - \frac{s}{P_2}$

Is the pole zero cancellation in mag or phase; should be both right?

LEC #2 APRIL 3'08

Fully Differential 2-stage OpAmp:



signal swing not ∝ imp ∴ signal goes less time @ low voltage

Notes: 1) Same $P1, P2, Z1$ as in non-differential version (Verify)

2) No pole zero doublet from systematic mismatches (unlike single ended version)

3) CMFB required (senses V_{oc}, but adjusts bias current in stage 1) → Other strategies possible

Op-Amp Topologies:

So far have covered ~~non-differential~~ single-ended (S.E) versions of simple one stage & and fully differential (F.D.) versions of simple one stage & two-stage opamps.

General observations:

1) Feedback applications
   - (i) accuracy depends on op-amp's DC gain.
   - (ii) BW depends on dominant pole
   - (iii) stability (marginal) depends on non-dominant pole & (zero, if lead compensated)

(ii), (iii) ⇒ For a given PM, BW depends on non-dominent pole.

2) Simple one-stage op amp (n-MOST diff pair)

$$A_{v0} \cong g_{mn}(r_{dsn} \| r_{dsp}), \quad p_d \cong -\frac{1}{(r_{dsn}\|r_{dsp})C_L}$$

In general C_L acts as comp. cap but if C_L & N.D poles start to influence

$$\omega_u \cong \frac{g_{mn}}{C_L} \qquad |p_{nd}| >> p_d \text{ (F.D. version)}$$

① doesn't hold for C.S. w/

3) Simple 2-stage op amp (p-MOST diff pair)

$$A_{v0} \cong \underbrace{g_{mp}(r_{dsn1} \| r_{dsp1})}_{\text{DC gain of stage1}} \underbrace{g_{mn}(r_{dsn2} \| r_{dsp2})}_{\text{DC gain of stage 2}}$$

$$p_d \cong -\frac{1}{g_{mn}(r_{dsn1}\|r_{dsp1})(r_{dsn2}\|r_{dsp2})C_C}$$

$$p_{nd} \cong -\frac{g_{mn}}{C_L}, \quad z_1 \cong \frac{1}{C_C\left(\frac{1}{g_{mn}} - R_C\right)}$$

②

$\omega_u = \frac{g_{mp}}{C_C}$ → After compensation → Assumes $|p_{nd}|, |z_1| > \omega_u$

①, ② ⇒ 2-stage op-amp has higher gain but lower BW for a given C_L and PM than one stage op-amp.

Why? p_{nd} limited by C_L in two stage op-amp (not so in one-stage op-amp) ③

⇒ Want to increase gain in one-stage op-amp (yet avoid problem like ③)

<u>Telescopic Op-Amp</u>

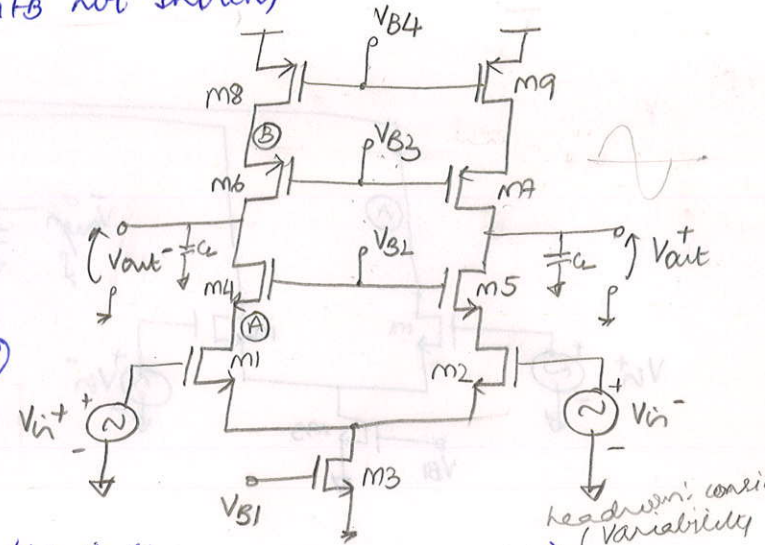
Idea: Use cascoding

Telescopic Op-Amp

(This config can't drive resistive loads)

Idea: Use cascoding

Ex F.D. version (CMFB not shown)



Headroom: consider (variability of chi.

DC gain:

$$A_{v0} = g_{m1}(r_{on} \| r_{op})$$

where

$$\left.\begin{aligned} r_{on} &\cong r_{ds1} r_{ds4}(g_{m4}+g_{mb4}) \\ r_{op} &\cong r_{ds8} r_{ds6}(g_{m6}+g_{mb6}) \end{aligned}\right\} (4)$$

$$p_d \cong \frac{-1}{(r_{on} \| r_{op}) C_L}$$

$p_{nd} \cong -\frac{g_{m4}}{C_A}$ (can verify heuristics: $\frac{v_B}{v_{id}}$ is small)

$\Rightarrow$ gain improved by factor of $g_m r_{ds}$ but p_{nd} is still indep. of C_L

Q What's the catch?

A Headroom $pp\text{-}max\{V_{od}\} = 2\left[V_{DD} - 5V_{eff} - 3\underbrace{V_{margin}}\right]$

(max p-p swing of v_{od}) ↳ ∴ it is differential

margin b/w V_{B4} & V_{B3}, margin b/w V_{B2} & V_{IC}, " " & V_{IC} & V_{B1}

e.g. Suppose $V_{eff} = 0.2V \cong$ edge of strong inversion

$V_{margin} = 0.1V$; $V_{DD} = 2.7V$ } (5)

Then $pp\text{-}max\{V_{od}\} = 2.8V$

In simple one-stage opamp,

$$pp\text{-}max\{V_{od}\} = 2\left[V_{DD} - 3V_{eff} - V_{margin}\right]$$

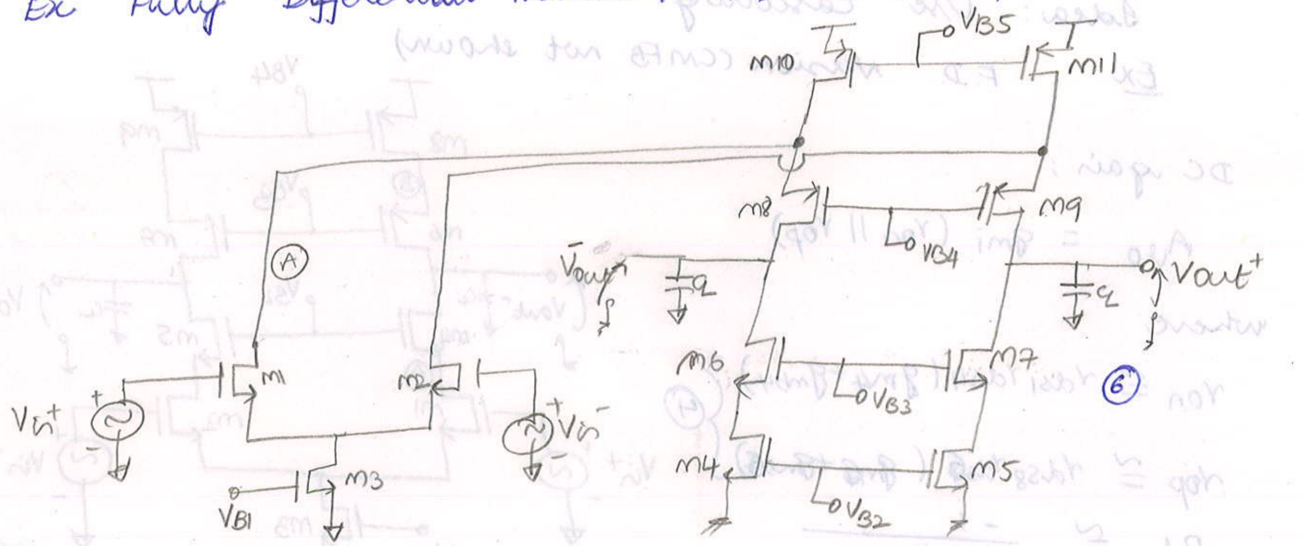
So (5) $\Rightarrow$ $= 4.1V$

In 2-stage op-amp, $pp\text{-}max\{V_{od}\} = 2[V_{DD} - 2V_{eff}]$ (swing is determined by 2nd stage)

So (5) $\Rightarrow$ $= 4.6V$

Folded Cascode Op-Amp:

Ex Fully Differential version (CMFB not shown)



$A_{v0} \cong g_{m1}(r_{on}||r_{op})$ where r_{on}, r_{op} are analogous to 4 (similar values)

$p_d \cong \frac{-1}{(r_{on}||r_{op})C_L}$, $p_{nd} = \frac{-g_{m8}}{C_A}$

$pp\text{-}max\{v_{od}\} = 2[V_{DD} - 4V_{eff} - 2V_{margin}]$

$(\because$ 5 $\Rightarrow pp\text{-}max\{v_{od}\} = 3.4V)$

$\Rightarrow$ Better headroom, similar A_{v0}, p_d, p_{nd} expressions to 3

Q Whats the catch?

1) C_A larger in 6 than in 3, and $g_{mp} < g_{mn}$ for same power $\Rightarrow$ 6 has lower B/W

2) "folding" requires extra set of bias currents
$\Rightarrow$ 6 has higher power consumption

3) More transistors in 6 $\Rightarrow$ more noise w.r.t. 3

Conclusion:

Telescopic & folded cascode op-amps have

1) BW benefits of simple one-stage amplifiers

2) gain " ______________ " two-stage " ______ "

3) Reduced headroom w.r.t. both 1 and 2 stage opamps

headroom: available headroom ; I/P signal is accurately placed
~~MDB~~ may take away some margin.

LECTURE # 3, APRIL 8'08

Gain Boosting:

Highest DC gain op-amps so far

(i) two stage

(ii) cascode (telescopic or folded)

found $\overbrace{\text{DC gain}}^{A_{v0}}$ $\propto g_m^2 r_{ds}^2$

In practice (typical CMOS process, $V_{DD} \le 2.7V$, $I_{tot} < 5mA$)

hard to achieve $A_{v0dB} \equiv 20\log(A_{v0}) \gtrsim 70dB$

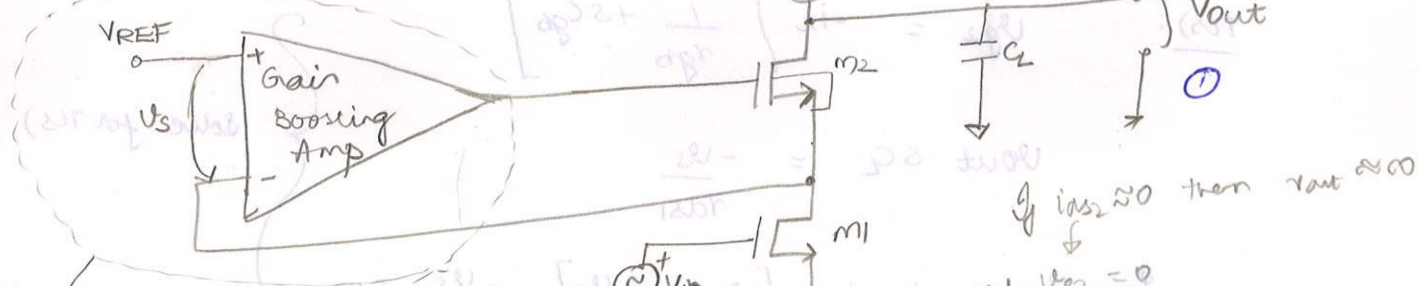
But often need $A_{v0dB} \ge 90dB$

In general, $A_{v0} = g_m r_{out}$ in each stage

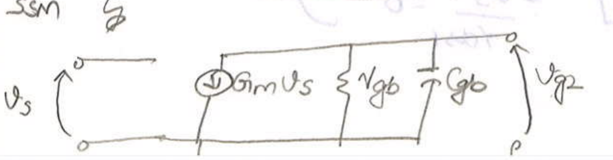
cascode ③ $\Rightarrow$ $r_{out} \propto g_m r_{ds}^2$ ($g_m r_{ds1} r_{ds2}$)

Gain boosting $\equiv$ use of local feedback (i.e. inside op-amp) to increase r_{out}.

Ex Non Differential Version: (In practice use differential version)



If $i_{ds2} \approx 0$ then $r_{out} \approx \infty$
↓
need $v_{s2} = 0$



Simplifying assumption:

C_L and C_{gb} are only caps that contribute poles & zeros with magnitudes $< \omega_u$ } ②

Application of A.G.R. to ① & ②:

Let $Y_j \equiv G_m v_s \equiv$ ref source

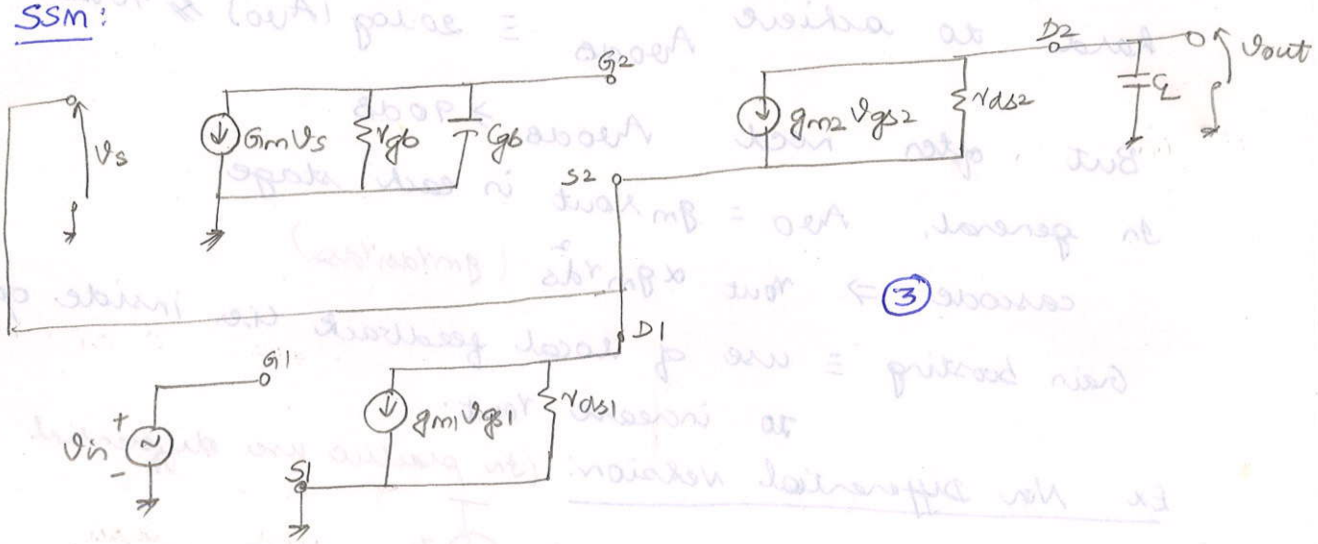
Then $A(s) = A_\infty(s)\dfrac{T(s)}{1+T(s)} + A_0(s)\dfrac{1}{1+T(s)}$

$T(s) = -\dfrac{G_m v_s(s)}{i_x}$ where $v_{in} = 0$ & Y_j is replaced by an indep. test source i_x.

$A_\infty(s) = A_v(s)\big|_{G_m \to \infty}$, $A_0(s) = A_v(s)\big|_{G_m \to 0}$

② ⇒ To find $A_v(j\omega)$ for $|\omega| < \omega_u$, can neglect all caps except C_L & C_{gb}.

SSM:



T(s): ①

$v_{g2} = -i_x\left[\dfrac{1}{r_{gb}} + sC_{gb}\right]^{-1}$

$v_{out}\, sC_L = -\dfrac{v_s}{r_{ds1}}$

$g_{m2}\underbrace{v_{gs2}}_{\equiv v_{g2} - v_{s2}} + \dfrac{1}{r_{ds2}}\left[v_{out} - v_s\right] - \dfrac{v_s}{r_{ds1}} = 0$

} solve for T(s)

$$\therefore\ g_{m2}\left[-i_x\left(\frac{1}{r_{gb}}+sC_{gb}\right)^{-1}-v_s\right]+\frac{1}{r_{ds2}}\left[\frac{-v_s}{sC_L r_{ds1}}-v_s\right]-\frac{v_s}{r_{ds1}}=0$$

$$\therefore\ v_s\left[g_{m2}+\frac{1}{r_{ds1}r_{ds2}sC_L}+\underbrace{\frac{1}{r_{ds1}}+\frac{1}{r_{ds2}}}_{\text{assume} \ll g_{m2}}\right]=\frac{-i_x g_{m2}}{\frac{1}{r_{gb}}+sC_{gb}}$$

$$\therefore\ T(s)\cong\frac{G_m g_{m2}}{\left(\frac{1}{r_{gb}}+sC_{gb}\right)\left(g_{m2}+\frac{1}{r_{ds1}r_{ds2}sC_L}\right)}$$

$$=A_{gb0}\,\frac{sC_L r_{ds1}\,g_{m2}r_{ds2}}{(1+sC_{gb}r_{gb})\,(1+sC_L r_{ds1}\,g_{m2}r_{ds2})}\qquad (4)$$

where $A_{gb0}=G_m r_{gb}$ (DC gain of gain boosting amp)

$\underline{A_\infty(s)}$:

$$v_{g2}=-G_m v_s\underbrace{\left(\frac{1}{r_{gb}}+sC_{gb}\right)^{-1}}_{\text{finite (assume stable neg feedback)}}$$

$v_s \propto \frac{v_{g2}}{G_m}$

$G_m\to\infty,\ v_s\to 0$

$\Rightarrow$ must have $v_s\to 0$ as $G_m\to\infty$

$\Rightarrow\ \boxed{A_\infty(s)=-g_{m1}\frac{1}{sC_L}}$ ($\because v_s=0$, no current through r_{ds1}, but g_{m1} generated current is present $\Rightarrow$ this current should flow through C_L)

$\underline{A_0(s)}$:

$G_m\to 0\Rightarrow$ (3) $\to$ c.s. amp. w/ cascode stage

Already know $A_0(s)=-g_{m1}\left[\frac{1}{r_{ocasc}}+sC_L\right]^{-1}$

$$=-g_{m1}r_{ocasc}\left(\frac{1}{1+r_{ocasc}sC_L}\right)$$

where $r_{ocasc}\cong g_{m2}r_{ds2}r_{ds1}$

Let $A_{orig0} = -g_{m1} r_{ocasc}$ (DC gain of "original" cascoded CS amp without gain boosting)

$P_{orig} = \dfrac{-1}{r_{ocasc} C_L}$ (dominant pole of original cascoded CS amp)

$P_{gb} = \dfrac{-1}{r_{gb} C_{gb}}$ (dominant pole of gain boosting amp.)

$$T(s) = A_{gb0} \frac{s C_L r_{ocasc}}{(1 - s/P_{gb})(1 - s/P_{orig})}$$

$$A_0(s) \frac{T(s)}{1+T(s)} = \frac{|A_{gb0} A_{orig0}|}{(1 - s/P_{gb})(1 - s/P_{orig}) - s/(P_{orig}/A_{gb0})}$$

$$A_0(s) \frac{1}{1+T(s)} = \frac{-|A_{orig0}| (1 - s/P_{gb})}{(1 - s/P_{gb})(1 - s/P_{orig}) - s/(P_{orig}/A_{gb0})}$$

$\therefore$ Assuming $A_{gb0}, |A_{orig0}| \gg 1$ Then $A(s) \cong$

$$\frac{-A_{gb0} |A_{orig0}| \left(1 - s/A_{gb0} P_{gb}\right) \frac{1}{A_{gb0}}\left(1 - \frac{s}{P_{gb}}\right)}{1 - s\underbrace{\left[\frac{1}{P_{gb}} + \frac{1+A_{gb0}}{P_{orig}}\right]}_{\cong \frac{A_{gb0}}{P_{orig}} \text{ if } P_{gb} \gtrsim \frac{10 P_{orig}}{A_{gb0}}} + s^2/P_{gb}P_{orig}}$$

Claim: $1 + \beta s + \alpha s^2 \cong (1+\beta s)\left(1 + \frac{\alpha}{\beta} s\right)$ if $\alpha, \beta \in \mathbb{R}$ & $\beta^2 \gg 4\alpha$ } holds for denom. of 6

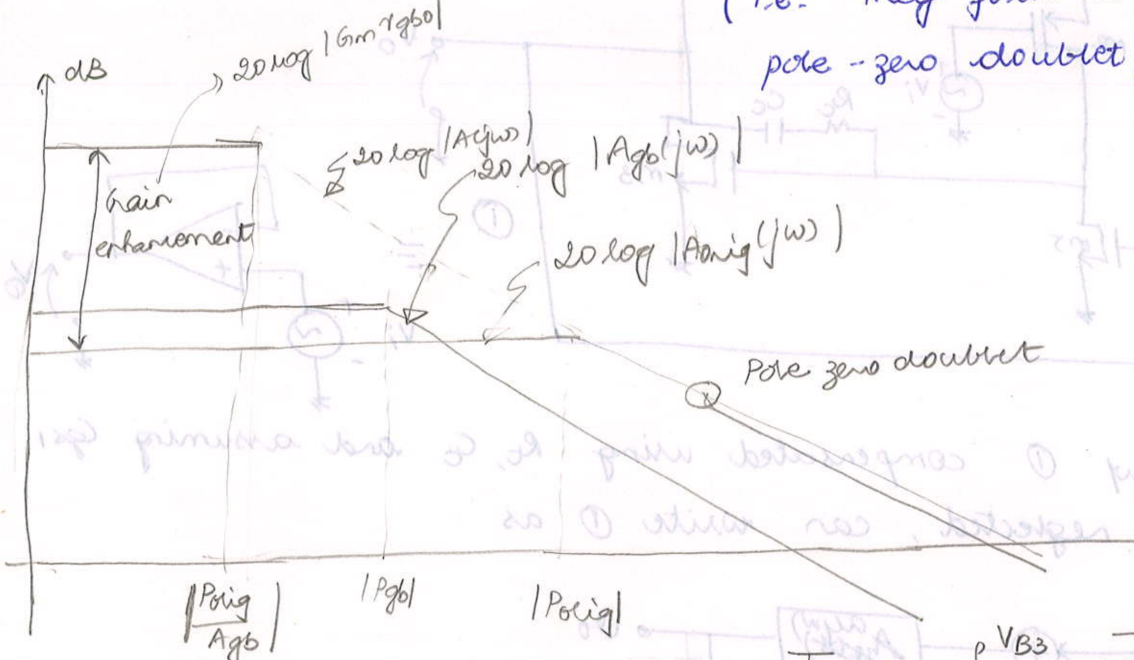
Prove as exercise.

$$\therefore \quad A(s) \cong -A_{gb0}\,|A_{orig0}| \; \frac{1 - s/(A_{gb0} P_{gb})}{\left(1 - s/(P_{orig}/A_{gb0})\right)\left(1 - s/(P_{gb} A_{gb0})\right)}$$

Poles: $\dfrac{P_{orig}}{A_{gb0}}$, $P_{gb} A_{gb0}$

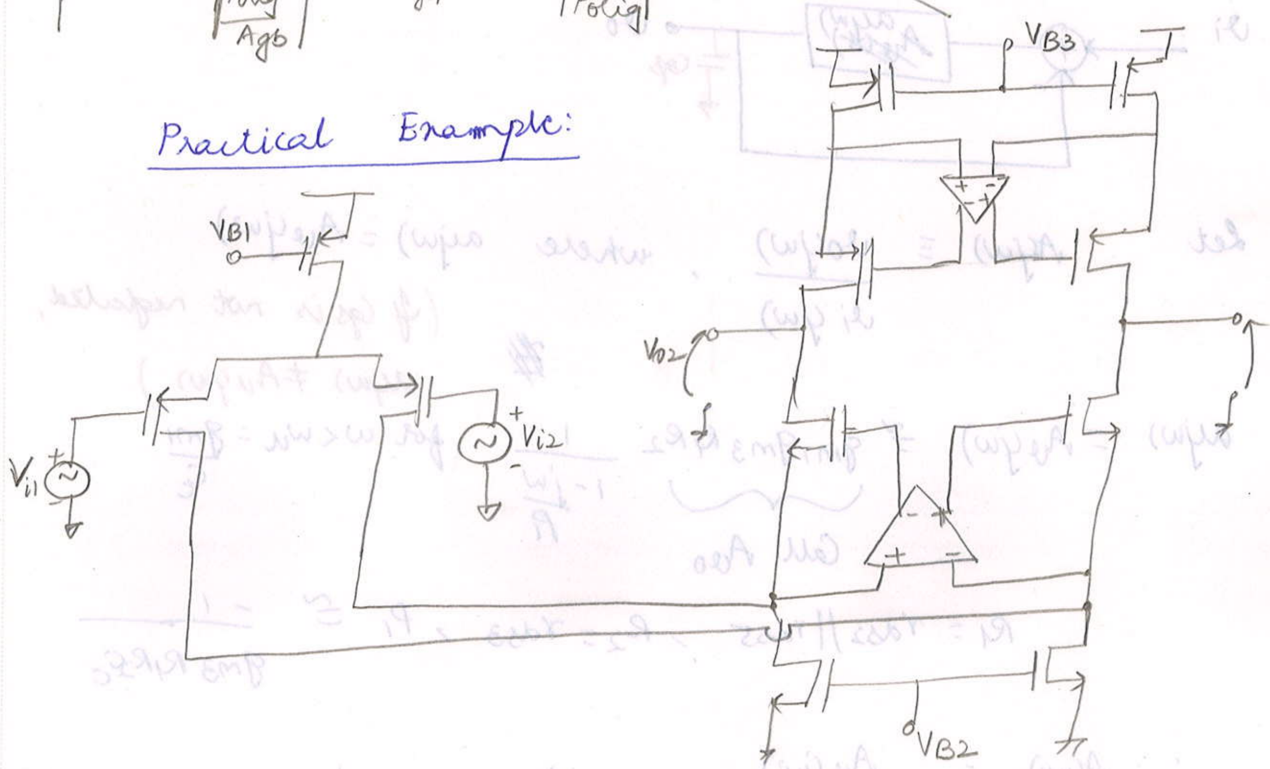
zero: $P_{gb} \cdot A_{gb0}$

Approximations ⇒ these do not cancel exactly (i.e. they form a very close pole-zero doublet)



(IEEE: Klaus Bult (Gain Boosting) Harry Li / MIT

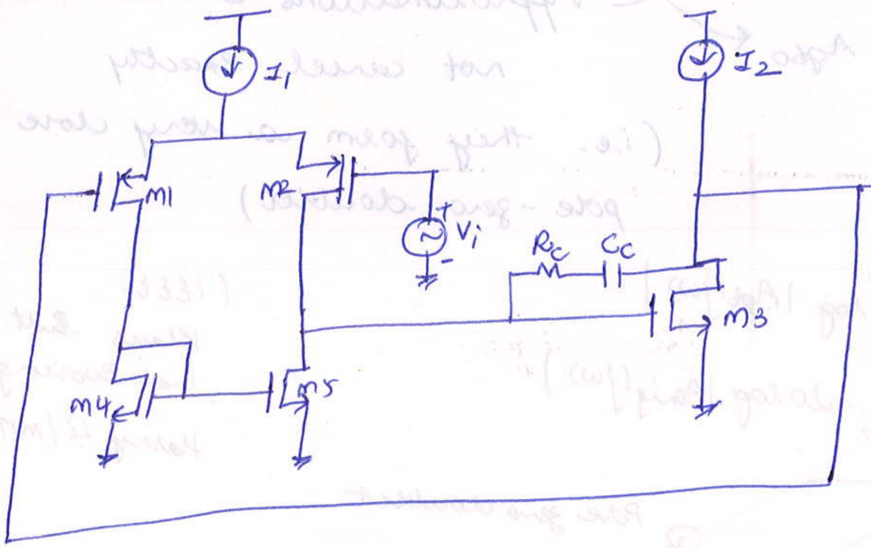
Practical Example:



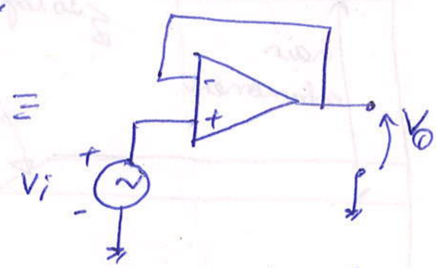
LEC #4 APR 10'08

Slew Rate Limiting:

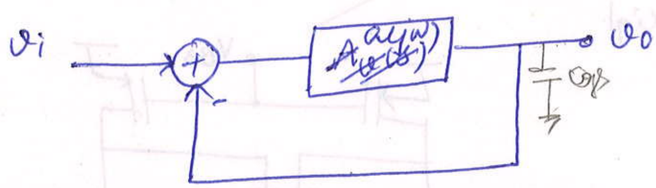
Ex 2-stage op-amp in voltage follower configuration



① ≡



Recall w/ ① compensated using R_c, C_c and assuming C_{gs1} can be neglected, can write ① as



Let $A(j\omega) \equiv \dfrac{v_o(j\omega)}{v_i(j\omega)}$, where $a(j\omega) = A_v(j\omega)$

(If C_{gs} is not neglected, $a(j\omega) \neq A_v(j\omega)$)

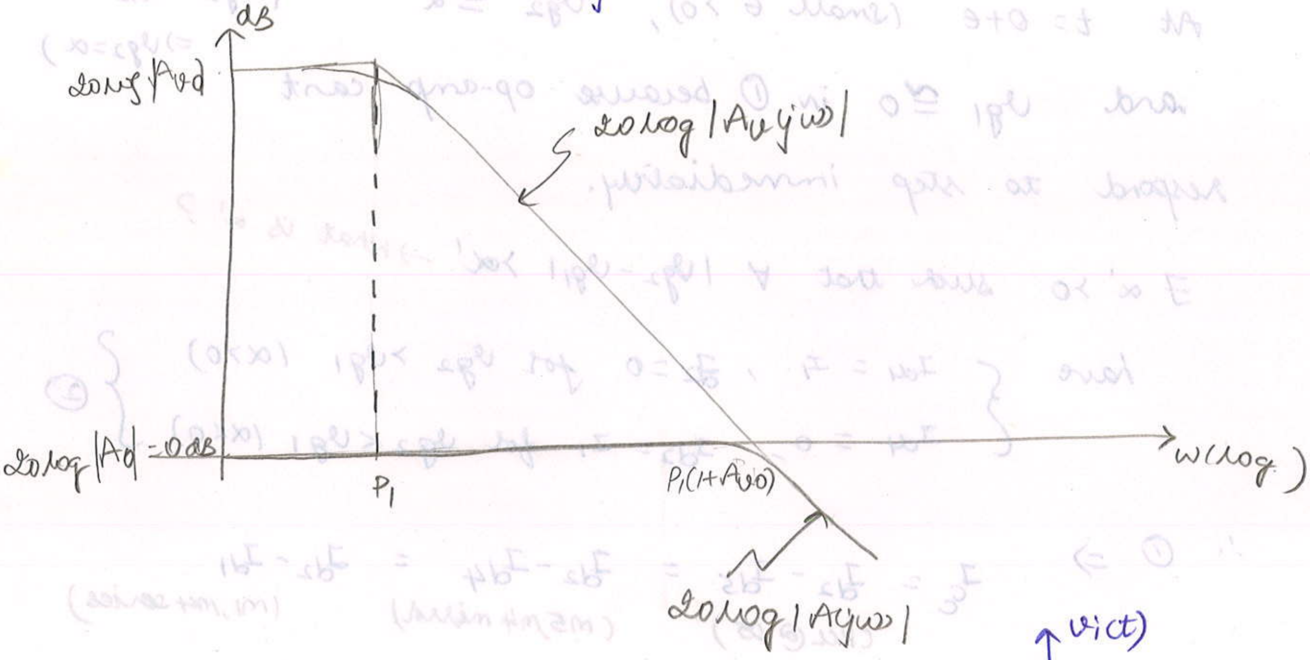
$$a(j\omega) = A_v(j\omega) \cong \underbrace{g_{m1} g_{m3} R_1 R_2}_{\text{Call } A_{v0}} \frac{1}{1 - \frac{j\omega}{P_1}} \quad \text{for } \omega < \omega_u = \frac{g_{m1}}{C_c}$$

$$R_1 = r_{ds2} \| r_{ds5}, \quad R_2 = r_{ds3}, \quad P_1 \cong \frac{-1}{g_{m3} R_1 R_2 C_c}$$

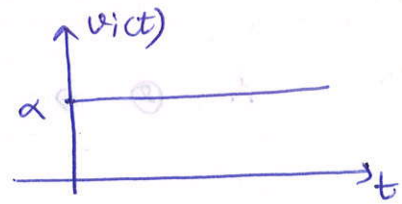
$$\therefore \quad A(j\omega) = \frac{A_v(j\omega)}{1 + A_v(j\omega)} = \frac{A_{v0}}{1 + A_{v0}} \cdot \frac{1}{1 - \frac{j\omega}{P_1(1 + A_{v0})}} \quad \text{(verify)}$$

(B/W enhances by the same amount

$= A_0 \dfrac{1}{1+j\omega\tau}$ where $\tau = \dfrac{-1}{P_1(1+A_{vo})}$



Let $v_i(t) = \alpha u(t) = \alpha \cdot (\text{step fn})$

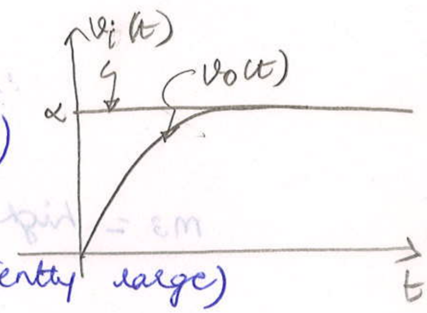


$\therefore \; v_i(s) = \dfrac{\alpha}{s}$

$\therefore \; v_o(s) = \dfrac{\alpha}{s} \cdot \dfrac{1}{1+s\tau} \quad (\because A_0 = 1) \quad = \alpha \cdot \dfrac{1}{s} + \dfrac{\alpha(-\tau)}{1+s\tau}$ (By P.F.E)

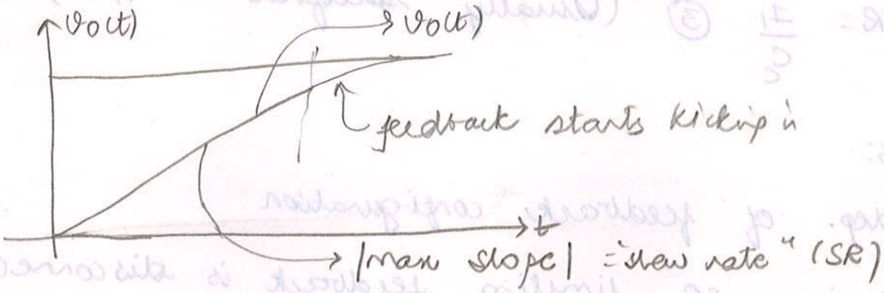
$= \dfrac{\alpha}{s} - \dfrac{\alpha}{s+\frac{1}{\tau}}$

$\therefore \; v_o(t) = \alpha u(t)\left(1 - e^{-t/\tau}\right)$



But ("slew rate limiting" (when α = sufficiently large)

$\Rightarrow$



Reason for SR limiting:

At $t = 0 + \epsilon$ (small $\epsilon > 0$), $v_{g2} \cong \alpha$ ($V_{g2} = V_{G2} + \alpha \Rightarrow v_{g2} = \alpha$)

and $v_{g1} \cong 0$ in ① because op-amp can't respond to step immediately.

$\exists\, \alpha' > 0$ such that $\forall\, |v_{g2} - v_{g1}| > \alpha'$ → what is α'?

have
$$\left.\begin{cases} I_{d1} = I_1,\ I_{d2} = 0 & \text{for } v_{g2} > v_{g1}\ (\alpha > 0) \\ I_{d1} = 0,\ I_{d2} = I_1 & \text{for } v_{g2} < v_{g1}\ (\alpha < 0) \end{cases}\right\} ②$$

∴ ① ⇒
$$I_{C_c} = \underset{(\text{KCL @ D5})}{I_{d2} - I_{d5}} = \underset{(M5, M4\ \text{mirror})}{I_{d2} - I_{d4}} = \underset{(M1, M4\ \text{series})}{I_{d2} - I_{d1}}$$

∴ ② ⇒
$$I_{C_c} = \begin{cases} I_1 & \text{if } v_{g2} - v_{g1} < -\alpha' \\ -I_1 & \text{if } v_{g2} - v_{g1} > \alpha' \end{cases}$$

i.e. $|I_{C_c}| = I_1$ when $|v_{g2} - v_{g1}| > \alpha'$

Recall $I_{C_c} = C_c \dfrac{dv_{cc}}{dt}$

$$\therefore \left|\frac{dv_{cc}}{dt}\right| = \frac{I_1}{C_c} = \text{const. slope.}$$

$M3$ = high gain C.S. stage $\Rightarrow \dfrac{dv_o}{dt} \cong \dfrac{dv_{c_c}}{dt}$ = const slope

$\therefore\ SR = \dfrac{I_1}{C_c}$ ③ (Usually specified in V/μs)

Observations:

1) SR = indep. of feedback configuration

Why? During SR limiting feedback is disconnected because first stage acts like a const. current source $\forall$ inputs w/ $|v_{g2} - v_{g1}| > \alpha'$

OH 2) SR limiting always increases settling time

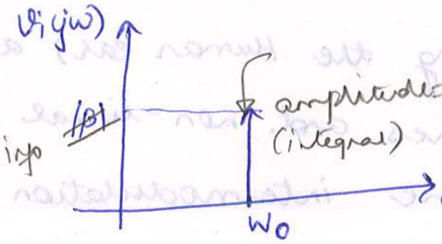
3) SR limiting introduces harmonic distortion

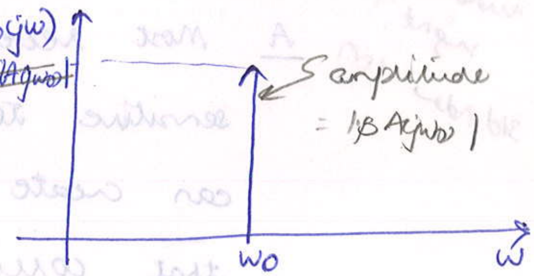
$\Leftrightarrow$ Behaves as a non-linear system $\quad \frac{1}{s\tau} \quad \frac{1}{s^2}$

Recall:

$v_i(t) = \beta \sin(w_0 t) \longrightarrow$ LTI system $A(jw)$ $\longrightarrow v_o(t) = \beta |A(jw_0)| \sin(w_0 t + \measuredangle A(jw_0))$

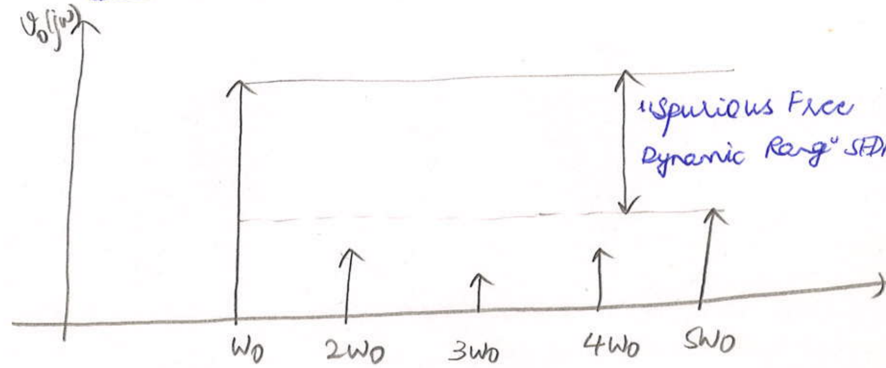
F.T. doesn't retain phase info



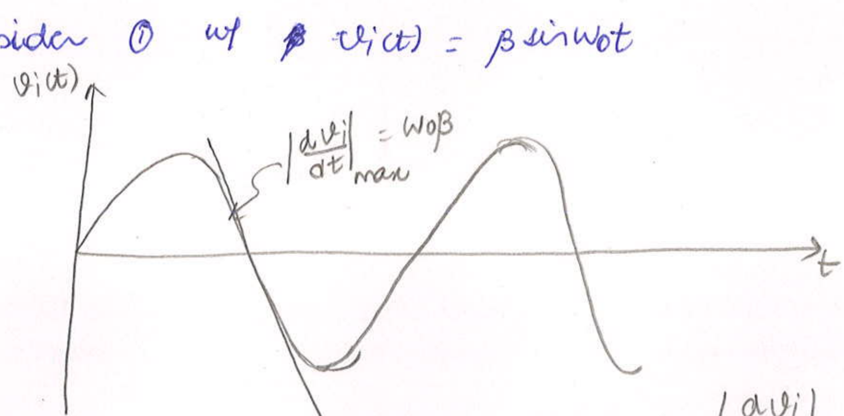


But $\quad v_i(t) = \beta \sin(w_0 t) \longrightarrow$ non-LTI system $\longrightarrow v_0 = \sum_{k=-\infty}^{\infty} \beta_k e^{jkw_0 t}$

Spurious tones don't decrease in amplitude?

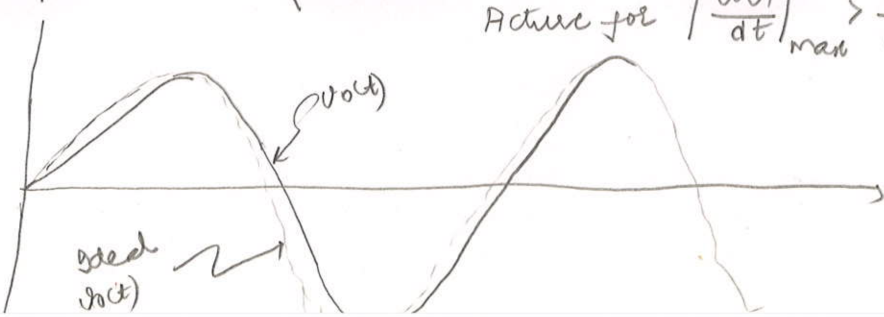


Ex $\quad$ Consider ① w/ $v_i(t) = \beta \sin w_0 t$



Active for $\left|\frac{dv_i}{dt}\right|_{max} > \frac{I_1}{C_c}$

non-LTI system $\longrightarrow$



Note: $v_o(t)$ = periodic but not sinusoidal

$\therefore$ $v_o(t)$ has Fourier series representation that $\Rightarrow$ harmonic distortion.

Internod IM can happen in wide band too right?

3rd order IM

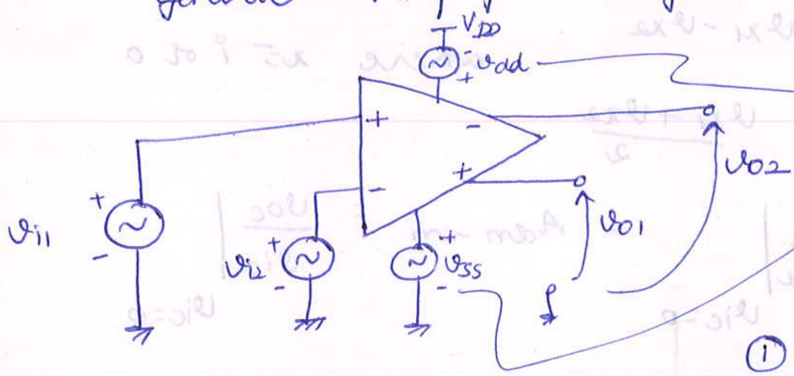
Q Why is non linear distortion usually worse than linear distortion?

A Most receivers (including the Human ear) are very sensitive to spurious tones and non-linear distortion can create spurious tone intermodulation products that corrupt signal of interest.

LEC #5, APRIL 15 '08

Op-Amp Design: Matching considerations

Generic Amplifier Stage:



⇒ Undesired ripple and noise on power supply lines.

①

For amps w/ differential i/p but single ended o/p use ① w/ $v_O = v_{O1} - v_{O2}$

Ideally, want $v_{i1} - v_{i2} = 0 \Rightarrow v_{O1} - v_{O2} = 0$ ②

If amplifier circuitry is perfectly symmetric and like components are perfectly matched, ② holds.

Otherwise, have a non-zero offset voltage, v_{OS}.

↳ why would there be v_{OS} for D_2S?

Def $v_{OS} = v_{i1} - v_{i2}$ (w/ $v_{i1} = -v_{i2}$) such that $v_{O1} - v_{O2} = 0$ in ①

Q Significance of $v_{OS} \neq 0$? (Offset builds up in amp. chain)

↳ This is DC, then why doesn't it get amplified?

A Feedback ckts. ideally set $v_{i1} = v_{i2} + \epsilon$ where $\epsilon \to 0$ as |loop gain| $\to \infty$. But for $v_{OS} \neq 0$, $\epsilon \to v_{OS}$ as " "

$\Rightarrow v_{OS}$ shows up as part of o/p signal

Similarly, circuit asymmetry & imperfect matching of ~~to~~ like components decreases CMRR & PSRR

CM ≡ common-mode, RR ≡ rejection ratio, PS ≡ power supply

Recall:

$$v_{od} = A_{dm} v_{id} + A_{cm-dm} v_{ic}$$

③

$$v_{oc} = A_{dm-cm} v_{id} + A_{cm} v_{ic}$$

where $v_{xd} = v_{x1} - v_{x2}$ where $x = i$ or o

& $v_{xc} = \dfrac{v_{x1} + v_{x2}}{2}$

$\therefore A_{dm} = \left.\dfrac{v_{od}}{v_{id}}\right|_{v_{ic}=0}$, $A_{dm-cm} = \left.\dfrac{v_{oc}}{v_{id}}\right|_{v_{ic}=0}$

$A_{cm-dm} = \left.\dfrac{v_{od}}{v_{ic}}\right|_{v_{id}=0}$, $A_{cm} = \left.\dfrac{v_{oc}}{v_{ic}}\right|_{v_{id}=0}$

Def 1: $CMRR = \dfrac{A_{dm}}{A_{cm-dm}}$

Ideally, want $CMRR = \infty$ (Need CMRR↑ so that are ↑,

Def 1: $PSRR_{DD} = \dfrac{A_{dm}}{A_{dd}}$ where $A_{dd} = \left.\dfrac{v_{od}}{v_{dd}}\right|_{v_{id}, v_{ic}=0}$

Why $PSRR_{DD}$ is more important than $PSRR_{SS}$?

$PSRR_{SS} = \dfrac{A_{dm}}{A_{ss}}$ where $A_{ss} = \left.\dfrac{v_{od}}{v_{ss}}\right|_{v_{id}, v_{ic}=0}$

2 stage gain during desensitization

Ideally, want $PSRR_{XX} = \infty$

LHP, RHP zero

Fact: Usually, 1st stage of op-amp most significantly affects v_{os}, CMRR, $PSRR_{XX}$

e.g. Suppose stage 1 has gain A_1, offset voltage $= v_{os1}$

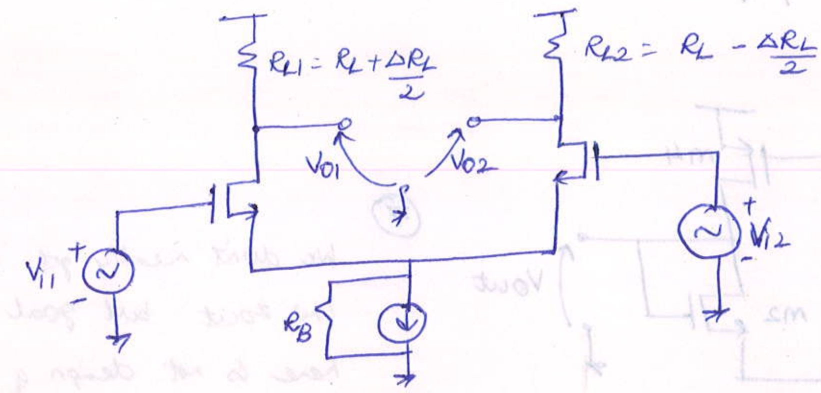
" ______ " 2 " ______ " A_2, " ______ " $= v_{os2}$

Then both stages together have $v_{os} = v_{os1} + \dfrac{v_{os2}}{A_1}$

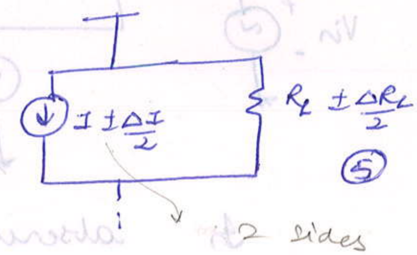
$\therefore$ If $|A_1| \gg 1$, then $v_{os} \approx v_{os1}$

Similarly for CMRR, $PSRR_{XX}$ (Verify) (In BJTs A_2 typ ↑↑ due to g_m↑ ⇒ 1st stage matters more)

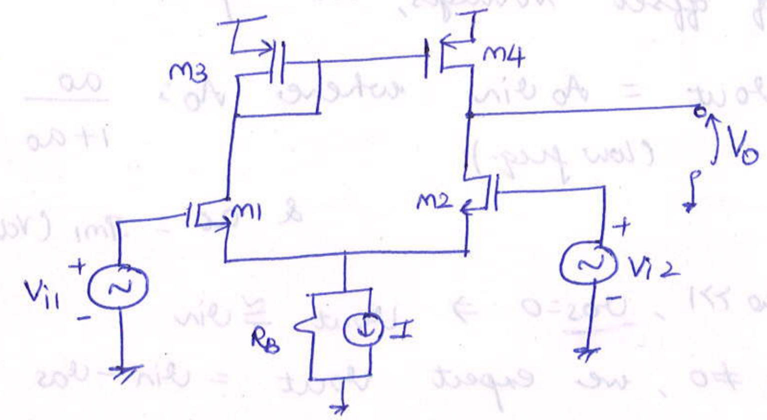
<u>Typical Op Amp 1st stages:</u>



or ④ w/ $R_L \pm \frac{\Delta R_L}{2}$ replaced by



or



Note: 1) ④, ⑤ symmetric circuits

∴ Only component mismatches (i.e. $M_1 \neq M_2$, $\Delta R_L \neq 0$, $\Delta I \neq 0$)

⇒ $V_{OS} \neq 0$, CMRR, $PSRR_{XX} \neq \infty$

$\neq \infty$ ∵ r_{tail} is present. A_{CM}

2) ⑥ = asymmetric circuit Check CMRR defn. in 164 notes

∴ Component mismatches & circuit asymmetry

⇒ $V_{OS} \neq 0$, CMRR, $PSRR_{XX} \neq \infty$

<u>Def:</u> "Systematic Offset" refers to effects of circuit asymmetry

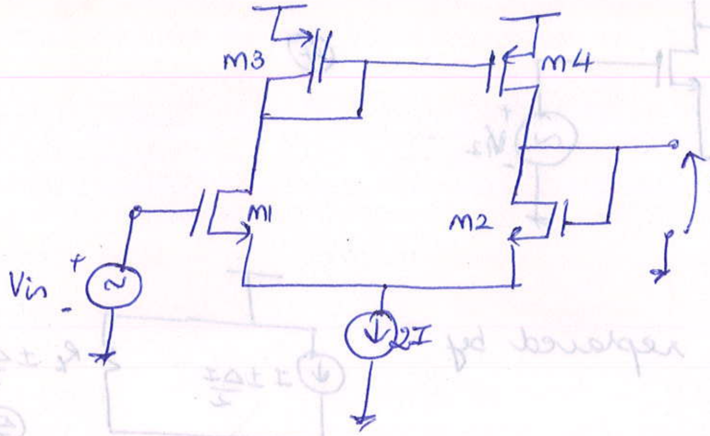
"Random Offset" " ________________ " component mismatches.

~~e.g.~~ <u>systematic</u>

e.g. Systematic Offset Voltage

vs Random " ________ "

<u>Ex</u> Consider amplifier in (6) connected as a voltage follower:



(7)

We don't really get low Zout but goal here is not design of voltage follower

In absence of offset voltages, we expect

$$v_{out} = A_0 v_{in} \quad \text{where } A_0 = \frac{a_0}{1+a_0}$$

(low freq.)

$$\& \quad a_0 = g_{m1}(r_{ds2} \| r_{ds4})$$

$\therefore$ Assuming $a_0 \gg 1$, $v_{os} = 0 \Rightarrow v_{out} \cong v_{in}$

Thus for $v_{os} \neq 0$, we expect $v_{out} = v_{in} - v_{os}$

This definition of offset voltage is consistent w/ that presented above for fully diff. amps.

For $V_{out} = V_{d1}$, circuit is symmetrically biased $\Rightarrow v_{os} = v_{osr} \equiv$ random offset voltage component

(This is done by setting V_{in} appropriately)

For $V_{out} \neq V_{d1}$, circuit is asymmetrically biased

(↳ I sets $V_{D1} = V_{D2}$? Then $V_{d1} = V_{d2}$ determines symmetry?)

$$\Rightarrow v_{os} = v_{osr} + \underbrace{v_{oss}}_{\text{systematic offset voltage}}$$

<u>Calculation of v_{osr}:</u>

$\therefore v_{osr} \equiv v_{in} - v_{out}$ when <u>V_{in} is such that $V_{d1} = V_{out}$</u> (condition for symmetric biasing)

Let $V_{TP} = (V_{T3} + V_{T4})/2$, $\Delta V_{TP} = V_{T3} - V_{T4}$

$K_P = (K_3 + K_4)/2$, $\Delta K_P = K_3 - K_4$

Similarly for V_{Tn}, K_n

HW Prob $\Rightarrow$ $$\frac{I_{d3} - I_{d4}}{I} \cong -\frac{2}{V_{GSP} - V_{TP}} \cdot \Delta V_{TP} + \frac{\Delta k_p}{k_p}$$

(holds for $V_{d1} = V_{out}$)

w/ $v_{in} = v_{out}$ & $V_{d1} = V_{out}$, can verify (exercise)

$$I_{d1} - I_{d2} = g_{mn}\left(-\Delta V_{Tn} + \frac{V_{GSn} - V_{Tn}}{2} \cdot \frac{\Delta k_n}{k_n}\right)$$

$$\left(\text{where } g_{mn} = \frac{g_{m1} + g_{m2}}{2}\right)$$

KCL $\Rightarrow$ $$(I_{d1} - I_{d2})\Big|_{V_{i1} = V_{i2}} + v_{osr}\, g_{mn} = I_{d3} - I_{d4}$$

$$\therefore \quad v_{osr} = \frac{1}{g_{mn}}\left[I_{d3} - I_{d4} - (I_{d1} - I_{d2})\right]$$

$$\therefore \quad v_{osr} = \frac{V_{GSn} - V_{Tn}}{2I}\left[\frac{-2\Delta V_{TP}}{V_{GSP} - V_{TP}} I + \frac{\Delta k_p}{k_p} I\right]$$

$$- \frac{1}{g_{mn}}\left[g_{mn}\left(-\Delta V_{Tn} + \frac{V_{GSn} - V_{Tn}}{2} \cdot \frac{\Delta k_n}{k_n}\right)\right]$$

$$\therefore \quad v_{osr} = -\frac{V_{GSn} - V_{Tn}}{V_{GSP} - V_{TP}} \cdot \Delta V_{TP} + \left(\frac{V_{GSn} - V_{Tn}}{2}\right)\left(\frac{\Delta k_p}{k_p} - \frac{\Delta k_n}{k_n}\right)$$

$+ \Delta V_{Tn}$

Reduce OD of i/p pair

Increase OD of o/p pair

<u>Calculation of v_{oss}</u> :

Consider (7) w/ feedback removed

& $V_{G1} = V_{G2} = V_{IN}$ $\quad (V_{IN} = V_{in} - v_{in})$ $\Big\}$ (8)

Unless M1 – M4 are such that $V_{DG2} = 0$ (i.e. feedback is removed) in circuit w/o feedback, it follows that $V_{out} \neq V_{in}$ in (7). because the feedback will adjust V_{G2} such that $V_{DG2} = 0$

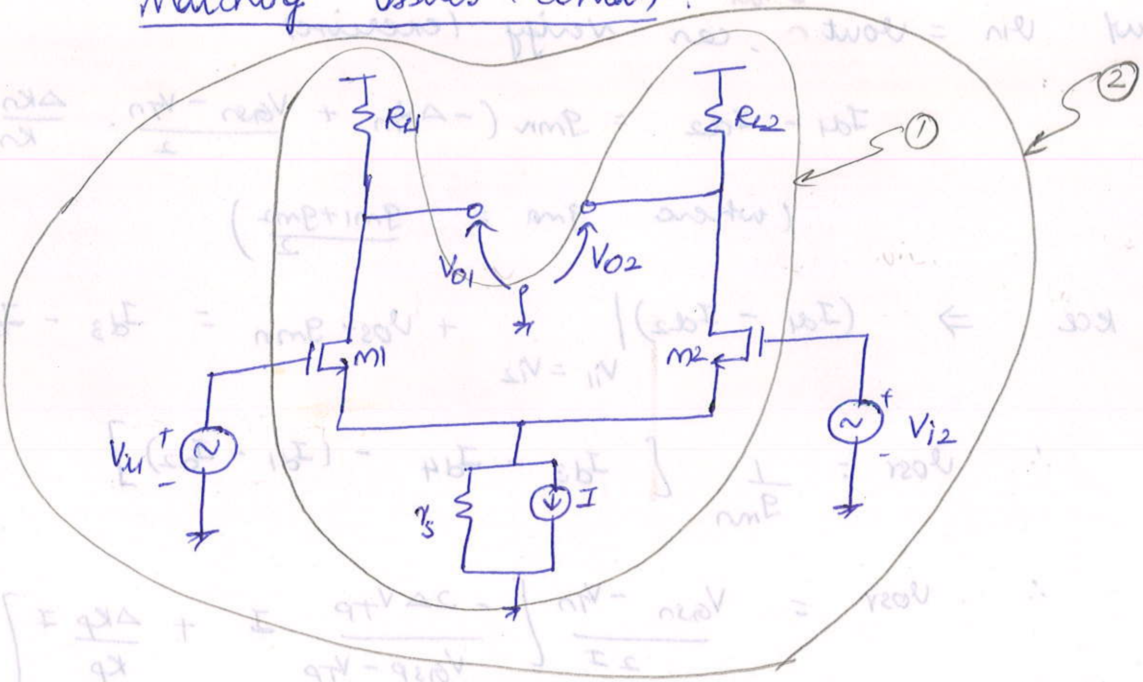
Know from previous work $\Rightarrow$ $v_{out} = g_{mn}(r_{dsn} \| r_{dsp})(v_{in} - v_{out})$

$$\therefore \quad V_{DG2}\Big|_{(8)} \cong g_{mn}(r_{dsn} \| r_{dsp})\, v_{oss}$$

$$\therefore \quad v_{oss} = \frac{V_{DG2}|_{(8)}}{g_{mn}}\left(\frac{1}{r_{dsn}} + \frac{1}{r_{dsp}}\right) = V_{DG2}\Big|_{(8)} \frac{V_{GS2} - V_{Tn}}{2 I_{D2}}\left(\frac{1}{r_{dsn}} + \frac{1}{r_{dsp}}\right)$$

LECTURE #6 APRIL 22 '08

Matching Issues (contd.):



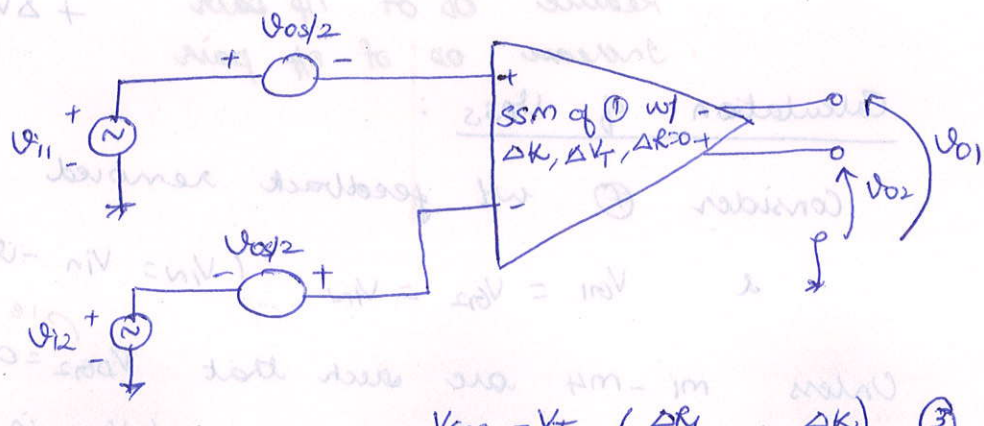
where $R_{L1} \equiv R_L + \frac{\Delta R}{2}$ M1: $K_1 = K + \frac{\Delta K}{2}$, $V_{T1} = V_T + \frac{\Delta V_T}{2}$

$R_{L2} \equiv R_L - \frac{\Delta R}{2}$ M2: $K_2 = K - \frac{\Delta K}{2}$, $V_{T2} = V_T - \frac{\Delta V_T}{2}$

Process variations $\Rightarrow \Delta R, \Delta K, \Delta V_T \neq 0$ in "matched" devices

$\Rightarrow v_{os} \neq 0$

S.S.M. of 2 $\equiv$



(Note that w/ this representation, $A_{cm\,dm} = 0$ $\gamma_{CMRR} = \infty$)

Showed $\quad v_{os} \cong \Delta V_T + \frac{V_{GS} - V_T}{2}\left(\frac{\Delta R_L}{R_L} + \frac{\Delta K}{K}\right) \quad$ 3

Remarks

1) $\Delta V_T, \Delta R_L, \Delta K$ not known in (advance) practice. They are typically assumed to be independent, zero mean, Gaussian random variables.

$\Rightarrow$ v_{os} = zero mean, Gaussian R.V.

$\Rightarrow$ v_{OS}, ΔV_T, ΔR_L, ΔK are described by their variances

$$\sigma_{v_{OS}}^2, \quad \sigma_{V_T}^2, \quad \sigma_{R_L}^2, \quad \sigma_k^2$$

Polarity doesn't matter.

where $\sigma_x^2 = E[X^2]$ (for a zero mean R.V. x)

But can we cancel some R.Vs statistically in ③?

$\therefore$ ③ $\Rightarrow$ $$\sigma_{v_{OS}}^2 = \underbrace{\sigma_{V_T}^2}_{\text{most imp parameter.}} + \frac{(V_{GS}-V_T)^2}{4}\left(\frac{\sigma_{R_L}^2}{R_L^2} + \frac{\sigma_k^2}{k^2}\right) \quad ④$$

Is it ~~atts~~ valid for all types of RVs?

2) ③ $\Rightarrow$ ④ only if R.Vs are uncorrelated

$\Rightarrow$ Only an approx.

formulae not for calculation but for qualitative insight.

3) ④ $\Rightarrow$ errors add in power not in amplitude

Modelling Parameter mismatches:

Let ~~x~~ M1, M2 be two xMOSTs (x = n or p), w/ nominal widths = W, nominal lengths = L, spacing between centers of "mass" = D

Experimental observations $\Rightarrow$ (Pelgrom et al, JSSC, Oct 89) ⑤

$$\left.\begin{aligned} \sigma_{V_{TO}}^2 &= \frac{A_{V_{TO}}^2}{WL} + S_{V_{TO}}^2 D^2 \\ \sigma_\gamma^2 &= \frac{A_\gamma^2}{WL} + S_\gamma^2 D^2 \end{aligned}\right\} \text{where } A_x, S_x \text{ are process dependent constants}$$

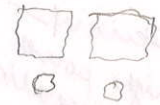
Recall: $V_T = V_{TO} + \gamma\left[\sqrt{\underset{\phi_0}{\cancel{2\phi_F}} + V_{SB}} - \sqrt{\underset{\phi_0}{2\phi_F}}\right]$

(Can approx $\sigma_{\phi_0}^2 \cong 0$)

Pmos $\Downarrow$ more process steps $\Downarrow$ more variance.

$$\frac{\sigma_K^2}{K^2} = \frac{A_W^2}{W^2L} + \frac{A_L^2}{WL^2} + \frac{A_\mu^2}{WL} + \frac{A_{cox}^2}{WL} + S_k^2 D \cong \frac{A_K^2}{WL} + S_k^2 D$$

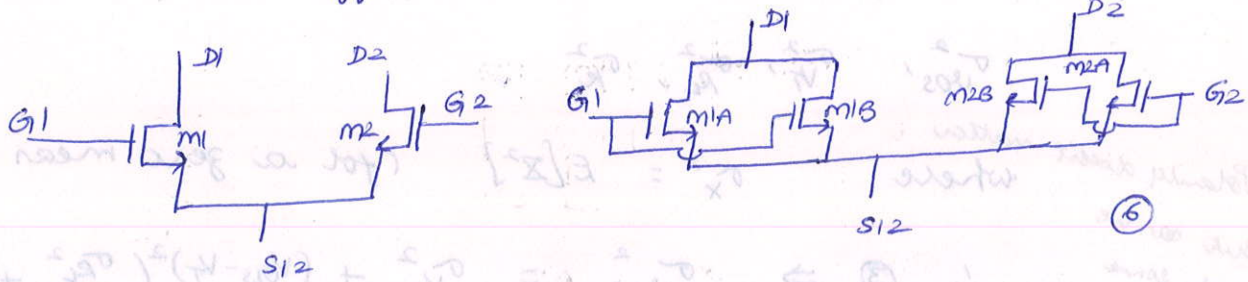
$\therefore$ $\left.\begin{array}{l}\text{Increasing } W, L \\ \text{Decreasing } D\end{array}\right\} \Rightarrow$ Better matching.



Layout considerations:

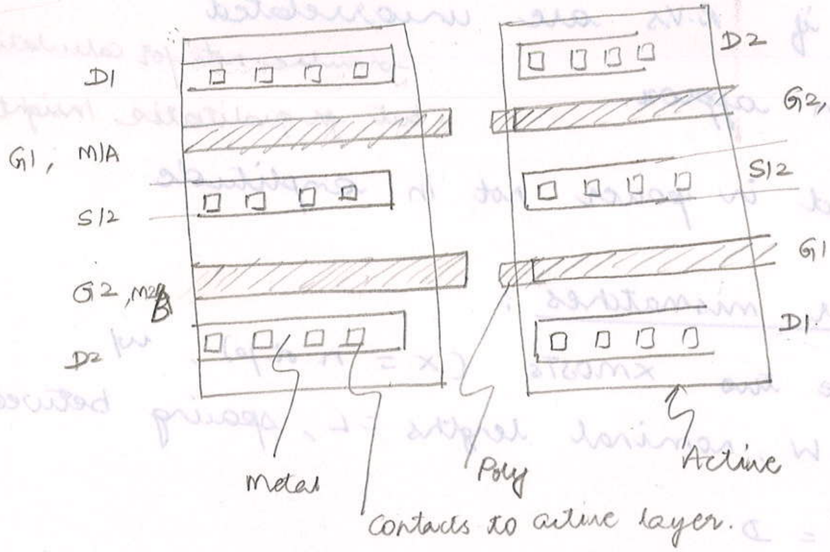
Can decrease effective D by interlacing stages.

Ex



(6)

where $W_{kx} = \frac{W_k}{2}$, $L_{kx} = L_k$ where $k=1,2$, $x = A$ or B.



(7) (Interconnects not shown)

(7) ≡ "common centroid layout"

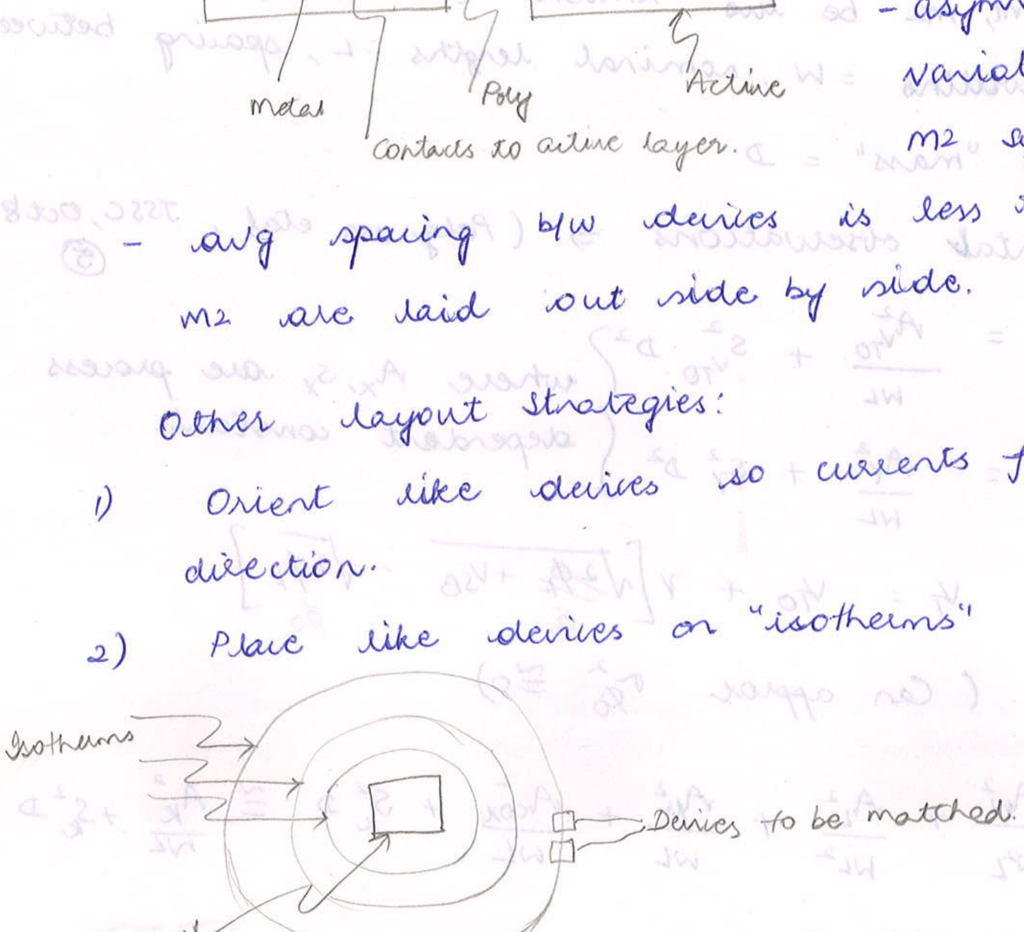
- asymmetric process variations affect M1 and M2 symmetrically.

- avg spacing b/w devices is less than if M1 and M2 are laid out side by side.

Other layout strategies:

1) Orient like devices so currents flow in same direction.

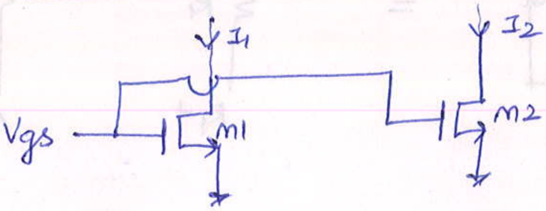
2) Place like devices on "isotherms"



LECTURE #7, APR 24'08 (Syllabus upto today)

(Numbering contd from last lecture)

Mismatch effects in current mirrors:



(8)

Assume M1 and M2 are nominally matched.

Process variations $\Rightarrow \Delta V_T, \Delta K \neq 0 \Rightarrow I_1 \neq I_2$

Let $I = \frac{1}{2}(I_1 + I_2)$, $\Delta I = I_1 - I_2$

HW (or Test) $\Rightarrow$ $$\frac{\Delta I}{I} = \frac{-2}{V_{GS} - V_T}\Delta V_T + \frac{\Delta K}{K} \quad (9)$$

(Assumes $V_{ds1} = V_{ds2}$)

(Else $I_{ds1} \neq I_{ds2}$ due to C.L.M.)

Note: Have $\frac{1}{V_{GS} - V_T}$ factor in (9) but

" $\frac{\quad}{V_{GS} - V_T}$ " " (3) (i.e. the result is diff. for SE & diff case)

[Proof for (9):

$$\Delta I = I_{ds1} - I_{ds2} = (1+\lambda V_{ds})\left[\frac{K_1}{2}(V_{gs} - V_{T1})^2 - \frac{K_2}{2}(V_{gs} - V_{T2})^2\right]$$

$$= \frac{(1+\lambda V_{ds})}{2}\left[\left(\overset{a}{K} + \overset{b}{\frac{\Delta K}{2}}\right)(\overset{c}{V_{gs}} - \overset{d}{V_{T1}})^2 - \left(\overset{a}{K} - \overset{b}{\frac{\Delta K}{2}}\right)(\overset{c}{V_{gs}} - \overset{e}{V_{T2}})^2\right]$$

$(a+b)(c-d)^2 - [(a-b)(c-e)^2]$
$= a(d^2 - e^2) - (d-e)2ca + 2bc^2 + b(d^2+e^2) - 2bc(d+e)$

$$= \frac{(1+\lambda V_{ds})}{2}\left[K(V_{T1}^2 - V_{T2}^2) - (V_{T1} - V_{T2})2V_{gs}(K) + 2\frac{\Delta K}{2}V_{gs}^2 + \frac{\Delta K}{2}(V_{T1}^2 + V_{T2}^2) - 2\frac{\Delta K}{2}V_{gs}(V_{T1}+V_{T2})\right]$$

$V_{T1}^2 + V_{T2}^2 = \left(V_T + \frac{\Delta V_T}{2}\right)^2 + \left(V_T - \frac{\Delta V_T}{2}\right)^2 = 2\left(V_T^2 + \frac{\Delta V_T^2}{4}\right)$

$$= \frac{(1+\lambda V_{ds})}{2}\left[K\, 2V_T\, \Delta V_T - 2\Delta V_T V_{gs}(K) + \Delta K V_{gs}^2 - \Delta K V_{gs} 2V_T + \frac{\Delta K}{2}2\left(V_T^2 + \cancel{\frac{\Delta V_T^2}{4}}\right)\right]$$

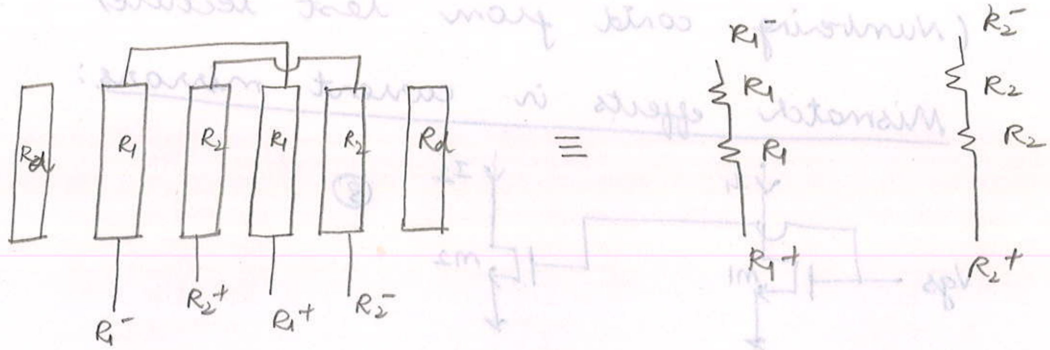
$$= \frac{(1+\lambda V_{ds})}{2}\left[2K\Delta V_T(V_T - V_{gs}) + \Delta K(V_{gs}^2 - V_{gs}2V_T + V_T^2)\right]$$

$$= \frac{(1+\lambda V_{ds})}{2}\left[-2K\Delta V_T V_{od} + \Delta K V_{od}^2\right] \quad ; \because I = \frac{K}{2}V_{OD}^2(1+\lambda V_{ds})$$

$$\Rightarrow \frac{\Delta I}{I} = \frac{-2\Delta V_T}{V_{od}} + \frac{\Delta K}{K} \;]$$

3) Use dummy devices to shield 1st and last interlaced devices from edge effects.

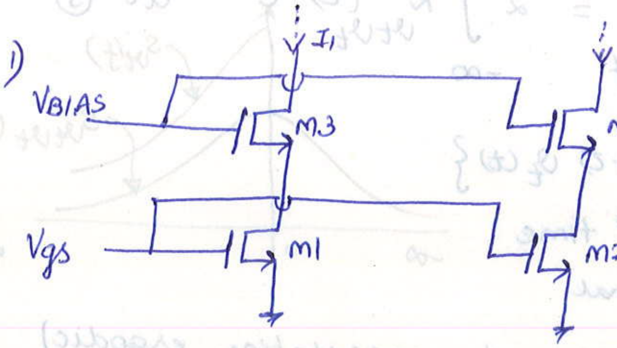
e.g.



4) Avoid asymmetric placement of metal lines around devices to be matched. (causes mechanical stress. what abt cap?)

Note: ⑨ holds for cascode current sources

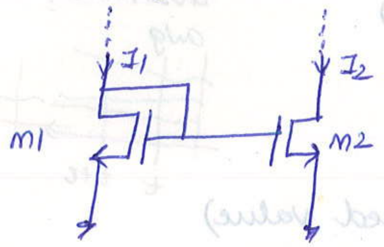
1)



Q Why?

A Because I_k $(k=1,2)$ is only a weak function of parameters in M3 and M4.

2) ⑨ holds for a current mirror provided $V_{ds_1} = V_{ds_2}$



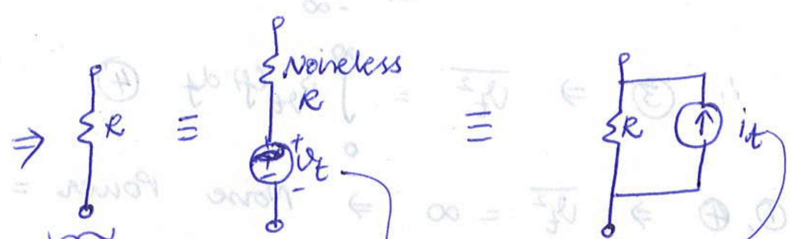
(Offset, CMRR, PSRR of op amps can be determined from these)

CIRCUIT NOISE (Reset numbering system)

Resistors:

Thermal noise of charge carriers

(⇒ "Thermal Noise" or "Johnson Noise" or "Nyquist Noise")



Actual Resistor

Equivalent models of noise behavior

v_t, i_t ≡ random processes (r.p's) (zero mean)

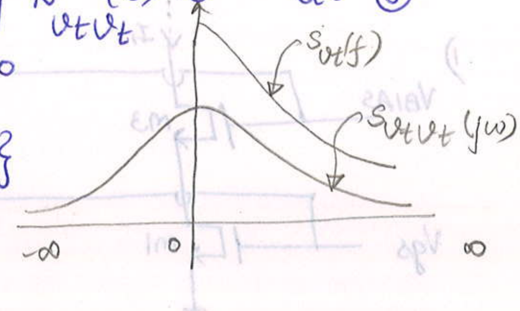
Can show: P.S.D. of $v_t(t)$ is: $S_{v_t}(f) = 4\hat{K}TR$ ①

" ______ " $i_t(t)$ is: $S_{i_t}(f) = \frac{4\hat{K}T}{R}$ ②

where $\hat{K} = 1.38 \times 10^{-23}$ $\underbrace{V.C}_{J}$ $/°K$. Boltzmann's const. (Note that $K = \mu C_{ox} \frac{W}{L}$)

T = Temperature in Kelvin.

$$S_{v_t}(f) \equiv 2 S_{v_t v_t}(j\omega)\Big|_{\omega = 2\pi f} \equiv 2\int_{-\infty}^{\infty} R_{v_t v_t}(\tau)\, e^{-j2\pi f\tau}\, d\tau \quad ③$$

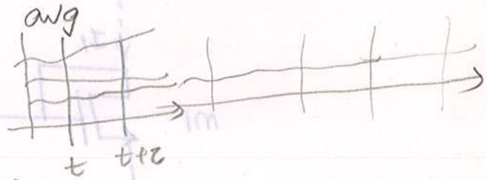


$$R_{v_t v_t}(\tau) \equiv E\{ v_t(t+\tau)\, v_t(t) \}$$

thermal   time

($v_t(t)$, $i_t(t)$ are assumed to be correlation ergodic)

ergodic → Ensemble avg = Time avg



$\overline{v_t^2}$ = mean squared value of $v_t(t)$

$$\equiv E[v_t^2] = R_{v_t v_t}(0)$$

($\therefore v_{t,rms} = \sqrt{\overline{v_t^2}}$ = root mean squared value)

Recall: $\overline{v_t^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{v_t v_t}(j\omega)\, d\omega$ $= \frac{1}{\cancel{2\pi}}\int_{-\infty}^{\infty} S_{v_t v_t}(j2\pi f)\, \cancel{2\pi}\, df$

$\therefore$ ③ $\Rightarrow \overline{v_t^2} = \int_0^{\infty} S_{v_t}(f)\, df$ ④ $\quad$ $\left(\because \int_{-\infty}^{0} S_{v_t}(f)\, df = 0 \right)$

But ①, ④ $\Rightarrow \overline{v_t^2} = \infty \Rightarrow$ Noise Power $= \infty$!

Obviously it is not true.

Actually, ①, ② = idealization ≡ "white noise"

More accurately:

$$S_{v_t}(f) = \frac{4hfR}{e^{hf/kT} - 1} \quad ⑤$$

where $h = 6.62 \times 10^{-34}$ J-sec, Planck's constant

$\therefore \overline{v_t^2} < \infty$

But $e^x \cong 1 + x$ for small x (e.g. $e^{0.2} - 1 = 0.22$)

$\frac{hf}{kT} < 0.2$ for $f < 1200$ GHz, $T = 290$K

$\therefore S_{v_t}(f) \cong 4kTR$ for $f < 1200$ GHz.

B/W of ckts., measmt. equipment << 1200 GHz (usually)

∴ White noise idealization ⇒ calculations give right answers but much simpler than if ⑤ used.

<u>Note:</u> $S_{v_t}(f)$ = one-sided PSD, valid for $0 \le f < \infty$

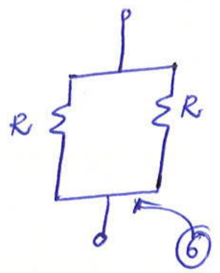
$S_{v_t v_t}(j\omega)$ = two-sided PSD, valid for $-\infty < \omega < \infty$

③ ⇒ " Total mean squared voltage in freq. band (f_1, f_2) "

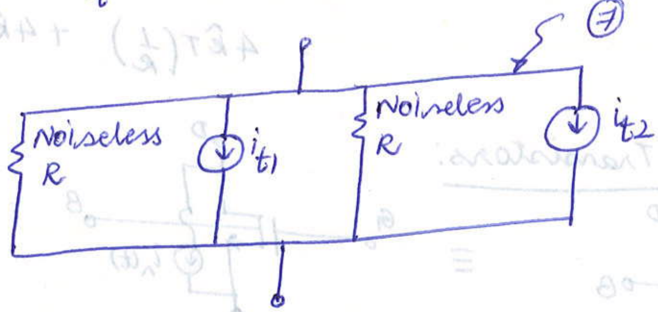
$$= \underbrace{\overline{v_t^2}}_{\text{re-use of notation}} = \int_{f_1}^{f_2} S_{v_t}(f)\, df$$

∴ For a resistor $\overline{v_t^2} = 4\hat{k}TR(f_2 - f_1)$

<u>Ex 1</u>

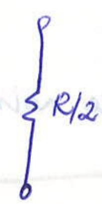


≡

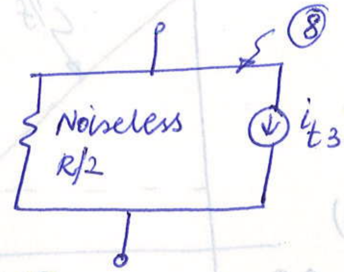


$$S_{i_{t1}}(f) = S_{i_{t2}}(f) = 4\hat{k}T\left(\frac{1}{R}\right)$$

⑥ ≡

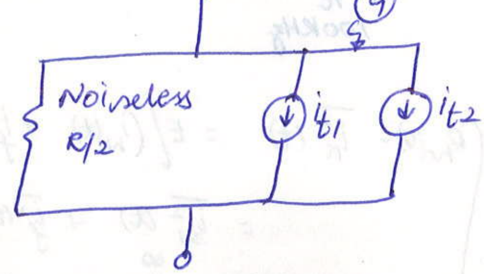


≡



$S_{i_{t3}}(f) = 4\hat{k}T\left(\frac{2}{R}\right)$

⑦ ≡



⑥ ≡ ⑦

$$\Rightarrow S_{i_{t3}}(f) = S_{i_{t1}}(f) + S_{i_{t2}}(f) \quad ⑩$$

= PSD of i_{t1} + PSD of i_{t2}

True ?

$$R_{i_{t3} i_{t3}}(\tau) = E\left[i_{t3}(t+\tau)\, i_{t3}(t)\right] = E\left[\left(i_{t1}(t+\tau) + i_{t2}(t+\tau)\right)\left(i_{t1}(t) + i_{t2}(t)\right)\right]$$

$$= R_{i_{t1} i_{t1}}(\tau) + R_{i_{t2} i_{t2}}(\tau) + E\left[i_{t1}(t+\tau)\, i_{t2}(t)\right] + E\left[i_{t2}(t+\tau)\, i_{t1}(t)\right]$$

<u>Fact:</u> Thermal motions in different resistors are uncorrelated

$$\Rightarrow E\left[i_{t1}(\tau_1)\; i_{t2}(\tau_2) \right] = E\left[i_{t1}(\tau_1) \right] E\left[i_{t2}(\tau_2) \right] \quad (11)$$

(<u>Aside:</u> For any R.P's i_{t1}, i_{t2} if (11) holds then i_{t1} and i_{t2} are said to be uncorrelated)

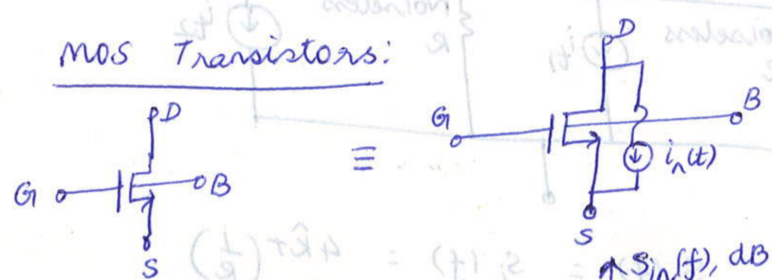
i_{t1} and i_{t2} ≡ zero mean

$$\Rightarrow R_{i_{t3}i_{t3}}(\tau) = R_{i_{t1}i_{t1}}(\tau) + R_{i_{t2}i_{t2}}(\tau)$$

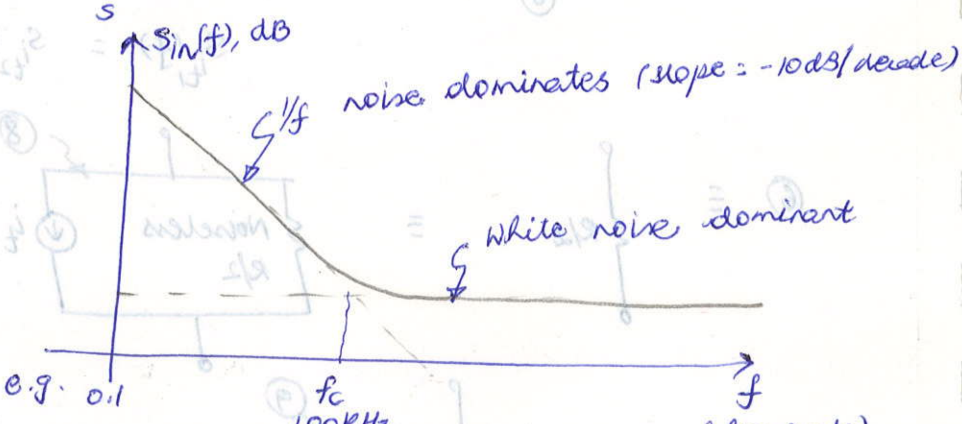
$$\therefore \quad S_{i_{t3}}(f) = S_{i_{t1}}(f) + S_{i_{t2}}(f)$$

$$= 4\hat{k}T\left(\tfrac{1}{R}\right) + 4\hat{k}T\left(\tfrac{1}{R}\right) = 4\hat{k}T\left(\tfrac{2}{R}\right) \Rightarrow (10) \text{ is correct}$$

<u>MOS Transistors:</u>



Typical Picture:



$$i_n(t) = \underbrace{i_w(t) + i_f(t)}$$

indep ($\Rightarrow$ uncorrelated) noise R.P's

$$\therefore \quad S_{i_n}(f) = S_{i_w}(f) + S_{i_f}(f)$$

$$R_{nn}(0) = \overline{i_n^2}(t) = E\left[\left(i_w(t) + i_f(t)\right)^2\right]$$

$$= \overline{i_w^2}(t) + \overline{i_f^2}(t)$$

$$S_{in}(f) \equiv 2\int_{-\infty}^{\infty} R_{nn}(0)\, d\tau$$

$$= 2\left[\int_{-\infty}^{\infty} R_{ww}(0)\, d\tau + \int_{-\infty}^{\infty} R_{ff}(0)\, d\tau\right]$$

$$= S_w(f) + S_{i_f}(f)$$

~ to (11) derivation

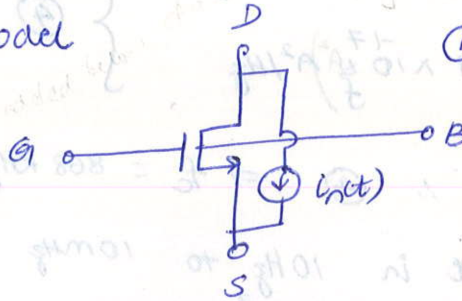
Note: 1/f noise is due to carriers in channel trapped @ [Si-SiO$_2$ interface

$\therefore$ By periodically turning off channel (possible in discrete systems like PLLs) can flatten 1/f noise.

($\because$ frees carriers)

MOS Transistors:

Noise model ①



$i_n(t) = \underbrace{i_w(t) + i_f(t)}$

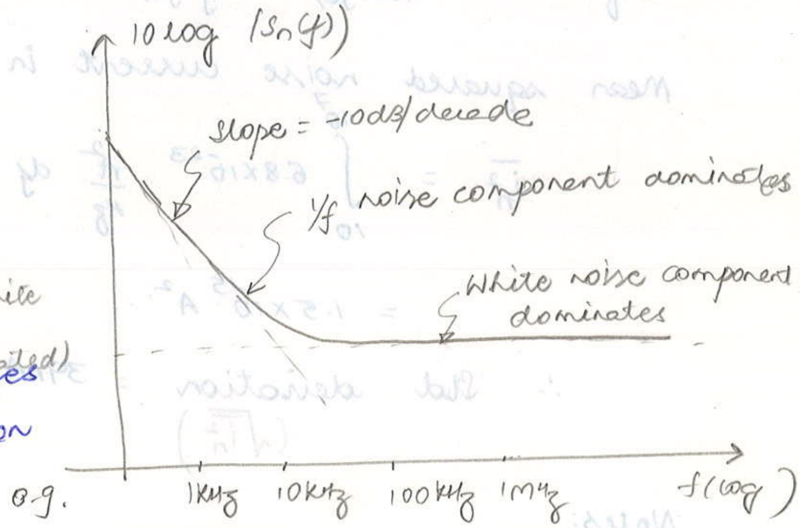
uncorrelated noise r.p's

$S_n(f) \equiv$ PSD of $i_n(t)$

$= S_{iw}(f) + S_{if}(f)$ ($\because$ 1/f noise & white noise are uncorrelated)

In strong inversion, $i_w(t)$ arises from thermal noise in inversion layer. (~ to that in resistor)

e.g.



$i_f(t)$ arises from mechanism not fully understood but related to presence of "traps" @ SiO_2 to Si interface which randomly exchanges carriers w/ channel.

Facts:

(process dependent, usually $1 \le \alpha \le 1.5$)

i) $S_{iw}(f) = \begin{cases} 4\hat{k}T\left(\frac{2}{3}\alpha g_m\right) & \text{(MOST in saturation)} \quad ② \\ 4\hat{k}T\alpha K(V_{GS}-V_T)\left(1-\frac{1}{3}\frac{V_{DS}}{V_{GS}-V_T}\right) & \text{(MOST in triode)} \quad ③ \end{cases}$

ii) $S_{if}(f) \cong \frac{\beta g_m^2}{WL}\frac{1}{f}$ (both triode & sat) ④

β = process dependent & sometimes bias dependent

(typ #s : $2\times10^{-9}\ V^2/\mu m^2$ for nMOSTs
$8\times10^{-11}\ V^2/\mu m^2$ for pMOSTs)

$\therefore$ 1/f noise can be reduced by 1) using pMOSTs instead of nMOSTs
2) using larger W & L

i.e. Let $\overline{i_{f1}^2} = \int_0^\infty S_{if}(f)\,df$ for W_1, L_1 MOST dimensions

Then, $\overline{i_{f2}^2} = \frac{1}{4}(\overline{i_{f1}^2})$ if $W_2 = 2W_1$ and $L_2 = 2L_1$

Ex2 nMOST w/ $V_{GS} - V_T = 2V$, $W = 200\mu m$, $L = 5\mu m$

$\mu_n C_{ox} = 68 \mu A/V^2$, $\alpha = 1.13$, $\beta = 2 \times 10^{-9} V^2 \mu m^2$

②, ③ $\Rightarrow$ $$\left.\begin{aligned} S_{iw}(f) &= 6.8 \times 10^{-23} \ A^2/Hz \\ S_{if}(f) &= \left(5.9 \times 10^{-17} \frac{A^2}{f}\right) A^2/Hz \end{aligned}\right\} ④$$

Def $\Rightarrow$ $S_{iw}(f_c) = S_{if}(f_c)$ & ④ $\Rightarrow f_c = 868\ KHz$

Mean squared noise current in 10 Hz to 10 MHz band,

$$\overline{i_n^2} = \int_{10}^{10^7} 6.8 \times 10^{-23} \frac{A^2}{Hz}\, df + \int_{10}^{10^7} 5.9 \times 10^{-17} A^2 \frac{1}{f}\, df$$

$$= 1.5 \times 10^{-15} A^2$$

$\therefore$ Std deviation $\left(\sqrt{\overline{i_n^2}}\right) = 39nA$ (gaussian distributed)

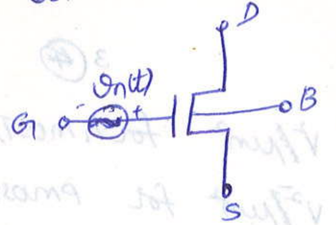
Notes:

1) ① doesnot model noise from extrinsic parasitic resistance of drain & source

gate noise?

2) ① does not model induced gate noise
$\equiv$ noise coupled from noisy channel to gate @ very high frequencies

3) Sometimes ① is modelled as:



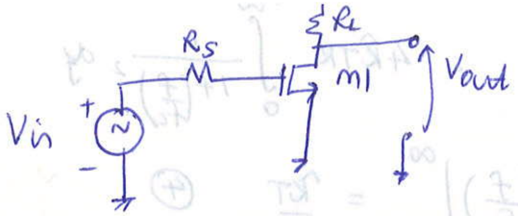
⑤ w/ $S_{vn}(f) = \frac{1}{g_m^2} S_{in}(f)$

Q Is ⑤ $\equiv$ ① in all cases?

A No 1) Suppose circuit driving gate has high impedance
$\Rightarrow$ $v_n(t)$ partly dropped across the impedance
$\Rightarrow$ predicted noise in i_d is too low.

2) Suppose $V_{DS} = 0 \Rightarrow g_m = 0 \Rightarrow S_{vn}(f) = \infty$

Ex



R_S, R_L, M1 contribute uncorrelated noise

Since SSM is LTI, total output noise is

$$S_{n_{out}}(f) = S_{v_{R_S}}(f)\,|g_{m_1}R_L|^2 + S_{v_{R_L}}(f) + S_{i_n}(f)\,|R_L|^2$$

(assumes $R_L << r_{ds1}$)

$$\therefore S_{n_{out}}(f) = 4\hat{k}T(R_S g_m^2 R_L^2 + R_L) + S_{i_n}(f) R_L^2$$

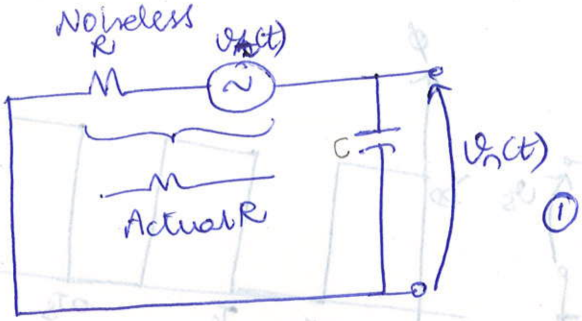
(assumes $f <<$ BW of ckt.)

Midterm: marks taken off for units & sanity check.

(all mixed signal ckts are limited by kT/C)

LEC #9, ~~APRIL~~ MAY 6 '08

KT/C Noise:



①

$S_{v_t}(f) = 4\hat{k}TR$

$\underbrace{S_{v_C}(f)}_{\text{cap noise PSD}} = 0$

(Ideal caps are noiseless
Actual " ______________ " for practical purposes)

$\frac{\frac{1}{sC}}{\frac{1}{sC}+R}$

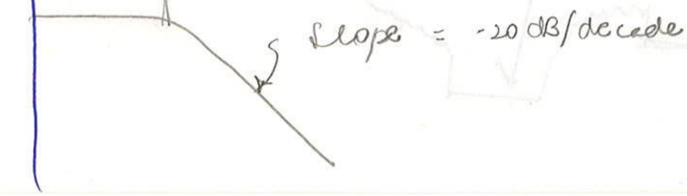
$\frac{1}{1+sCR}$

Let $H(j\omega) = \frac{v_n(j\omega)}{v_t(j\omega)}$. Then we know $S_{v_n}(f) = |H(j2\pi f)|^2 S_{v_t}(f)$

$$v_n(j\omega) = v_t(j\omega)\,\frac{1/j\omega C}{R + 1/j\omega C} \quad \therefore H(j\omega) = \frac{1}{1+j\omega CR}$$ ②

$$\therefore |H(j2\pi f)|^2 = \frac{1}{1+(\omega CR)^2}$$

$$\therefore S_{v_n}(f) = \frac{4\hat{k}TR}{1+(2\pi f RC)^2} = \frac{4\hat{k}TR}{1+(f/f_c)^2}$$ ③ $f_c = \frac{1}{2\pi RC}$



$$\overline{v_n^2(t)} = \int_0^\infty S_{v_n}(f)\,df = 4\hat{k}TR\int_0^\infty \frac{1}{1+\left(\frac{f}{f_c}\right)^2}\,df$$

$$= 4\hat{k}TR\,f_c \tan^{-1}\left(\frac{f}{f_c}\right)\Big|_0^\infty = \frac{\hat{k}T}{C} \quad (4)$$

$$4\hat{k}T\cancel{R}\frac{1}{2\cancel{\pi}\cancel{R}C}\frac{\cancel{\pi}}{2}$$

$\Rightarrow$ $\overline{v_n^2(t)}$ does not depend on R!

<u>Q</u> Why?

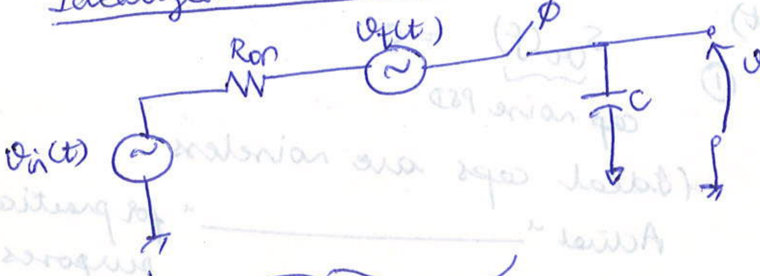
<u>A</u> Increasing R increases the $\overline{v_t^2(t)}$ but decreases bandwidth of RC filter $\Rightarrow$ total m.s. noise voltage across C is independent of R.

<u>Q</u> Significance?

<u>A</u> Can reduce noise in practice only by increasing C.

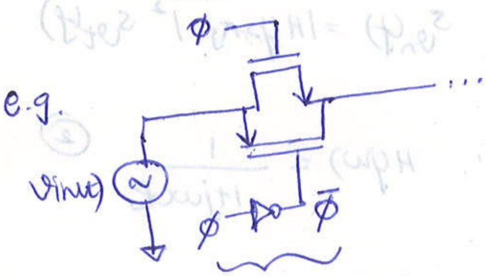
$\Rightarrow$ Higher power consumption.

<u>Idealized S/H circuit:</u>



$(\alpha < 1)$

e.g.

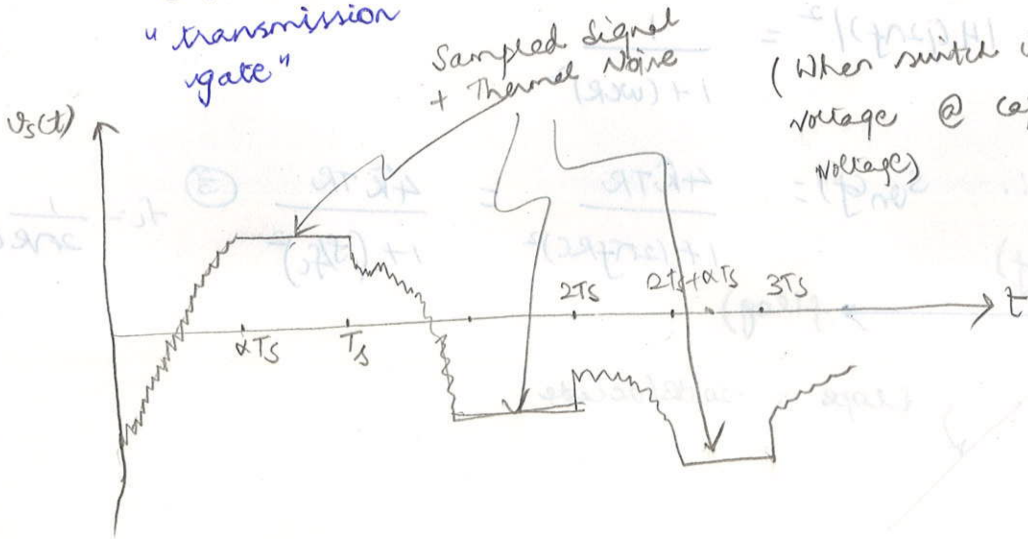


"transmission gate"

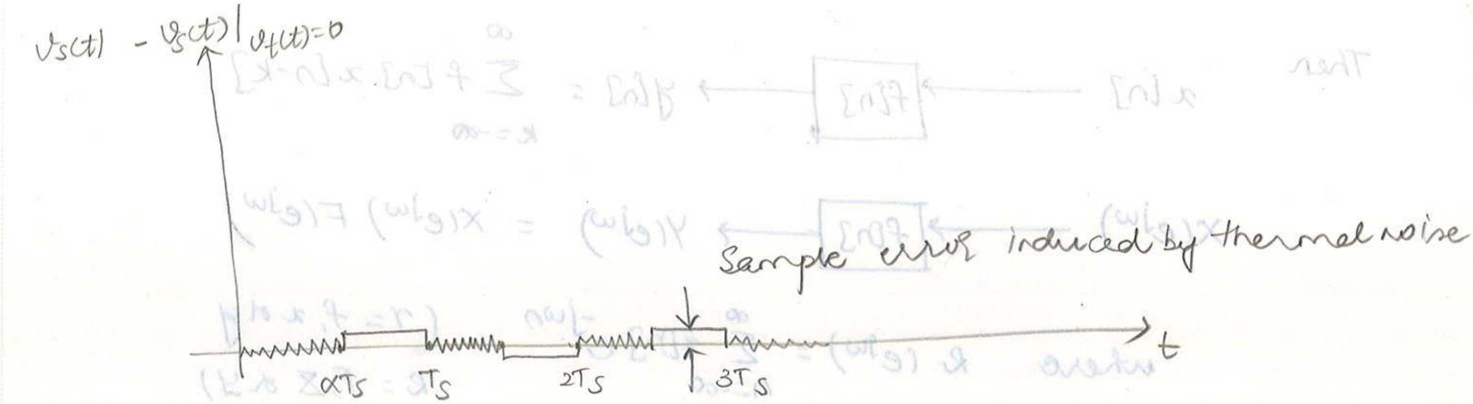
$R_{on} = R_s + R_{transgate}\Big|_{\phi = V_{DD}}$

voltage source R

1) Why this condition
2) passgate resistance diff. for $\phi = 1, 0$



(When switch is OFF, voltage @ cap is sampled voltage)



Let $x[n] = v_s(nT_s + \alpha T_s)\Big|_{v_t(t)=0} \equiv$ "ideal sampled signal"

$\hat{x}[n] = v_s(nT_s + \alpha T_s) \equiv$ "actual ______"

∴ "Sampling error from thermal noise"

$\equiv e[n] = \hat{x}[n] - x[n]$

Discrete time random process

(Zero-mean - why?) $v_t(t) \to$ zero mean
$LTI \to v_n(t) \Rightarrow$ zero mean.

i.e., $\hat{x}[n] = x[n] + e[n]$

Interested in : $R_{ee}[k] = E\{x[n]\, x[n+k]\} \equiv$ "discrete-time autocorrelation"

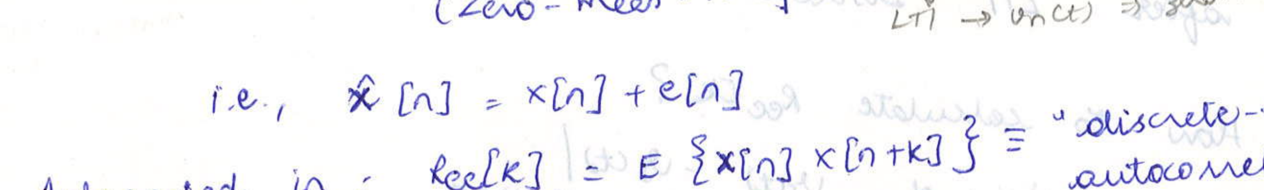
& $S_{ee}(e^{j\omega}) = \sum_{k=-\infty}^{\infty} R_{ee}[k]\, e^{-j\omega k} \equiv$ "discrete PSD"

Then $\overline{e^2[n]} = E[e^2[n]] = R_{ee}[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{ee}(e^{j\omega})\, d\omega$ | Assumption: noise is W.S.S. true for thermal case | → By Parseval's theorem

We are interested in PSD. Why?

Ex Let $f[n]$ = impulse response of an LTI discrete-time filter

i.e.



Then $x[n] \longrightarrow \boxed{f[n]} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} f[n]\, x[n-k]$

$X(e^{j\omega}) \longrightarrow \boxed{f[n]} \longrightarrow Y(e^{j\omega}) = X(e^{j\omega})\, F(e^{j\omega})$

where $R(e^{j\omega}) = \sum_{n=-\infty}^{\infty} r[n]\, e^{-j\omega n}$ ($r = f, x$ or y; $R = F, X$ or Y)

In our case:

$e[n] \longrightarrow \boxed{f[n]} \longrightarrow \mathcal{E}[n]$

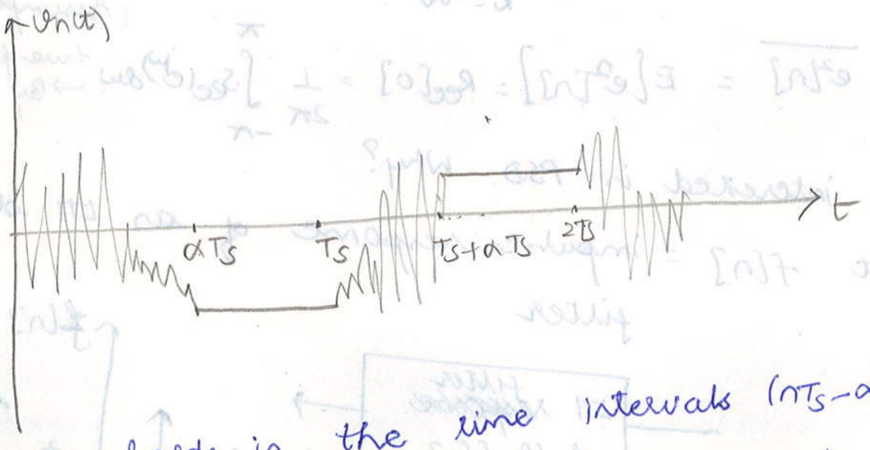
$e[n]$: W.S.S. r.p.; $\mathcal{E}[n]$: W.S.S. r.p

where $S_{\mathcal{E}\mathcal{E}}(e^{j\omega}) = |F(e^{j\omega})|^2\, S_{ee}(e^{j\omega})$

$\therefore$ Knowing $S_{\mathcal{E}\mathcal{E}}(e^{j\omega})$ allows us to calculate m.s. error after LTI Discrete Time Signal Processing (DSP)

How to calculate $R_{ee}[k]$?

Closeup view of $\underbrace{v_s(t) - v_s(t)\big|_{v_t(t)=0}}_{\text{call } v_n(t)}$



Idea: Ignore holds in the time intervals $(nT_s - \alpha T_s, nT_s]$ $n=1,2,\dots$

Outside these intervals, the resulting signal has same statistics as the signal.

$$v_n'(t) = v_n(t)\big|_{\phi = V_{DD}} + \sum_{k=-\infty}^{\infty} \beta[k]\, h(t - k\alpha T_s) \quad (5)$$

when switch is closed

$v_n'(t)$ has same statistics as $v_n(t)$ + extra transfer fns where $v_n(t)\big|_{\phi = V_{DD}}$ corresponds to system 1

β gives same signal b/w holds as if holds had not been there.

$\beta[k]$ = r.p.

$h[t]$ = impulse response of 2

$$h(t) = u(t)\,\frac{1}{RC}\,e^{-t/RC}$$

$H(s) = \dfrac{1/RC}{s + \frac{1}{RC}}$ $\Rightarrow h(t) = \frac{1}{RC}e^{-t/RC}u(t)$

In practice, R_{on}, C chosen such that $h(t) \cong 0$ for $t \geq \alpha T_s$
(else we have settling error in samples)

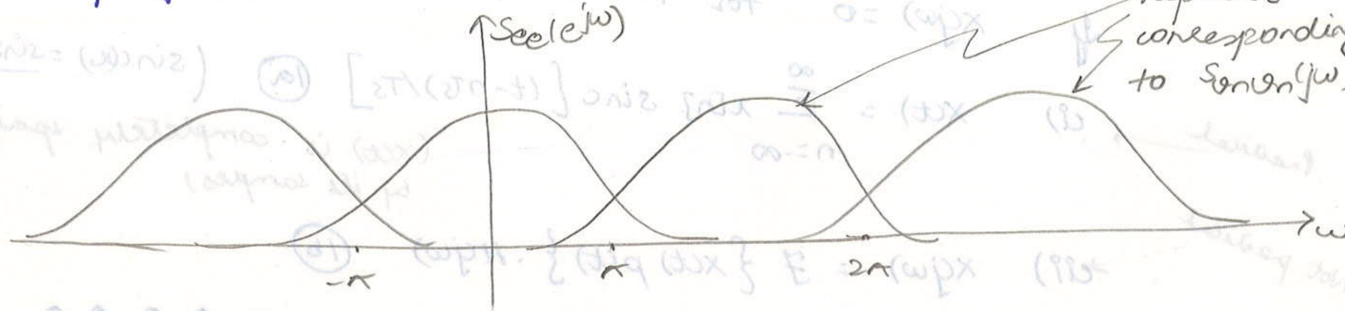
(settling time small, else would be just LPFing i/p)

$\therefore$ Can model $e[n]$ as:

$$e[n] = v_n(nT_s)\big|_{\phi = V_{DD}}$$ can ignore the αT because W.S.S.

$$\therefore \quad R_{ee}[k] = R_{v_n v_n}(kT_s)$$

— Sampling theorem $\Rightarrow$ $S_{ee}(e^{j\omega}) = \dfrac{1}{T_s}\displaystyle\sum_{k=-\infty}^{\infty} S_{v_n v_n}\left[j\left(\frac{\omega}{T_s}\right) - j\left(\frac{2\pi k}{T_s}\right)\right]$



$$\overline{e^2[n]} = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{ee}(e^{j\omega})\,d\omega$$

$$= \frac{1}{2\pi T_s}\int_{-\infty}^{\infty} S_{v_n v_n}\left(\frac{j\omega}{T_s}\right)d\omega = \frac{2\cdot 1}{2\pi T_s}\int_{0}^{\infty} S_{v_n v_n}\left(\frac{j\omega}{T_s}\right)d\omega$$

(Recall: $S_{v_n}(f) = 2\,S_{v_n v_n}(2\pi f)$ for $f > 0$)

$$\therefore \quad S_{v_n v_n}\left(\frac{j\omega}{T_s}\right) = \frac{1}{2}\,S_{v_n}\left(\frac{\omega}{2\pi T_s}\right) \quad \omega > 0$$

$$\overline{e^2[n]} = \frac{1}{2\pi T_s}\int_{0}^{\infty} S_{v_n}\left(\frac{\omega}{2\pi T_s}\right)d\omega$$

let $\Omega = \dfrac{\omega}{2\pi T_s}$

$\Rightarrow d\omega = 2\pi T_s\, d\Omega$

$$= \int_{0}^{\infty} S_{v_n}(\Omega)\,d\Omega = \frac{kT}{C}$$

($\Rightarrow$ whether sampling is done or not, noise variance is not changed.)

RECT #10 MAY 8 '08

(CT ⇒ Area ↑
Power ↑)

Ex $T = 290K$, $C = 1pF$; if have 1V RMS signal ⇒ limited to 12.3 bits

For $C = 100pF$, limited to 15.7 bits

(Oversampling ⇒ more caps in time)

Switched cap ckts.
(Continuous amp, discrete time)

SWITCHED CAPACITOR CIRCUITS:

Preliminaries:

Let $x(t) \equiv$ analog signal (continuous time)

$x[n] = x(nT_s)$, $n = .., -1, 0, 1, ..$ $\equiv$ discrete-time signal

Recall Sampling Theorem:

If $X(j\omega) = 0$ for $|\omega| > \omega_m$, and $T_s < \frac{\pi}{\omega_m}$, then

Practical → (i) $x(t) = \sum_{n=-\infty}^{\infty} x[n] \,\text{sinc}\left[(t - nT_s)/T_s\right]$ (1a) $\left(\text{sinc}(x) = \frac{\sin \pi(x)}{x}\right)$

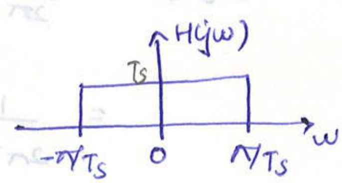
($x(t)$ is completely specified by its samples)

Not practical → (ii) $X(j\omega) = \mathcal{F}\{x(t)\, p(t)\} \cdot H(j\omega)$ (1b)

Ideal LPF not stable !?

where $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

& $H(j\omega) = \begin{cases} T_s & \text{if } |\omega| < \pi/T_s \\ 0 & \text{otherwise} \end{cases}$



Implication:

Let $H_c(j\omega)$ be the freq. resp. of an LTI cont.-time system. Then $\exists\, h[n]$, $n = .., -1, 0, 1, 2, ..$ S.T.

$x(t) \longrightarrow \boxed{H_c(j\omega)} \longrightarrow y(t)$

|||

$x(t) \longrightarrow \boxed{C/D} \xrightarrow{x[n]} \boxed{h[n] \text{ Imp. resp.}} \xrightarrow{y[n]} \boxed{D/C} \longrightarrow \hat{y}(t)$

C/D: $1/T_s$

D/C: Discrete to continuous time converter (via 1a)

C/D: Continuous-to discrete time converter (e.g. sample & hold ckt.)

where $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$ ②

and $\hat{y}(t) = y(t) \;\; \forall t$ provided $X(j\omega) = 0 \;\; \forall |\omega| > \pi/T_s$

<u>Defs:</u> (i) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$ ≡ Discrete-Time Fourier Transform (DTFT)

(This is continuous in freq. If it is sampled then it is DFT. FFT is method for calculating DFT)

Note: different use of ω from continuous time F.T.

(ii) $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$ ≡ Z-transform (if exists for some $\underbrace{R_0 < |z| < R_1}$, $R_0 < R_1$) (Always include ROC in Z-transform)

Called Region of Convergence (ROC)

Recall:

1. $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$ (if DTFT exists)

z^{-1} $e^{j\omega}$

complex.

2-d fn. → 2d variable

2. $X(e^{j\omega}) = X(z)\Big|_{z=e^{j\omega}}$; periodic -2π

$x[n] \in \mathbb{R} \Rightarrow X(e^{j\omega}) = X^*(e^{-j\omega})$

3. $X(e^{j\omega}) = \mathcal{F}\{x(t)\, p(t)\}\Big|_{T_s=1}$ (see ①) ③

C.T. F.T.

$\Rightarrow$ connection b/w $X(e^{j\omega})$ and $X(j\omega)$

periodic

band-limited

4. Properties of DTFT & Z-transform

e.g. $h[n] * x[n] \longleftrightarrow$ ~~$H(e^{j\omega})$~~ $H(e^{j\omega})\, X(e^{j\omega})$

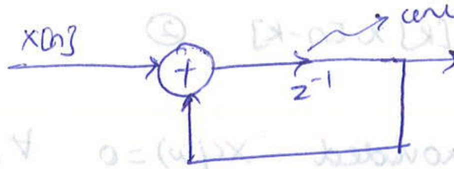
$H(z)\, X(z)$ ROC ⊃ (ROC of $H(z)$) ∩ (" $X(z)$)

<u>Q.</u> Why bother w/ Z-transforms instead of using DTFT only? & vice-versa?

(C.T. case: integrator analyzed by Laplace instead of F.T.)

<u>A</u> DTFT provides intuition and connection to continuous time F.T. but Z-transform often exists when DTFT does not.

Ex



unit delay. $V[n] \leftrightarrow V[z] \Rightarrow V[n-1] \leftrightarrow z^{-1}V[z]$

$y[n]$ = discrete time integrator

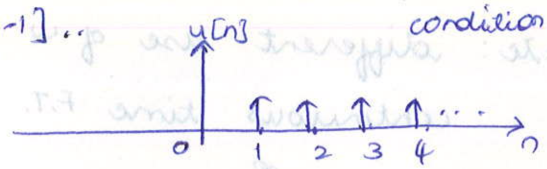
④'

$$y[n] = x[n-1] + y[n-1] = \sum_{k=n_0}^{n-1} x[k] + y[n_0] \quad (\text{any } n_0 < n)$$

Starting time

initial condition

④ $\Rightarrow h[n] = u[n-1]$..



$$\sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n} = \sum_{n=1}^{\infty} e^{-j\omega n} = \infty \Rightarrow H(e^{j\omega}) \text{ doesnot exist}$$

But, $$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n} = \sum_{n=1}^{\infty} z^{-n} = \frac{z^{-1}}{1-z^{-1}} \quad \underbrace{\forall\, |z|>1}_{R.O.C}$$

Note: ④ is not BIBO stable

(e.g. $x[n] = u[n] \Rightarrow y[n] = (n-1)u(n-1)$ (Verify)

$\Rightarrow y[n]$ = unbounded)

But $H(z)$ exists $\Rightarrow$ Can use freq. domain tools to analyze

(e.g. Convolution Theorem etc.)

REC #11 MAY 13 '08

Switched Capacitor circuits:

Let $H_c(j\omega)$ = freq. response of a continuous time filter

Let $x(t)$ = i/p signal w/ $X(j\omega) = 0 \;\; \forall \; |\omega| \geq \pi/T_s$ (i.e. $x(t)$ is band limited)

Def: (i) Continuous to discrete time converter

$$x(t) \longrightarrow \boxed{C/D} \longrightarrow x[n] = x(nT_s) \quad ①$$

(ii) Discrete to continuous time converter

$$y[n] \longrightarrow \boxed{D/C} \longrightarrow y(t) \quad ②$$

where $y(t) = \sum_{n=-\infty}^{\infty} y[n] \, \text{sinc}\left[(t - nT_s)/T_s\right]$

Sampling theorem implies:

$$x(t) \longrightarrow \boxed{H_c(j\omega)} \longrightarrow y(t) \quad ③$$

$$|||$$

can pre-filter/equalize to take care of

has sync roll off

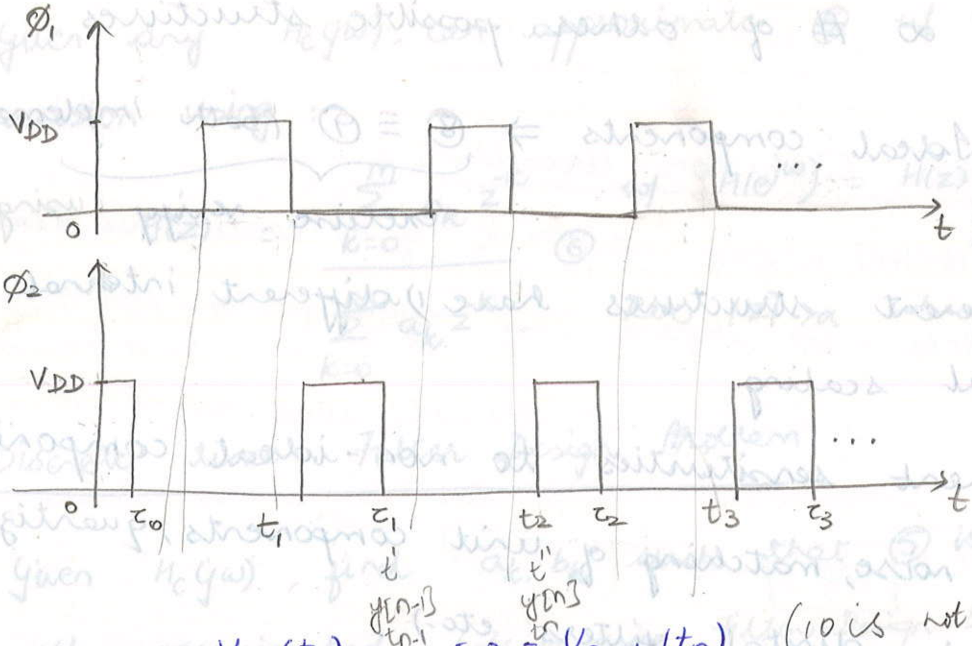
$$x(t) \longrightarrow \boxed{C/D} \xrightarrow{x[n]} \boxed{H(e^{j\omega})} \xrightarrow{y[n]} \boxed{D/C} \longrightarrow y(t) \quad ④$$

where $H(e^{j\omega}) = \begin{cases} H_c(j\omega/T_s) & |\omega| < \pi \\ \text{periodic elsewhere} \end{cases}$ ⑤

Facts:

1) Can well approximate ① using sample and hold (S/H) circuits

2) " ________________ " ② using S/H followed by continuous time analog filters.



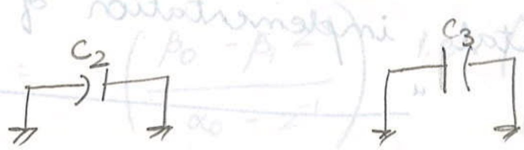
Implementation details soon

$x[n] \equiv V_{in}(t_n)$, $y[n] \equiv V_{out}(t_n)$ (10 is not 7-] ∵ ckt is diff. for $\phi_1 = 0$ & 1 ; $\phi_2 = 0$ & 1)

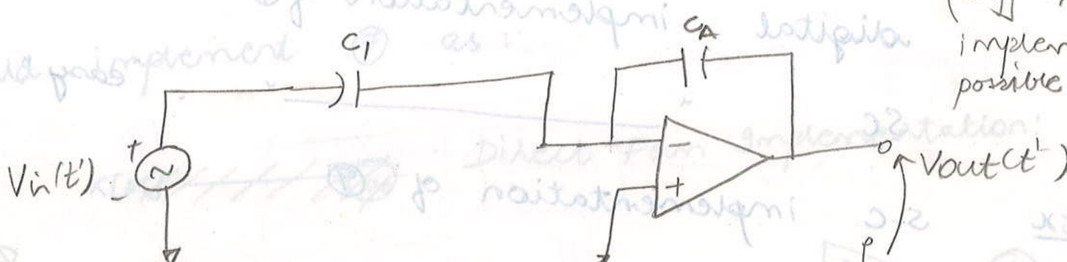
At $t' = \tau_{n-1} - \epsilon$; ($\epsilon > 0$, small)

(Cap notation)

bottom plate



(10) $\equiv$

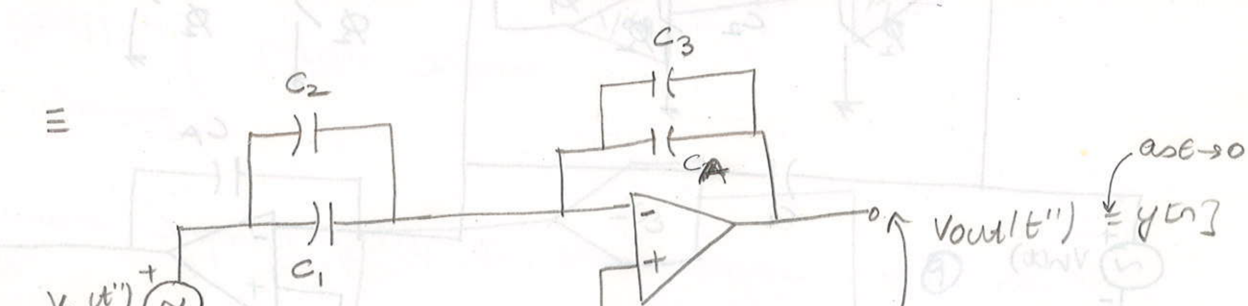


(Differential implementations possible & needed)

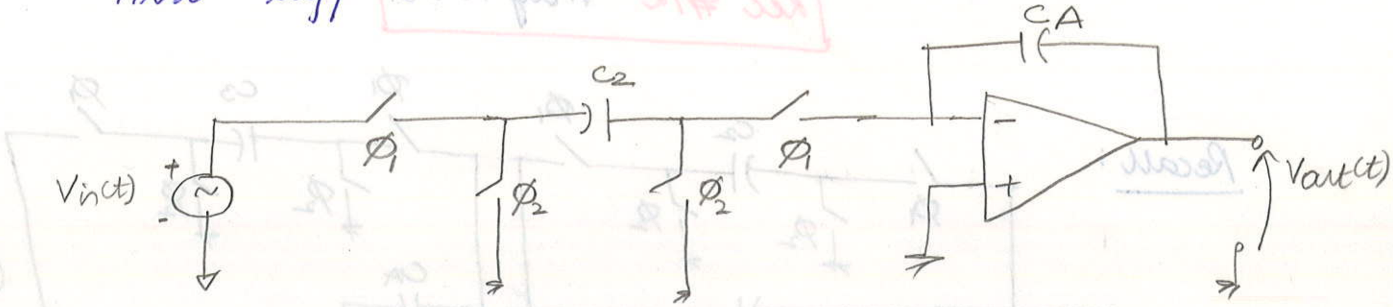
for noise rejection

At $t'' = t_n - \epsilon$:

(10) $\equiv$



First suppose $C_1 = C_3 = 0$. Then (10) becomes



place bottom plate less sensitive to parasitics

Consider charge transfer b/w t_{n-1} & t_n :

why is $Q_{C_A} = Q_{C_2}$?

- $V_{out}(t) = V_{out}(t_{n-1}) = y[n-1]$ $\forall t$ from t_{n-1} until ϕ_1 first goes high

- When ϕ_1 first goes high, charge on C_2 is zero. At time t_n, voltage across C_2 is $V_{in}(t_n) = x[n]$. So (assuming a virtual ground) it's total charge is $C_2 x[n]$

how is this defined? $t_n = t''$

- But all the current that charges C_2 also passes through C_A.

$C_A V_{out}$

$C_A(V_{in} - V_{out})$

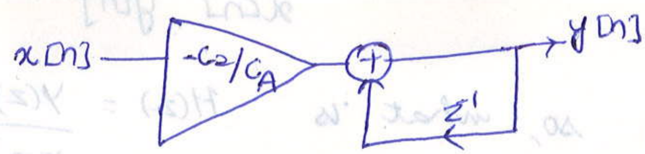
$\Rightarrow$ Charge on C_A changes by $C_2 x[n]$

$\Rightarrow$ Voltage across C_A changes by $C_2 x[n] / C_A$

$$\therefore \quad V_{out}(t_n) - V_{out}(t_{n-1}) = -\frac{C_2}{C_A} x[n]$$

$$\therefore \quad y[n] - y[n-1] = -\frac{C_2}{C_A} x[n]$$

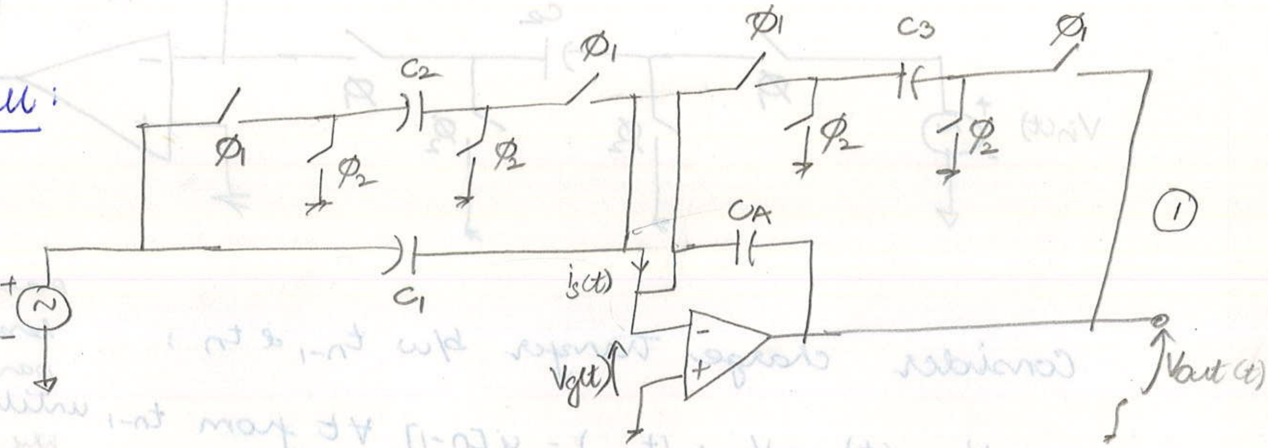
$$y[n] = y[n-1] - \frac{C_2}{C_A} x[n]$$

$\Rightarrow$ 

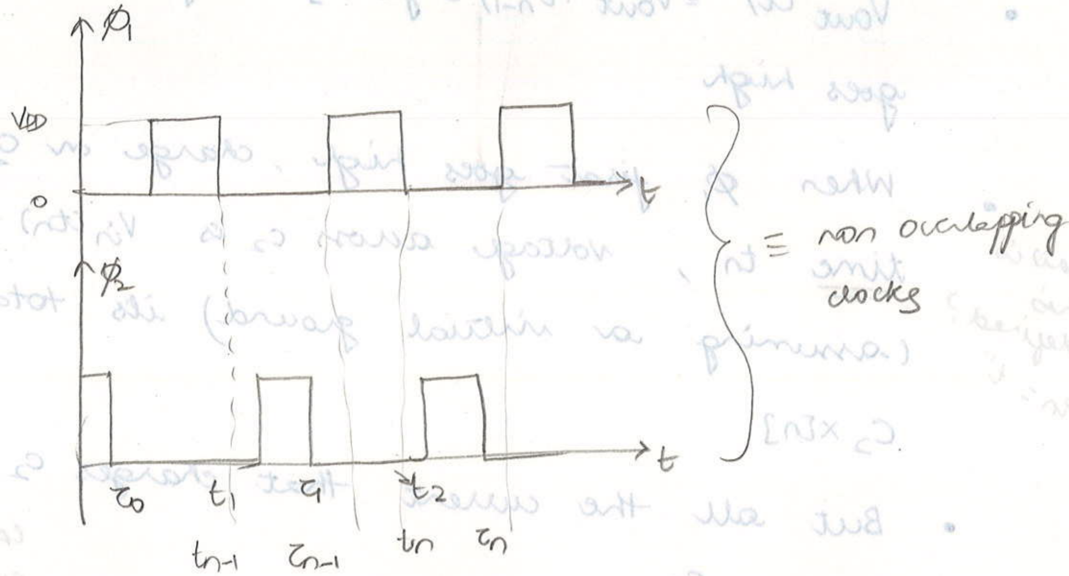
$$\therefore \; H(z) = -\frac{C_2}{C_A}\left(\frac{1}{1-z^{-1}}\right)$$

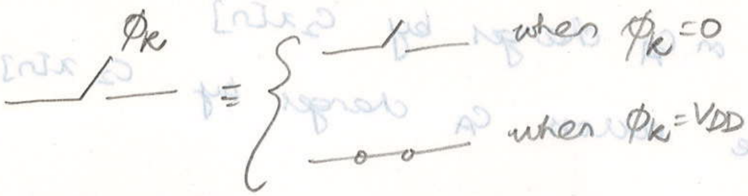
Rec #12 May 15'08

Recall:



where





Can view ① as discrete time circuit w/

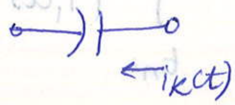
$x[n] = V_{in}(t_n)$, $y[n] = V_{out}(t_n)$ ②

Problems to solve: 1) Is ① LTI with respect to $x[n], y[n]$

2) If so, what is $H(z) = \frac{Y(z)}{X(z)}$

Solution: (Assuming ideal op-amp for now $\Rightarrow V_g(t)=0$)

Let $i_k(t)$ = current through C_k, $k=1,2,3,A$ in the following sense:



Similarly

$$i_s(t) = \begin{cases} -i_1(t) - i_2(t) & \text{when } \phi_1 = V_{DD} \\ -i_1(t) & \text{when } \phi_1 = 0 \end{cases} \quad (3)$$

it can't flow!

Similarly:

$$i_s(t) = \begin{cases} i_3(t) + i_A(t) & \text{when } \phi_1 = V_{DD} \\ i_A(t) & \text{when } \phi_1 = 0 \end{cases} \quad (4)$$

$i_s = ?$

$$\therefore \quad i_1(t) + i_2(t) + i_3(t) + i_A(t) = 0 \quad \text{when } \phi_1 = V_{DD} \quad (5)$$

$$i_1(t) + i_A(t) = 0 \quad \text{when } \phi_1 = 0$$

Recall: $i_k(t) = C_k \dfrac{dV_k}{dt}$

Let $q_k(t)$ = charge stored on C_k

$$\therefore \quad i_k(t) = \frac{dq_k(t)}{dt} \quad (6)$$

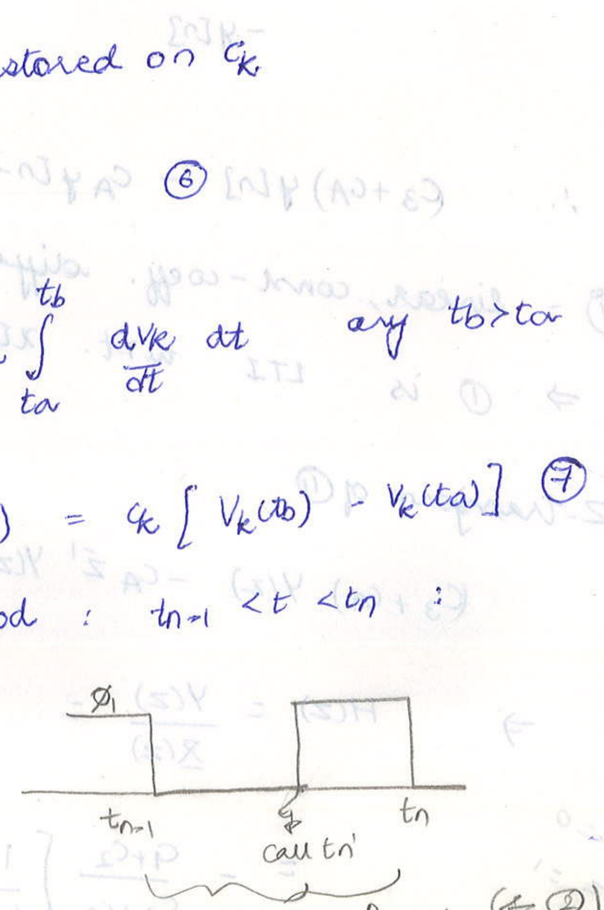
$$\therefore \int_{t_a}^{t_b} \frac{dq_k(t)}{dt}\, dt = C_k \int_{t_a}^{t_b} \frac{dV_k}{dt}\, dt \quad \text{any } t_b > t_a$$

$$\therefore \quad q_k(t_b) - q_k(t_a) = C_k \left[V_k(t_b) - V_k(t_a) \right] \quad (7)$$

Consider one sample period : $t_{n-1} < t < t_n$:

$t_{n-1} < t < t_n' \Rightarrow \phi_1 = 0$

$t_n' < t < t_n \Rightarrow \phi_1 = V_{DD}$



$$⑤ \Rightarrow \begin{cases} \displaystyle\int_{t_{n-1}}^{t_n'} \left(i_1(t) + i_A(t) \right) dt = 0 & (\phi_1 = 0) \quad ⑧ \\ \displaystyle\int_{t_n'}^{t_n} \left(i_1(t) + i_2(t) + i_3(t) + i_A(t) \right) dt = 0 & (\phi_1 = V_{DD}) \end{cases}$$

$$⑥ - ⑧ \Rightarrow \quad C_1 \left[V_1(t_n') - V_1(t_{n-1}) \right] + C_A \left[V_A(t_n') - V_A(t_{n-1}) \right] = 0 \quad ⑨$$

$$C_1 \left[V_1(t_n) - V_1(t_n') \right] + C_2 \left[V_2(t_n) - V_2(t_n') \right]$$

$$+ C_3 \left[V_3(t_n) - V_3(t_n') \right] + C_A \left[V_A(t_n) - V_A(t_n') \right] = 0 \quad ⑩$$

$⑨ + ⑩$

$$C_1 \Big[\overbrace{V_1(t_n) - V_1(t_{n-1})}^{\substack{-x[n] + x[n-1] \\ -V_{in}(t_n) + V_{in}(t_{n-1})}} \Big] + C_2 \Big[\overbrace{V_2(t_n) - V_2(t_n')}^{\substack{-x[n] \\ -V_{in}(t_n) + 0}} \Big]$$

$$+ C_3 \Big[\underbrace{V_3(t_n) - V_3(t_n')}_{\substack{-V_{out}[t_n] + 0 \\ -y[n]}} \Big] + C_A \Big[\underbrace{V_A(t_n) - V_A(t_{n-1})}_{\substack{-V_{out}(t_n) + V_{out}(t_{n-1}) \\ -y[n] + y[n-1]}} \Big] = 0$$

$$\therefore \quad (C_3 + C_A)\, y[n] - C_A\, y[n-1] = -(C_1 + C_2)\, x[n] + C_1\, x[n-1] \quad ⑪$$

(Const coeff. difference eqs $\Rightarrow$ linear)

⑪ = linear, const-coeff. difference equation

$\Rightarrow$ ① is LTI w.r.t. $x[n]$, $y[n]$

Z-transform of ⑪

$$(C_3 + C_A)\, Y(z) - C_A z^{-1} Y(z) = -(C_1 + C_2)\, X(z) + C_1 z^{-1} X(z)$$

$$\Rightarrow \quad H(z) = \frac{Y(z)}{X(z)} = \frac{-(C_1 + C_2) + C_1 z^{-1}}{(C_3 + C_A) - C_A z^{-1}}$$

$s = 0$

$z = 1$

$$= -\frac{C_1 + C_2}{C_3 + C_A} \left[\frac{1 - b_1 z^{-1}}{1 - a_1 z^{-1}} \right] \quad \text{where } b_1 = \frac{C_1}{C_1 + C_2}, \quad a_1 = \frac{C_A}{C_3 + C_A}$$

DC gain : $H(e^{j0}) = -\frac{C_1+C_2}{C_3+C_A}\left(\frac{1-b_1}{1-a_1}\right) = -\frac{C_2}{C_3}$

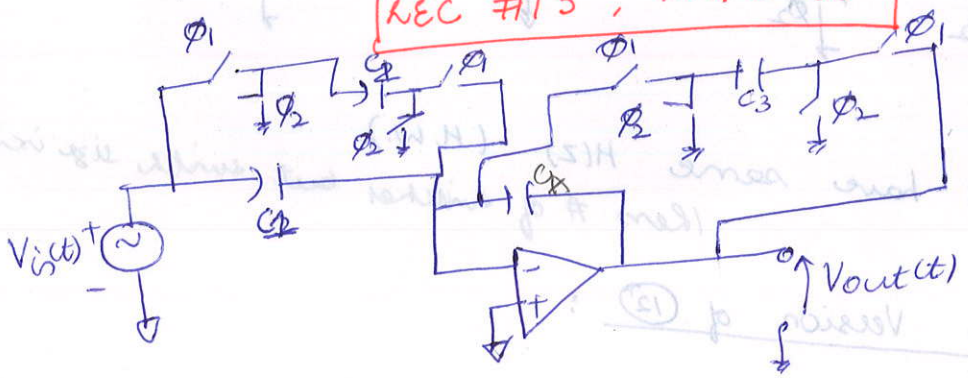
Q ROC of $H(z)$?

A ① is causal, single pole @ $z = a_1$

$\Rightarrow$ ROC $|z| > a_1$

REC #13 , MAY 22'08

(#ing contd.)



Last time found

$$H(z) = -\frac{C_1+C_2}{C_3+C_A}\left[\frac{1-b_1 z^{-1}}{1-a_1 z^{-1}}\right]$$

where $b_1 = \frac{C_1}{C_1+C_2}$

$a_1 = \frac{C_A}{C_3+C_A}$

Q Is ① BIBO stable?

A $C_3 > 0 \Leftrightarrow$ Yes. Why? Pole is at $z = a_1$ and $a_1 < 1$ $\Leftrightarrow C_3 > 0$

$\therefore$ Pole inside unit circle if $C_3 > 0$.

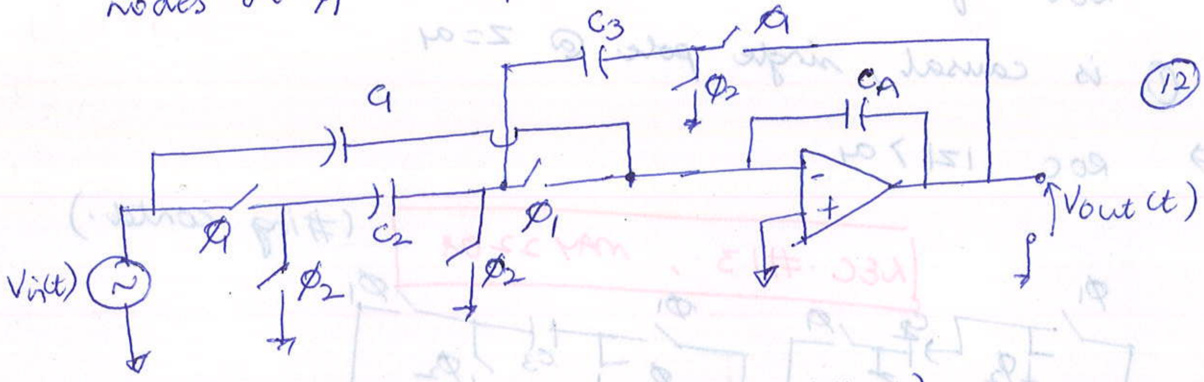
Notes 1) PM of opamp is ideal (180°), yet ① can still be unstable if $C_3 = 0$

2) In practice ① can be unstable if $C_3 > 0$ but if PM of opamp feedback config is too low

$\Rightarrow$ Must consider continuous time and discrete time stability in practice.

Switch Sharing

Top plates of C_2 and C_3 are always switched to same nodes on ϕ_1 and ϕ_2 ⇒ ① has redundant switches.

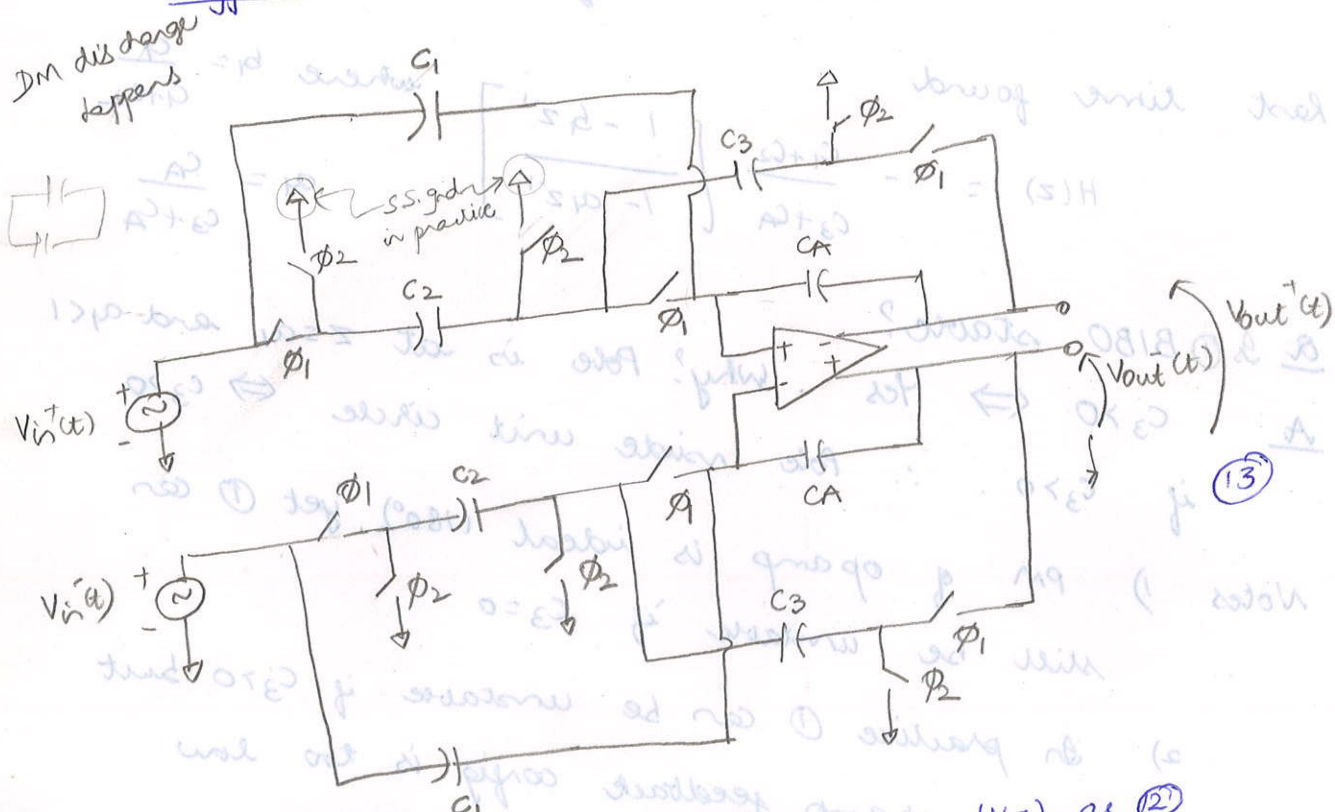


(12)

(11), (12) have same H(Z) (H.W.)

(Less # of switches but switch size increased)

Differential Version of (12) :



(13)

Exercise: Verify (13) has same H(z) as (12)

Topology ⇒ (13) $\cong$ 2 copies of (12)

But i) Only 1 opamp (except now w/ CMFB)

ii) Differential voltages ⇒ $4\times$ signal power, $2\times$ noise power } w.r.t. (12)

⇒ Can reduce capacitors by $1/2$ and maintain same SNR from $\frac{kT}{C}$ noise.

$\Delta = -3\,dB + 6\,dB = 3\,dB$, $\downarrow -3\,dB$, '0'

⇒ Switch sizes may also be reduced (for same settling performance).

⇒ (13) is larger than (12) but not 2X larger.

Benefits of (13) over (12)

1) Much better PSRR
2) All common mode noise sources tend to cancel.
3) Even order distortion terms " ______ ".
4) Can use cross coupling to achieve the effect of "negative capacitances"

Why would we need +ve or -ve poles & zeros

Recall, zero is @ $b_1 = \frac{C_1}{C_1+C_2}$ ⇒ Restricted to inside of unit circle (and ≥ 0) if $C_2 \geq 0$

But can "crosscouple" C_2 in (13) to achieve effective negative value for C_2 (HW or exam)

Reset Numbering

So far have S-C ckts to implement:

$$H_1(z) = -\alpha_1 \frac{1}{1-z^{-1}}$$

and $$H_2(z) = -\alpha_2 \left(\frac{1-b_1 z^{-1}}{1-a_1 z^{-1}}\right) \quad (1)$$

Now, we want a S.C. ckt to implement

$$H_3(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}} \quad (2)$$

Then, can achieve any rational transfer function by cascading $H_3(z)$ stages and $H_1(z)$ stages.

Observations:

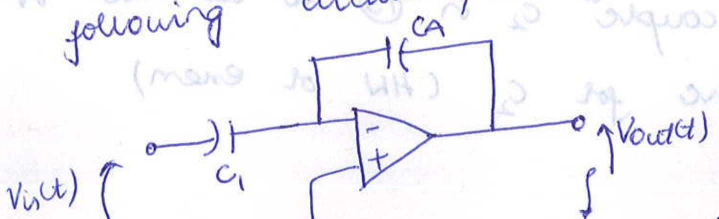
1) S-C principle ⇒ signal samples ≡ "charge packets"

2) Scheme depends on 1) good virtual grounds
2) No resistance paths to ground.

⇒ requires op amps w/ capacitive feedback

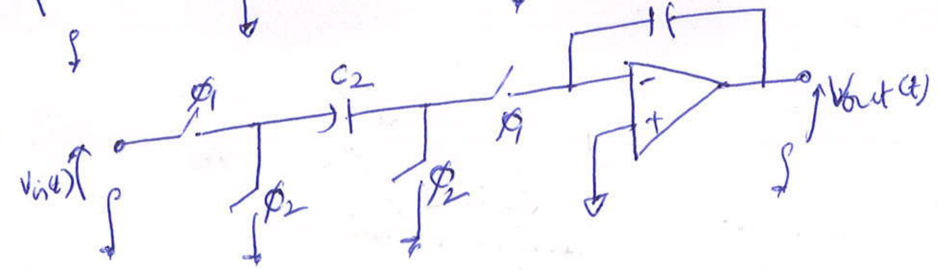
⇒ basic elements ≡ integrator.

3) So far, our S.C circuits have contained the following circuit portions:



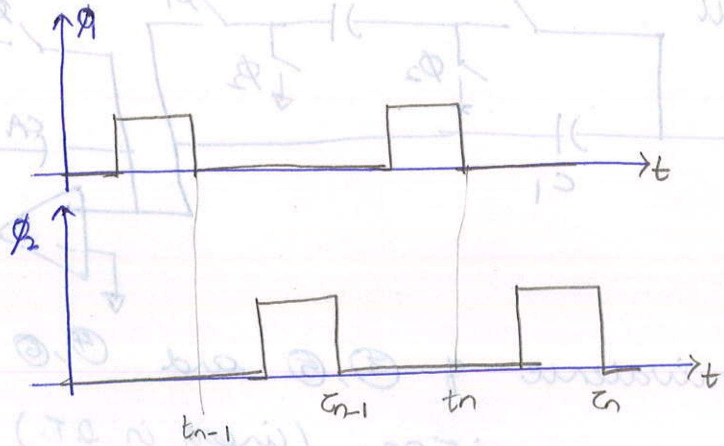
(3)

and



(4)

where



and $y[n] \equiv V_{out}(t_n)$, $x[n] \equiv V_{in}(t_n)$

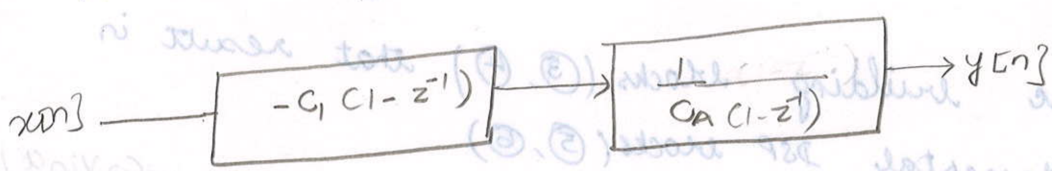
③ $$C_1\left[-V_{in}(t_n) + V_{in}(\tau_{n-1})\right] + C_A\left[-V_{out}(t_n) + V_{out}(t_{n-1})\right] = 0$$

$$\therefore \quad C_1\left[-x[n] + x[n-1]\right] = C_A\left[y[n] - y[n-1]\right]$$

$$\therefore \quad \frac{Y(z)}{X(z)} = -\frac{C_1}{C_A}(1-z^{-1})\frac{1}{(1-z^{-1})} = \frac{-C_1}{C_A} \quad (\text{for } ③)$$

caps to grd differentiator.)

$\therefore$ Can rewrite ③ as ⑤



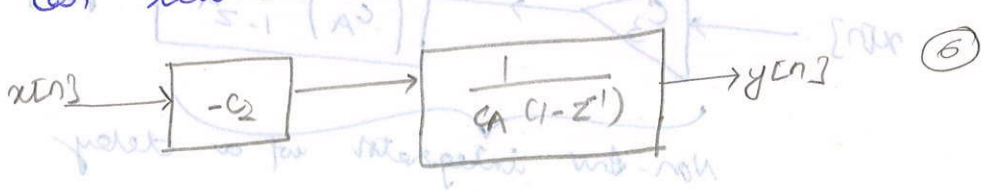
Similarly for ④

$$C_2\left[-V_{in}(t_n) + 0\right] = -C_A\left[-V_{out}(t_n) + V_{out}(t_{n-1})\right]$$

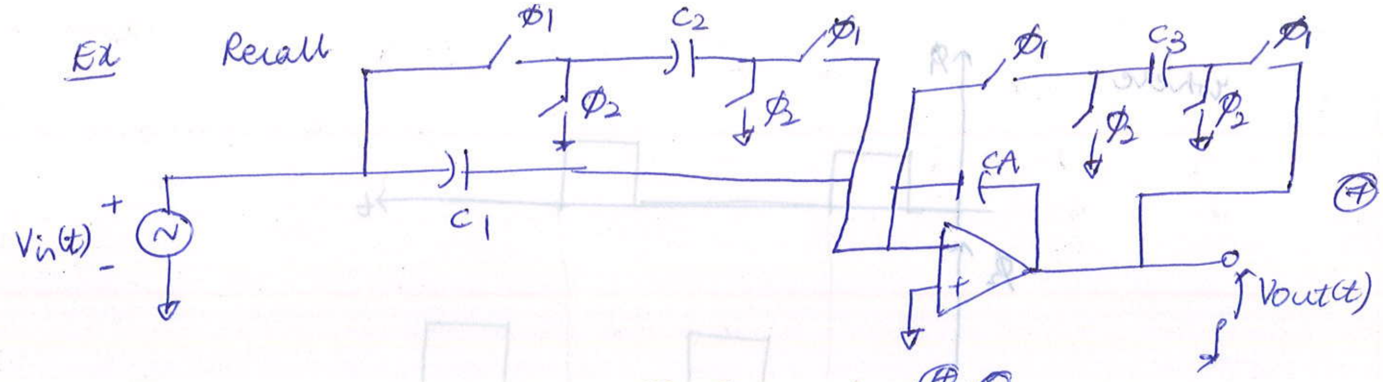
$$\therefore \quad -C_2\, x[n] = C_A\,(y[n] - y[n-1])$$

$$\Rightarrow Y(z) = \frac{-C_2}{C_A}\frac{X(z)}{(1-z^{-1})}$$

$\therefore$ Can rewrite ④ as ⑥

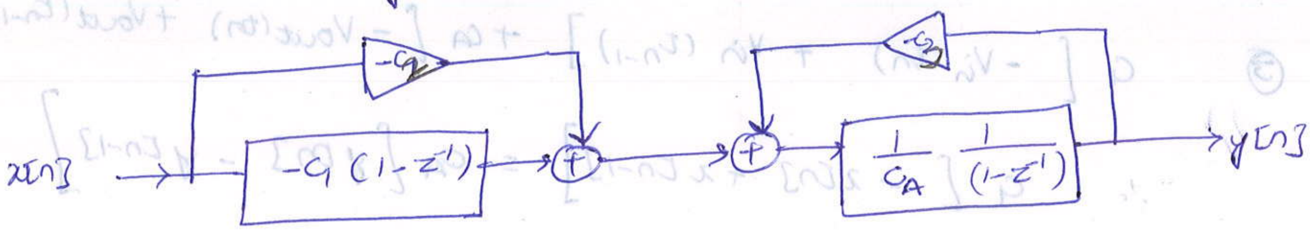


Ex Recall



(7)

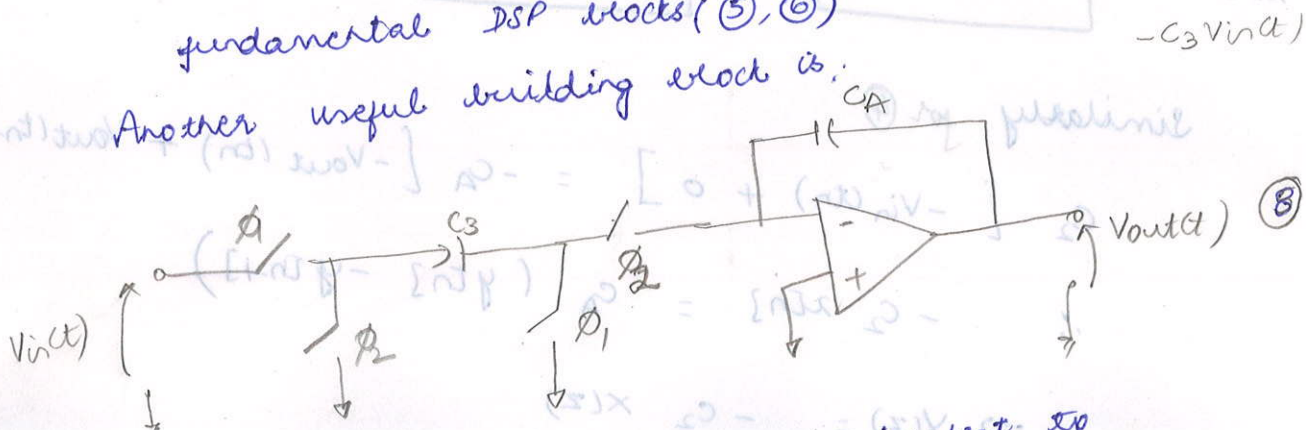
Using equivalence of (3), (5) and (4), (6) and using superposition (linear in D.T.), can immediately write (7) as:



$$\text{MGF} \Rightarrow H(z) = -\frac{C_1+C_2}{C_3+C_A}\left[\frac{1-\frac{C_1}{C_1+C_2}z^{-1}}{1-\frac{C_A}{C_3+C_A}z^{-1}}\right] \quad \text{as expected}$$

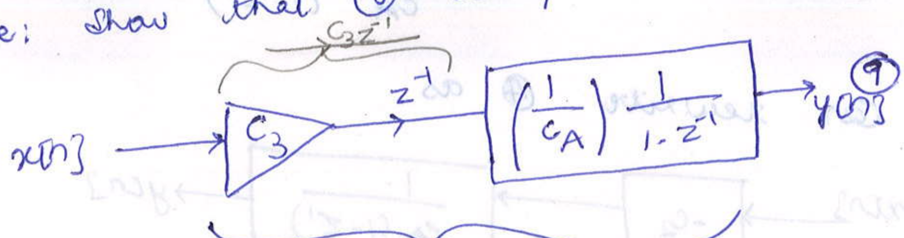
$\Rightarrow$ Have building blocks (3, 4) that result in fundamental DSP blocks (5, 6)

Another useful building block is:



(8)

Exercise: Show that (8) is equivalent to



(9)

Non-inv integrator w/ a delay

LEC #14, MAY 29 '08 (Continue #ing)

mean = 62

Biquad Synthesis:

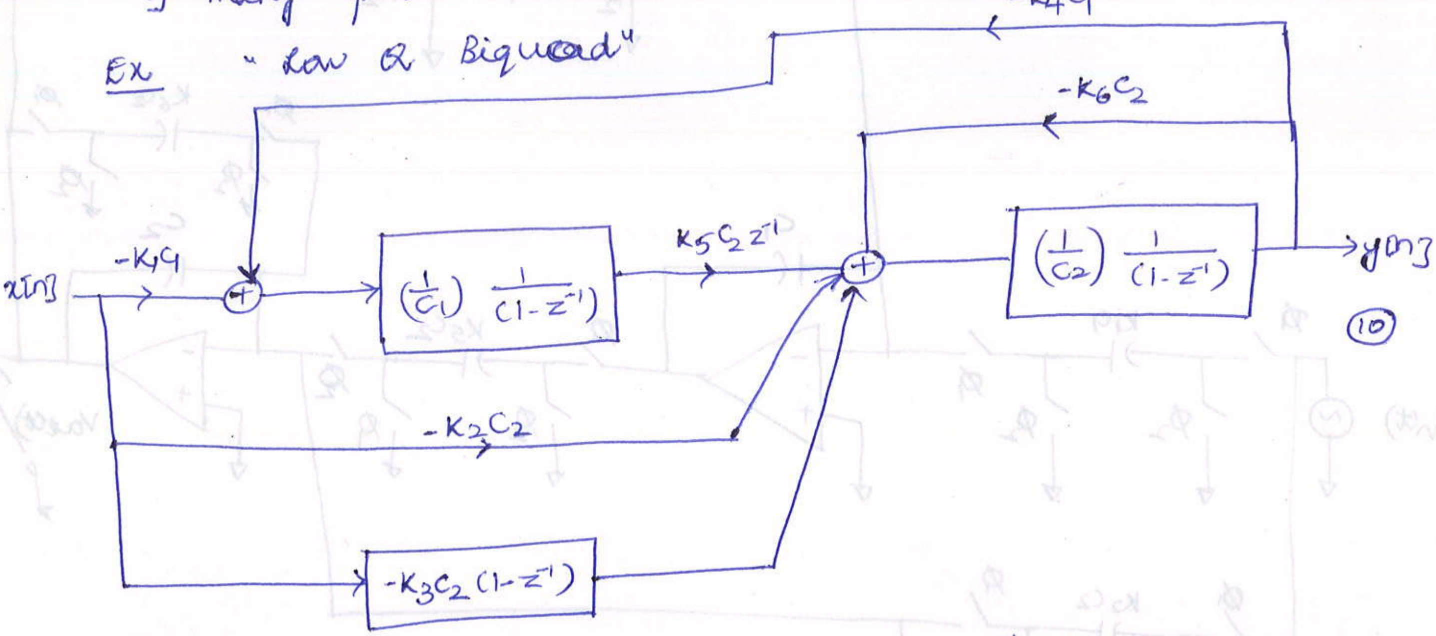
Goal: Use ⑤, ⑥, ⑨ & MGF to derive block diagram

w/ $H(z)$ = ② (2-zeros, 2-poles)

Then use ③, ④, ⑧ equivalences to find S-C circuit.

∃ many possibilities.

Ex "Low Q Biquad"



⑩

L_1 : $\frac{-K_6}{1-z^{-1}}$; L_2 : $\frac{-K_4K_5z^{-1}}{(1-z^{-1})^2}$

P_1 : $\frac{-K_1K_5z^{-1}}{(1-z^{-1})^2}$, P_2 : $\frac{-K_2}{1-z^{-1}}$; P_3 : $-K_3$

$$\therefore \quad \Delta = 1 + \frac{K_6}{1-z^{-1}} + \frac{K_4K_5z^{-1}}{(1-z^{-1})^2}$$

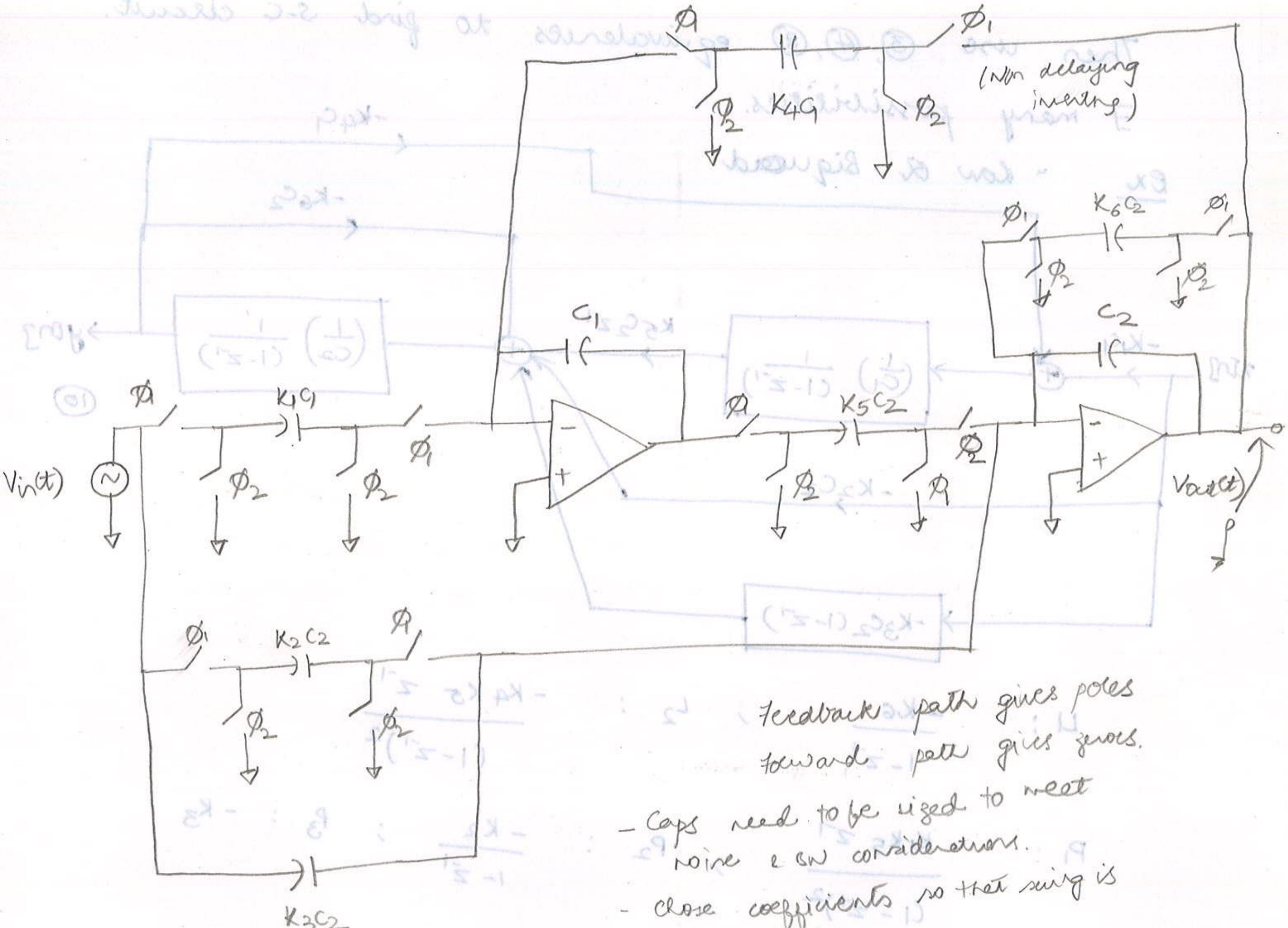
$$\therefore \quad H(z) = \frac{\frac{-K_1K_5z^{-1}}{(1-z^{-1})^2} - \frac{K_2}{1-z^{-1}} - K_3}{1 + \frac{K_6}{1-z^{-1}} + \frac{K_4K_5z^{-1}}{(1-z^{-1})^2}}$$

(All forward paths touch both loops & both loops touch each other.

$$\therefore \quad H(z) = \frac{-(K_2+K_3) + (K_2+2K_3-K_1K_5)z^{-1} - K_3 z^{-2}}{(1+K_6) + (K_4K_5 - K_6 - 2)z^{-1} + z^{-2}}$$

(has form of ②)

Using ③, ④ & ⑧, S-C equivalent circuit is :



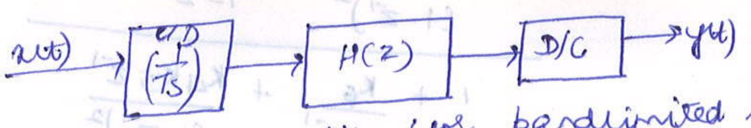
Feedback path gives poles
forward path gives zeros.

- Caps need to be sized to meet noise & BW considerations.
- Chose coefficients so that swing is not too high.

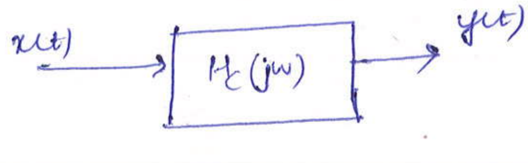
Why low-Q?

$$H(e^{j\omega}) = -\frac{K_1K_5 + 2jK_2\sin\left(\frac{\omega}{2}\right)e^{\frac{j\omega}{2}} - 4K_3\sin^2\left(\frac{\omega}{2}\right)}{K_4K_5 + 2jK_6\sin\left(\frac{\omega}{2}\right)e^{\frac{j\omega}{2}} - 4\sin^2\frac{\omega}{2}} \quad (11)$$

Recall



||| (for bandlimited signals x(t))



where $H_c(jw) = H(e^{jwT_s})$ $(|w| < \pi/T_s)$

Often $f_s = \frac{1}{T_s} \gg$ magnitudes of poles, zeros of $H_c(jw)$

$\therefore$ For w of interest, $wT_s \ll \pi$ (12)

(11), (12) $\Rightarrow$ $$H_c(jw) \cong -\frac{K_1K_5 + jK_2(wT_s) - K_3(wT_s)^2}{K_4K_5 + jK_6(wT_s) - (wT_s)^2} \quad (13)$$

Denominator of (13) $= T_s^2\left(\frac{K_4K_5}{T_s^2} + j\left(\frac{K_6}{T_s}\right)w - w^2\right)$

$= T_s^2\left(w_0^2 + j\left(\frac{w_0}{Q}\right)w - w^2\right)$

where Q = Quality Factor of Poles

w_0 = pole freq.

$w_0 = \frac{\sqrt{K_4K_5}}{T_s}$

$\frac{K_6}{T_s} = \frac{\sqrt{K_4K_5}}{T_s}\frac{1}{Q}$

$Q = \frac{\sqrt{K_4K_5}}{K_6}$

Could chose $K_4 \cong K_5 \cong w_0T_s$

but must have $K_6 \cong \frac{w_0T_s}{Q}$

Accurate placement of poles $\Rightarrow$ ratios of caps C_1, C_2, K_4C_1, K_5C_2 and K_6C_2 must be accurate.

Get best cap matching when we have similar sizes.

For $Q \leq 1$ cap spread $\cong \frac{1}{w_0T_s}$ (Recall $w_0T_s \ll \pi$, so $w_0T_s < 1$)

(largest/smallest)

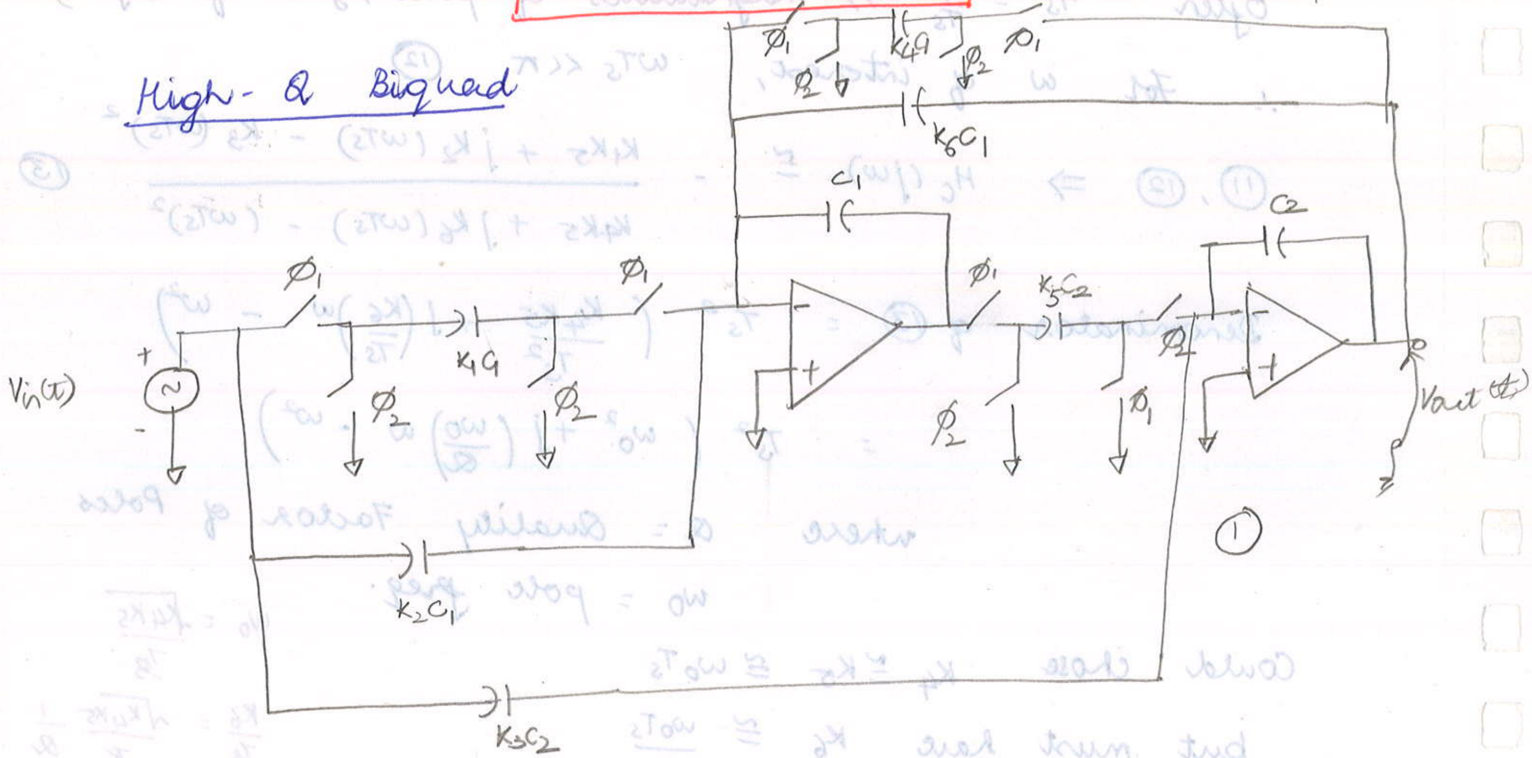
$Q \geq 1$ Cap spread $\cong \frac{Q}{w_0T_s}$

$\Rightarrow$ Matching becomes poor for high Q.

LEC #15, JUNE 3 '08

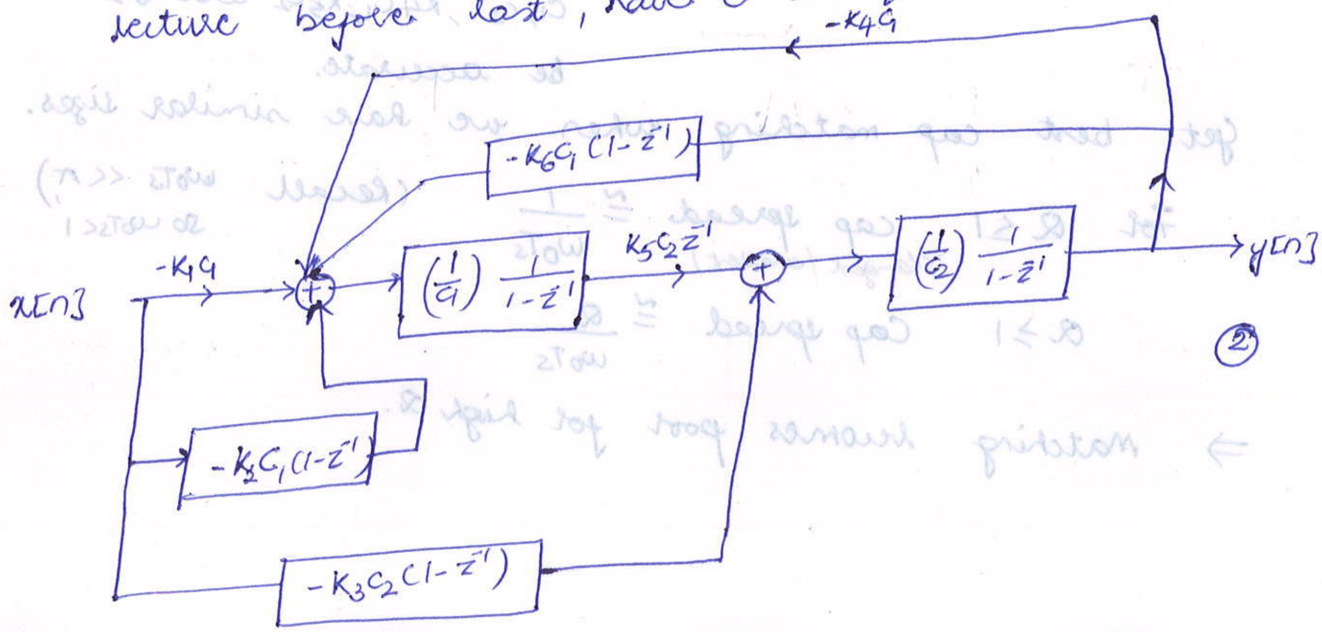
SW CAP NOISE ANALYSIS - FINAL.

High - Q Biquad



(1)

Using (3) ≡ (5), (4) ≡ (6), and (8) ≡ (9) equivalences from lecture before last, have (1) is equivalent to:



(2)

MGF $\Rightarrow$ $$H(z) = -\frac{K_3 + (K_1K_5 + K_2K_5 - 2K_3)z^{-1} + (K_3 - K_2K_5)z^{-2}}{1 + (K_4K_5 + K_5K_6 - 2)z^{-1} + (1 - K_5K_6)z^{-2}}$$ (3)

Exercise: Using approx analysis similar to that for the low-Q biquad, show that this version has lower cap spread for $Q > 1$

Ex:

$$H(z) = -\frac{0.288(1-z^{-1})z^{-1}}{1-(1.572)z^{-1}+(0.9429)z^{-2}}$$

$\frac{(z-1)}{z^2}$ ——— $\frac{p'}{2z}$ 0

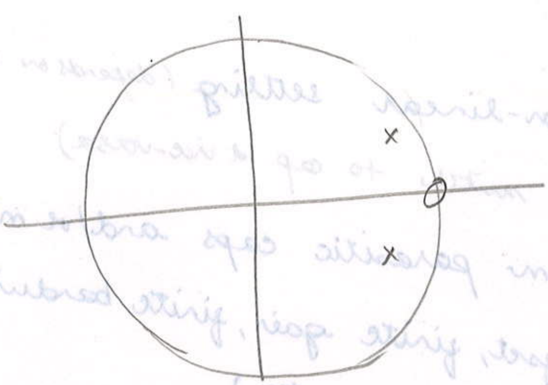
Poles: $z_1, z_2 = \dfrac{1.572 \pm \sqrt{1.572^2 - 4(0.9429)}}{2}$

$= 0.786 \pm j(0.570)$

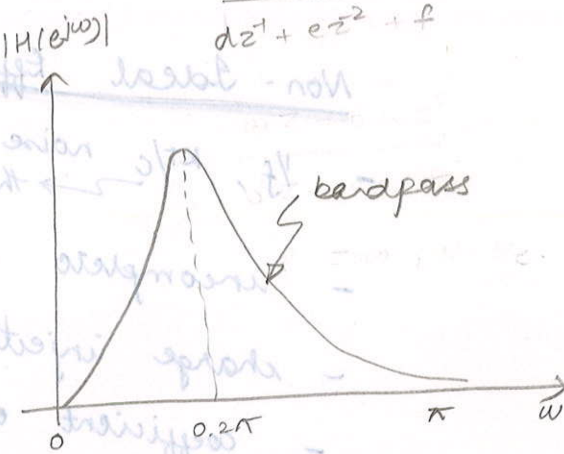
$= 0.971\, e^{\pm j(0.2\pi)}$

$$\frac{az^{-1}+bz^{-2}+c}{dz^{-1}+ez^{-2}+f}$$

Zero: $z_3 = 1$



$\Rightarrow$



Can show: corresponding $H_c(j\omega)$ has $Q \cong 10$

③ $\Rightarrow$ $K_4 = K_5 = 0.6090$, $K_3 = 0$ (can build switches w/ this anyway but not xtors)

$K_1 = 0$

$K_2 = 0.4729$, $K_6 = 0.0938$

w/ $C_1 = C_2$ chosen, cap spread is $\cong$ 11 to 1

Exercise: Show cap spread $\cong$ 17 to 1 for "low-Q" biquad.

Observations: (both biquads)

1) Have 6 constants, $K_1, K_2, \ldots, K_6$ but only need 5 coefficients.

2 pole locations, 2 zero locations, gain

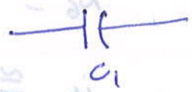
⇒ Can use extra degree of freedom to control signal swing at internal nodes.

2) "Low-Q" versus "High-Q" are guidelines (approx. valid). In practice, try both to verify min. spread in each case.

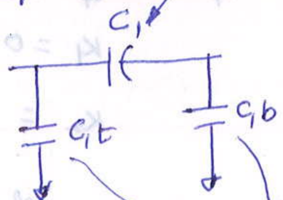
Non-Ideal Effects in S-C circuits:

- 1/f, KT/C noise → thermal
- incomplete linear/non-linear settling (depends on op-amp SR)
- charge injection (from switch to cap & vice-versa)
- coefficient errors from parasitic caps and/or mismatches.
- op-amp errors (offset, finite gain, finite bandwidth, SR limiting etc.)

Parasitic Capacitor Sensitivity:

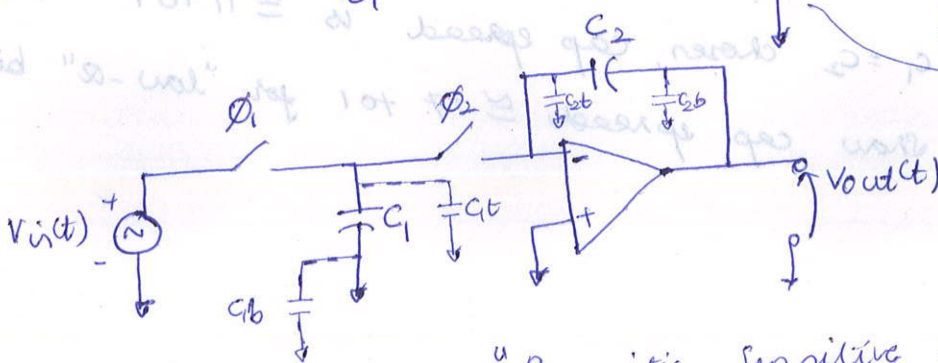
Ex 1 Want  get

Desired Cap



Why not include C_{1t} & C_{1b} in ckt. analysis? ~ model.

Parasitic caps.



≡ "Parasitic Sensitive S-C integrator"

C_{1b} shorted to ground

C_{2t} " ———— " virtual ground

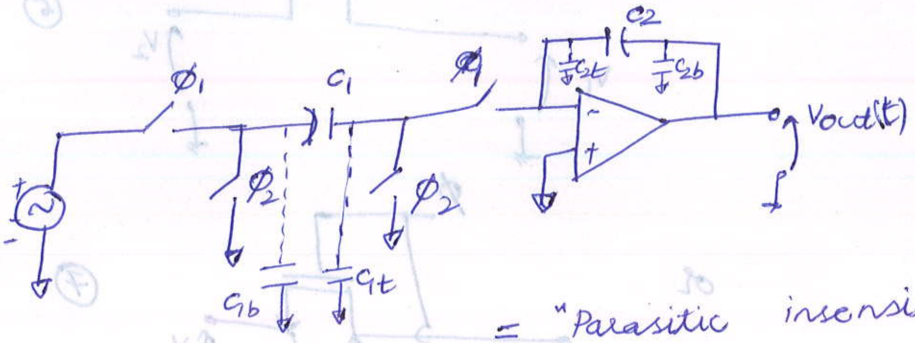
C_{2b} connected to opamp output

} no significant effect on transfer fn (for "good" op-amp) (If C_{2t} is really large, PM is bad)

But C_{1t} adds directly to C_1.

<u>Exercise:</u> Show $H(z) = -\left(\frac{C_1 + C_{1t}}{C_2}\right)\frac{z^{-1}}{1-z^{-1}}$ (including parasitic capacitors)

<u>Ex 2</u>



≡ "Parasitic insensitive inverting integrator"

Neglecting parasitic caps, we've seen

$$H(z) = -\left(\frac{C_1}{C_2}\right)\frac{1}{1-z^{-1}} \quad ④$$

<u>Exercise:</u> Show ④ holds even w/ parasitic caps.

Reasoning: C_{2t} and C_{2b} same as previous.

C_{1t} always set to ground or virtual ground.

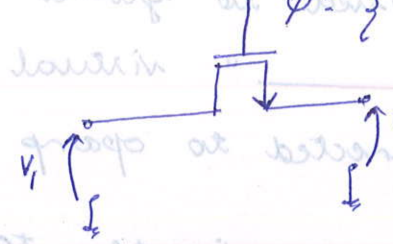
" : charge on C_{1b} never gets transferred to C_2.

Similar results for non-inverting integrator with delay from before.

Switch Resistance:

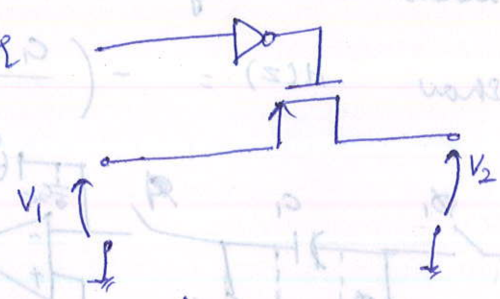
Switch implementations

$\phi = \begin{cases} V_{DD} & \text{when ON} \\ 0 & \text{when OFF} \end{cases}$



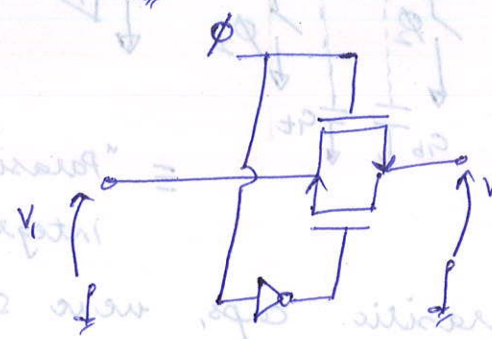
(5)

or



(6)

or



(7)

≡ "transmission gate"

(or "boot-strapped" w/ circuitry to make V_{GS} = const. when ON)

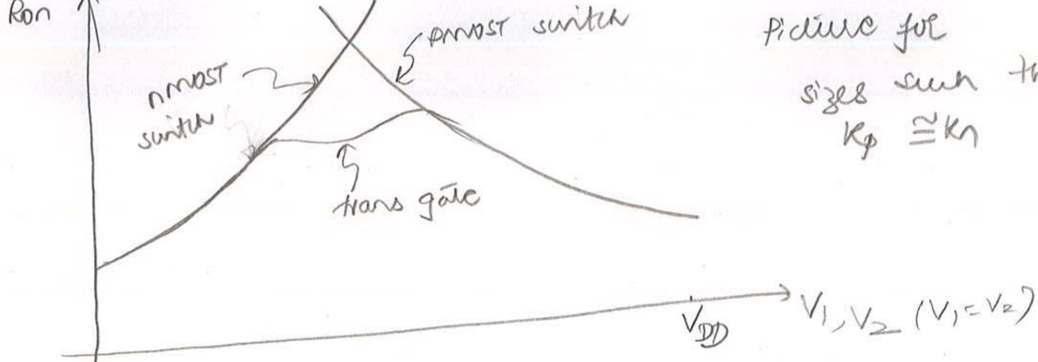
Goal: Want MOST in triode when ON
" " OFF when $\phi = 0$

∴ (5) only useful as a switch if $V_{DD} > \max(V_1 + V_{tn}, V_2 + V_{tn})$

(6) " " $0 < \min(V_1 - |V_{TP}|, V_2 - |V_{TP}|)$

(7) useful " for all $0 \le V_1, V_2 \le V_{DD}$

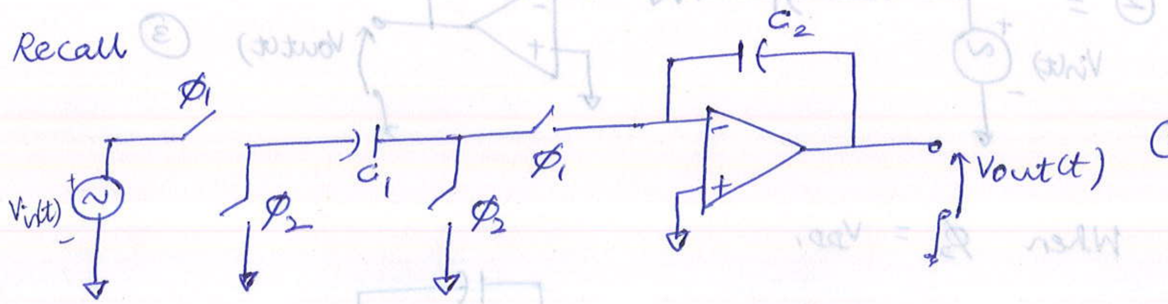
Recall for each MOST, $R_{on} = \dfrac{1}{K|V_{gs} - V_t|}$

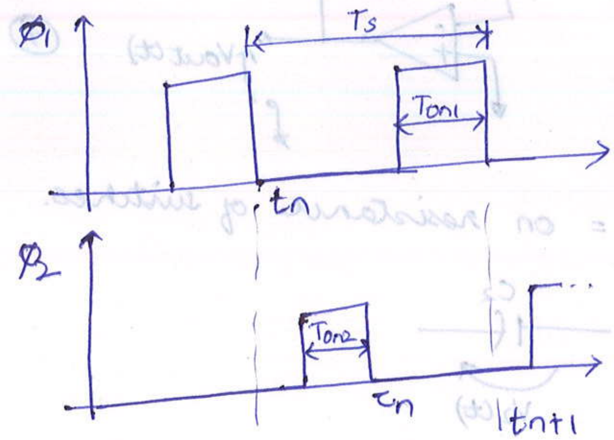


Picture for sizes such that $K_p \cong K_n$

LEC # 16 JUNE 5'08

Recall ③



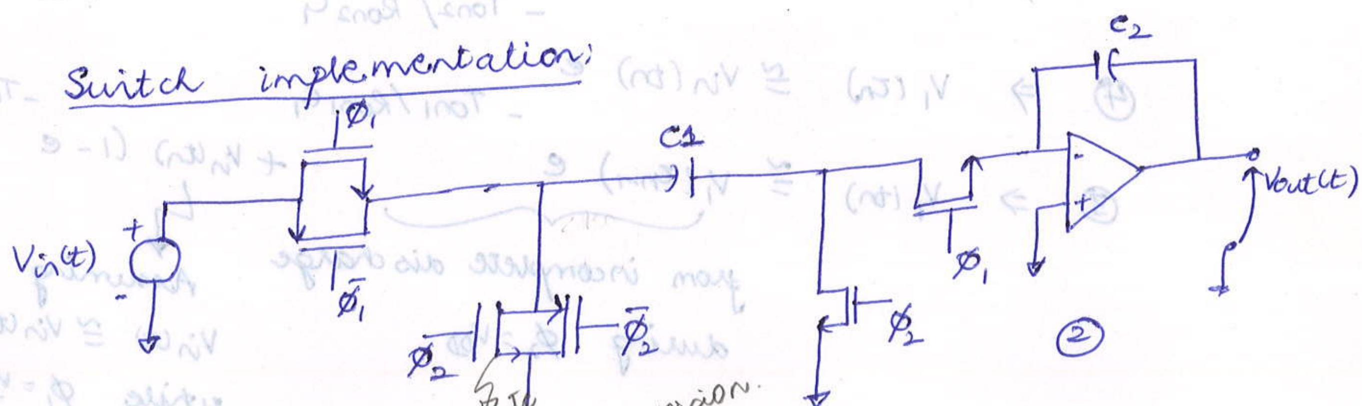


$x[n] \equiv V_{in}(t_n)$

$y[n] \equiv V_{out}(t_n)$

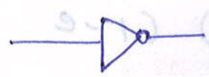
Know: Ideal components $\Rightarrow$ $H(z) = -\left(\frac{C_1}{C_2}\right)\frac{1}{1-z^{-1}}$

Switch implementation:



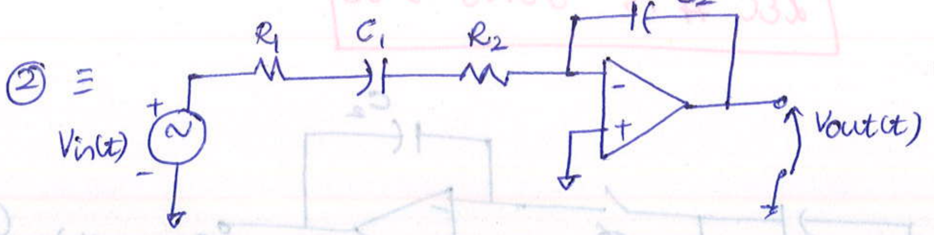
Need pass gate for differential version.

Note: Transmission gates only necessary for certain switches. Why?

where ϕ_k  $\bar{\phi}_k$ $\quad k = 1,2$

When $\phi_1 = V_{DD}$, (Note that R_{off} is still $\sim \infty$)

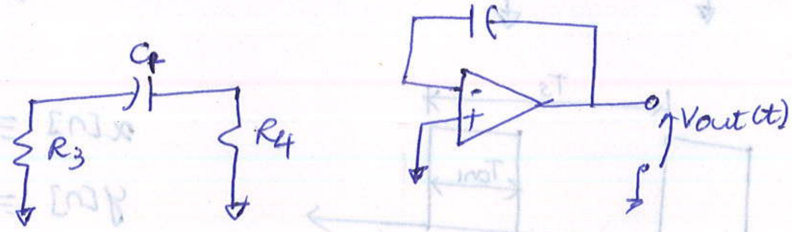
② ≡



③

When $\phi_2 = V_{DD}$,

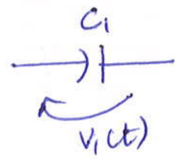
② ≡



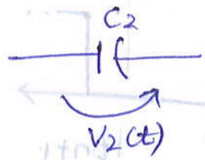
④

where $R_1, R_2, \ldots, R_4$ = on resistance of switches.

Define: 1)



,



2) $R_{on1} = R_1 + R_2$, $R_{on2} = R_3 + R_4$

④ $\Rightarrow$ $V_1(t_n) \cong V_{in}(t_n)\, e^{-T_{on2}/R_{on2}C_1}$

③ $\Rightarrow$ $V_1(t_n) \cong \underbrace{V_1(t_{n-1})\, e^{-T_{on1}/R_{on1}C_1}}_{\text{from incomplete discharge during } \phi_2 = V_{DD}} + V_{in}(t_n)\left(1 - e^{-T_{on1}/R_{on1}C_1}\right)$

Assuming $V_{in}(t) \cong V_{in}(t_n)$ while $\phi_1 = V_{DD}$.

$$= V_{in}(t_{n-1})\, \underbrace{e^{-T_{on2}/R_{on2}C_1}\, e^{-T_{on1}/R_{on1}C_1}}_{\text{very small} \Rightarrow \text{usually can neglect.}}$$

$$+ V_{in}(t_n)\left(1 - e^{-T_{on1}/R_{on1}C_1}\right)$$

Is it t_{n-1}?

$$C_1\left[V_1(t_n) - V_1(t_{n-1})\right] + C_2\left[V_2(t_n) - V_2(t_{n-1})\right] = 0$$

$$\therefore\ C_1\left\{\left[\underbrace{V_{in}(t_n)}_{x[n]}\ \underbrace{\left(1 - e^{-T_{on1}/R_{on1}C_1}\right)}_{\text{call } \alpha}\right] - \left[\underbrace{V_{in}(t_{n-1})}_{x[n-1]}\ \underbrace{e^{-T_{on2}/R_{on2}C_1}}_{\text{call } \beta}\right]\right\}$$

$$\cong C_2\left(y[n] - y[n-1]\right)$$

$$\therefore \quad H(z) = \frac{Y(z)}{X(z)} = -\left(\frac{C_1}{C_2}\right)\frac{\alpha - \beta z^{-1}}{1 - z^{-1}}$$

⇒ Even by adding exponential settling keeps it linear system.

$$= -\left(\frac{C_1}{C_2}\right)\frac{1}{1 - z^{-1}} \quad \text{as } T_{on1,2} \to \infty \text{ or } R_{on1,2} \to 0$$

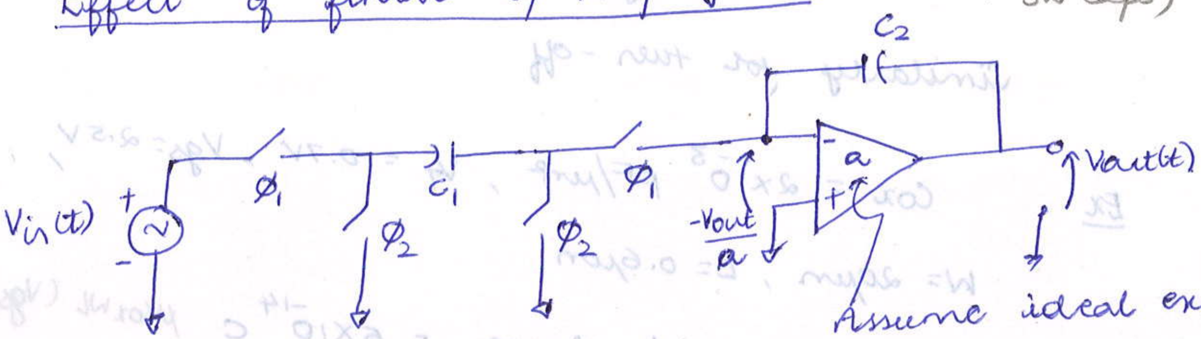
Observations:

1) Linear Settling ⇒ Linear error in transfer function (addition of a zero)

2) But R_{on1}, R_{on2} are non-linear functions of voltage.
⇒ don't have linear settling in reality
but can get close (e.g. using "boot-strapping" techniques)

(Slew rate limiting sw caps)

Effect of finite Op Amp Gain:



(5)

Assume ideal except $a \neq \infty$

$$C_1\left[V_{in}(t_n) + \frac{1}{a}V_{out}(t_n)\right] + C_2\left[V_{out}(t_n) + \frac{V_{out}(t_n)}{a} - V_{out}(t_{n-1}) - \frac{V_{out}(t_{n-1})}{a}\right] = 0$$

What if 'a' is not const but has freq. response

$$\Rightarrow H(z) = -\left(\frac{C_1}{C_2}\right)\frac{\gamma}{1 - \left(1 + \frac{1}{a}\right)\gamma z^{-1}}$$

where $\gamma = \left[1 + \frac{1 + C_1/C_2}{a}\right]^{-1}$ (6)

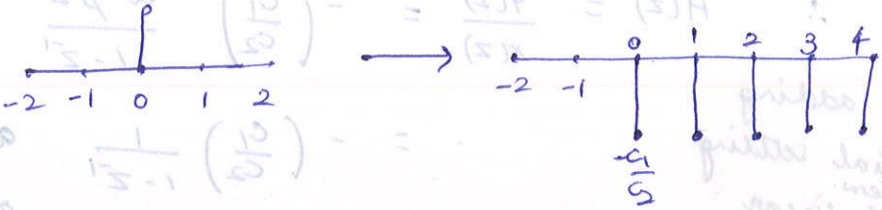
Pole: $z_p = \left(1 + \frac{1}{a}\right)\gamma$

(6) ⇒ $\gamma < \left(1 + \frac{1}{a}\right)^{-1} \Rightarrow |z_p| < 1$ for $a < \infty$ (>0)

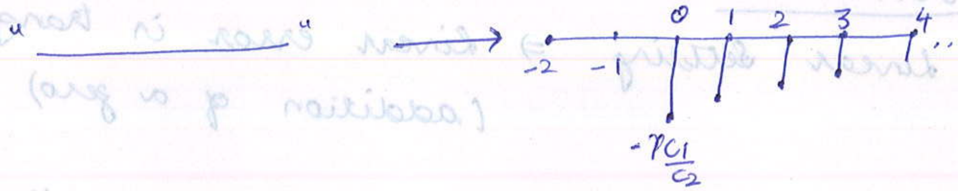
⇒ have "leaky integrator"

Idea

Ideal



Leaky



Charge Injection:

When MOSTs used as S-C switches:

turn-on $\Rightarrow$ terminals must supply channel charge and parasitic cap charges

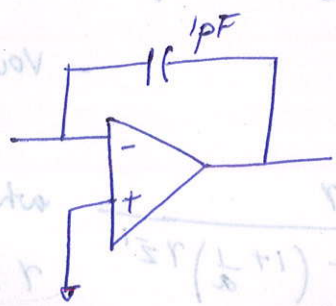
Similarly for turn-off

Ex $C_{ox} = 2\times10^{-3}\ pF/\mu m^2$, $V_T = 0.7V$, $V_{gs} = 2.5V$,

$W = 20\mu m$, $L = 0.6\mu m$

$\Rightarrow Q \equiv$ channel charge $= 5\times10^{-14}\ C$ $\quad \mu C_{ox} WL\,(V_{gs}-V_t)$

If half this charge gets dumped into summing node of:

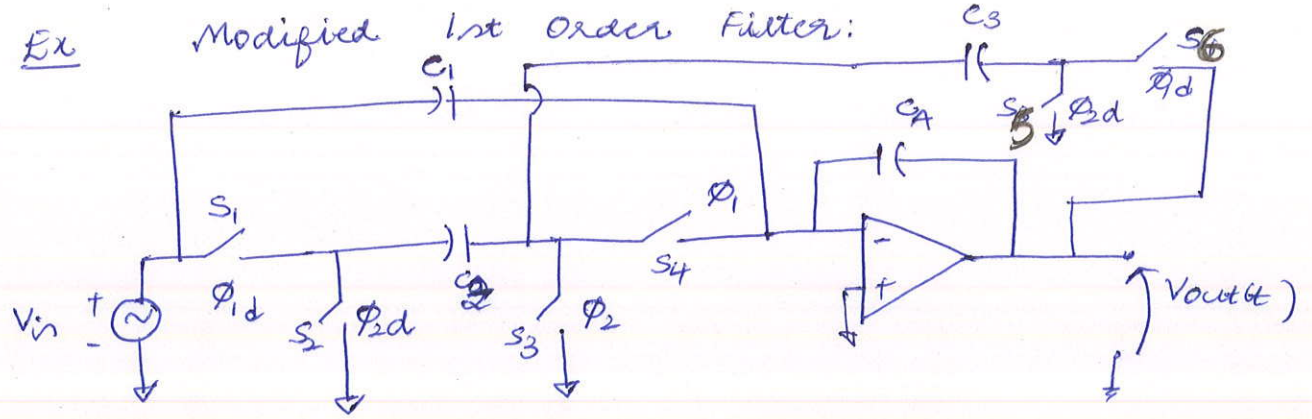


$\Rightarrow$ resulting error in V_{out} is

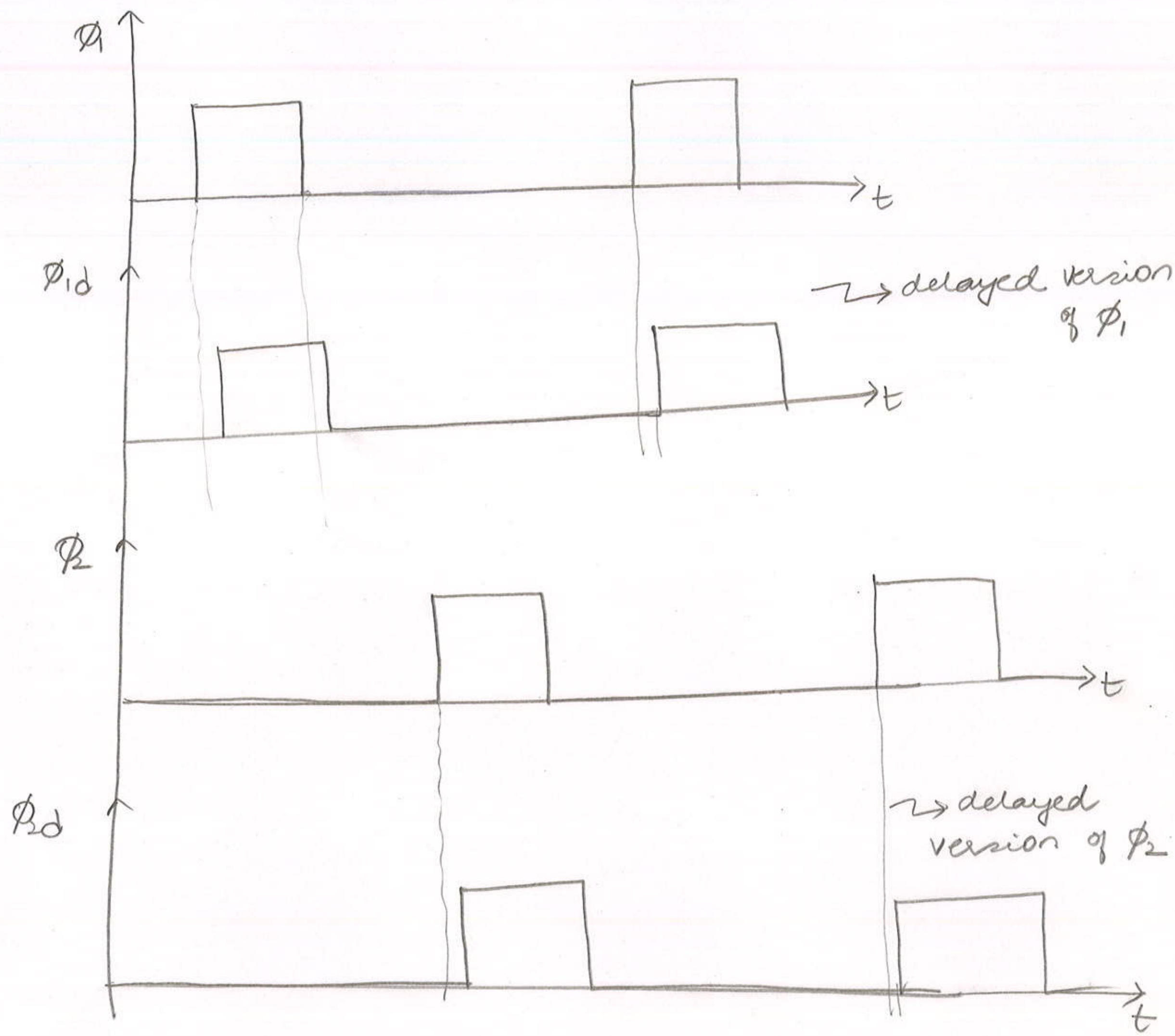
$\frac{1}{2}(5\cdot10^{-14}C)/(1\cdot10^{-13}F) \cong 25mV$

10^{-12}?

Ex Modified 1st Order Filter:



where



Idea: Open S_4 before S_1, S_6

Open S_3 before S_2, S_5

Why? S & D of MOST used for S_3 (S_4) are always @ SS ground

$\Rightarrow$ Charge injected is indep of signal. Once S_3 & S_4 are open, top plates of C_2, C_3 are tied

When S1 and S6 are opened, have signal dependent charge injection, but error is quickly eliminated because of low impedances @ i/p and o/p.