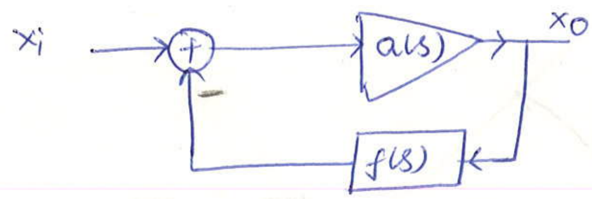
(CLN ... = "poles T(s)" ...

LEC #13 FEB 26'08

Gain & Phase Margin:



$\Leftrightarrow \quad A(s) = \dfrac{a(s)}{1 + \underbrace{a(s)f(s)}_{\equiv T(s)}}$

Assume : No pole of $a(s)$ is a zero of $f(s)$ (vice-versa?)
(True for analog ckts. but not for DSP).

Nyquist criterion ⇒ 2 special cases:

i) Suppose $\measuredangle T(0) = 0$, $T(s)$ has no RHP poles, and $|\measuredangle T(j\omega)| = \pi$ has no more than one positive solution, $\omega = \omega_\pi$ ①

Then $A(s)$ is stable iff $|T(j\omega_\pi)| < 1$ ③

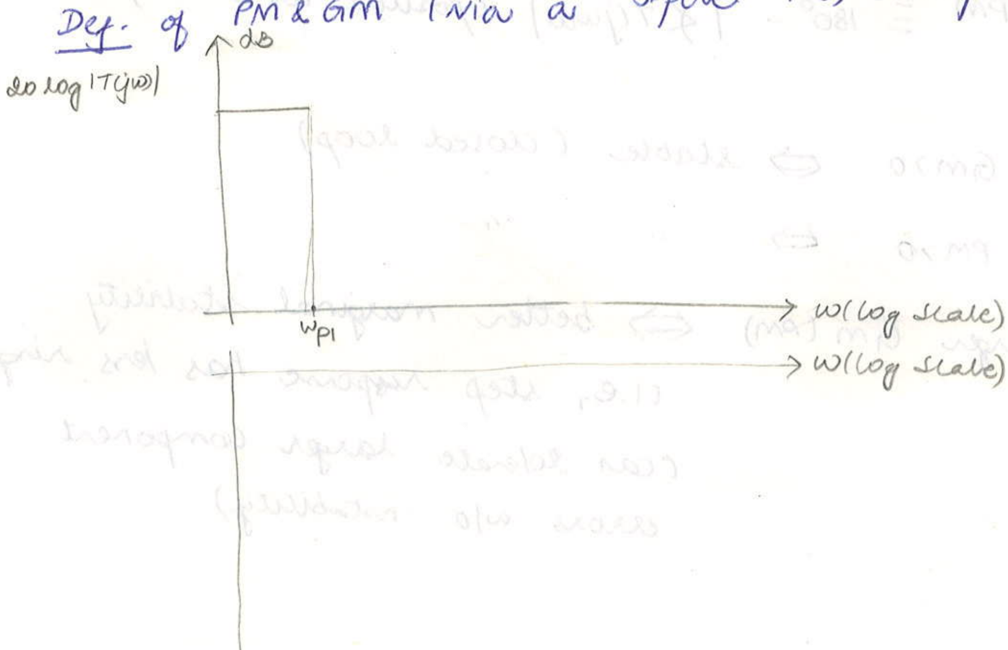
ii) Suppose $\measuredangle T(0) = 0$, $|T(0)| > 1$, $T(s)$ has no RHP poles, and $|T(j\omega)| = 1$ has no more than one positive solution, $\omega = \omega_u$ ② ← unity gain

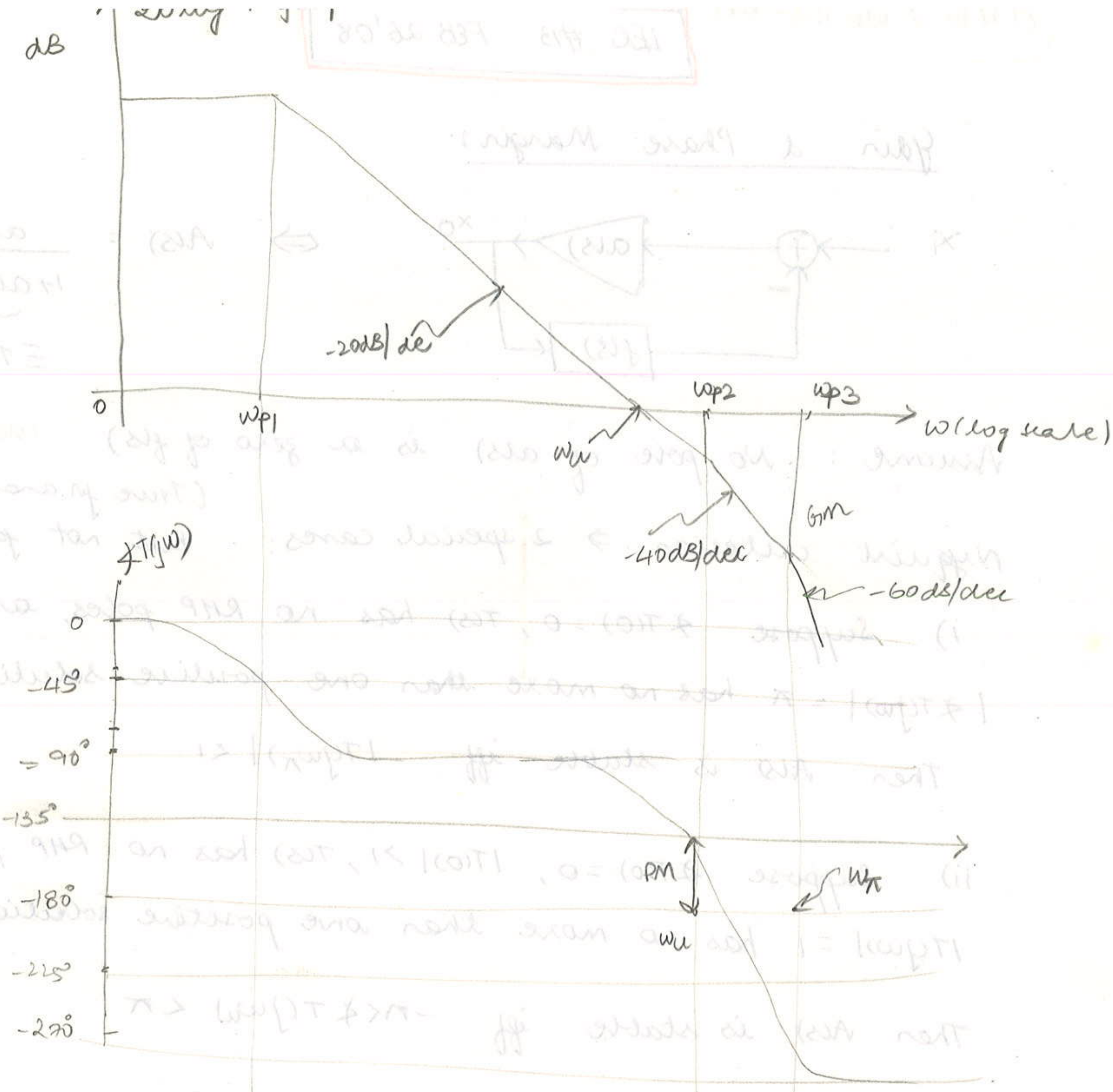
Then $A(s)$ is stable iff $-\pi < \measuredangle T(j\omega_u) < \pi$

i) ⇒ Basis of "gain margin" (GM) definition (soon)

ii) → " " "phase margin" (PM) " ".

Def. of PM & GM (via a 3-pole $T(s)$ example)





Why are there not extensions of Barkhausen's criterion?

i.e. $GM \equiv 20\log\left|\frac{1}{T(j\omega_\pi)}\right|$ (positive as shown)

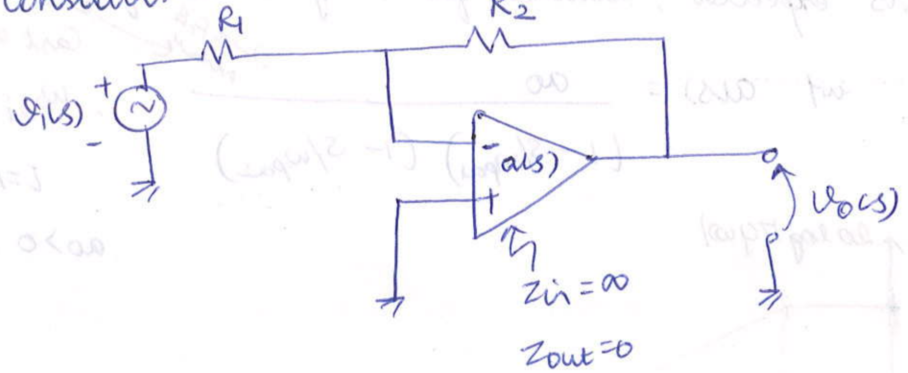
$PM \equiv 180° - |\measuredangle T(j\omega_u)|$ (positive as shown)

$\therefore$ ①, $GM > 0 \iff$ stable (closed loop)

②, $PM > 0 \iff$ " "

Also, larger GM (PM) $\iff$ better marginal stability
(i.e., step response has less ringing)
(can tolerate larger component errors w/o instability)

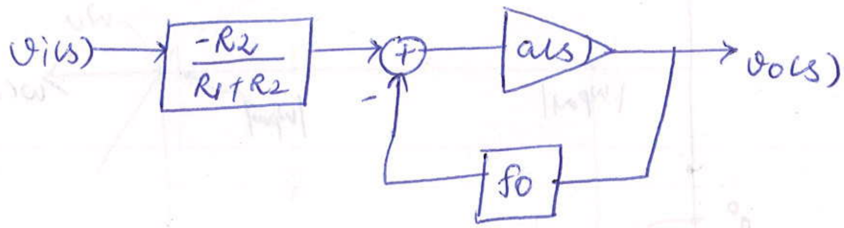
Consider,



③

Previously found,

③ $\equiv$



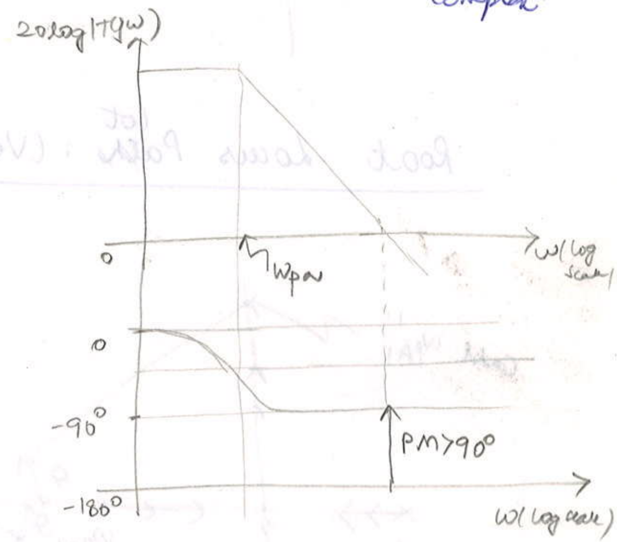
where $f_0 = \frac{R_1}{R_1+R_2}$ $(0 < f_0 < 1)$

<u>Ex 1</u> ③ w/ $a(s) = \frac{a_0}{(1 - s/\omega_{pa})}$, $\omega_{pa} < 0$ $(\in \mathbb{R})$

one pole can't be complex

$a_0 > 0$

$\therefore T(j\omega) = \frac{a_0 f_0}{(1 - j\omega/\omega_{pa})}$



Depending on $a_0 f_0$,

$90^\circ < PM < 180^\circ$

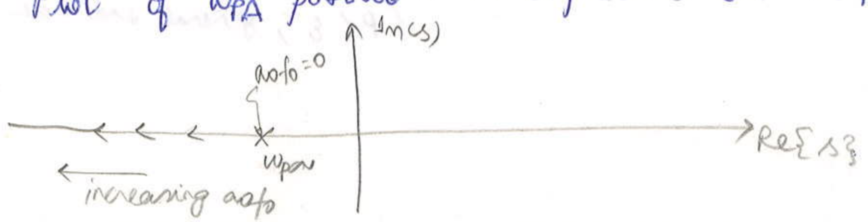
e.g. at $PM = 180^\circ$
$a_0 f_0 \to 1$

at $PM = 90^\circ$
$a_0 f_0 \to \infty$

Recall; $A(s) \equiv \frac{v_o(s)}{v_i(s)} = A_0 \frac{1}{1 - s/\omega_{pA}}$ where $A_0 \in \mathbb{R}$

$\omega_{pA} = (1 + a_0 f_0)\,\omega_{pa}$

Plot of ω_{pA} position in s-plane as $a_0 f_0$ increases from 0 to ∞



$\equiv$ "root locus" plot

∴ As expected, stable for any choice of $a_0 f_0 > 0$.

Ex 2 (3) w/ $a(s) = \dfrac{a_0}{(1 - s/\omega_{pa1})(1 - s/\omega_{pa2})}$ ⇒ -ve F.B. needed

Can't define <0 for complex #

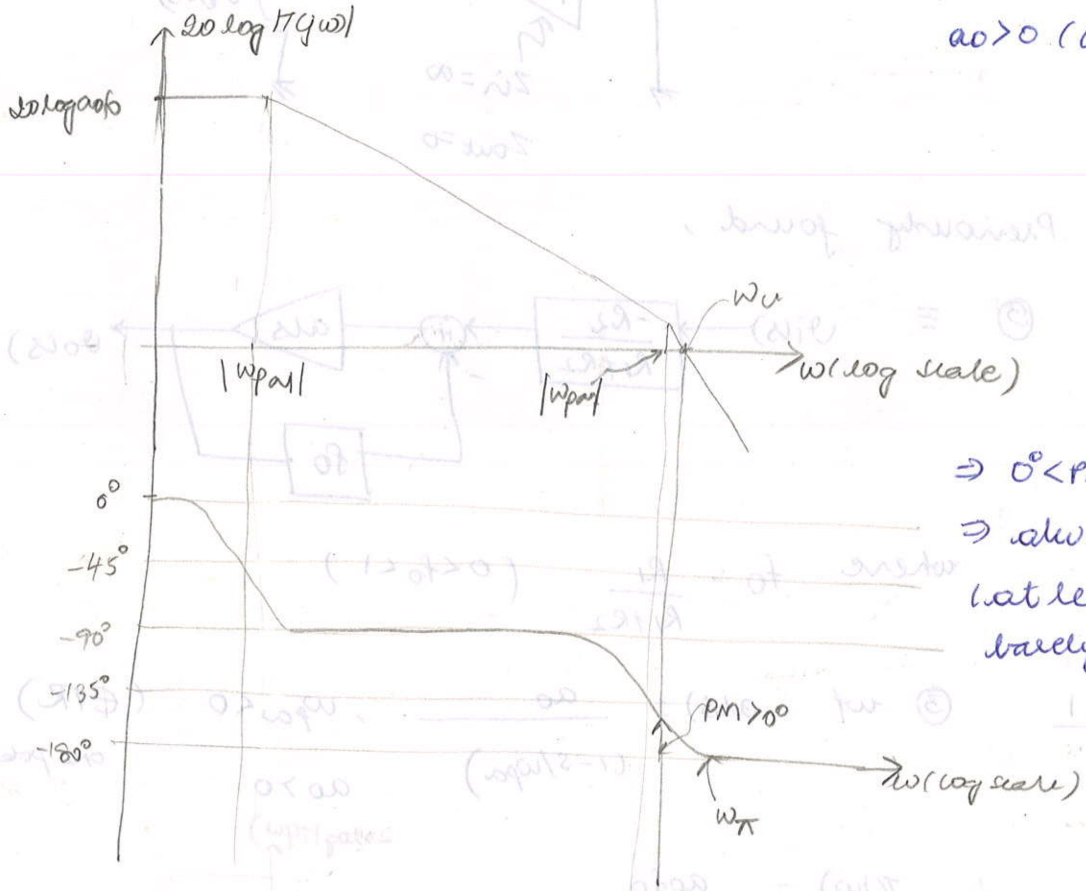
$\omega_{pai} < 0$ $(\in \mathbb{R})$

$i = 1, 2$

$a_0 > 0$ $(\in \mathbb{R})$

e.g.

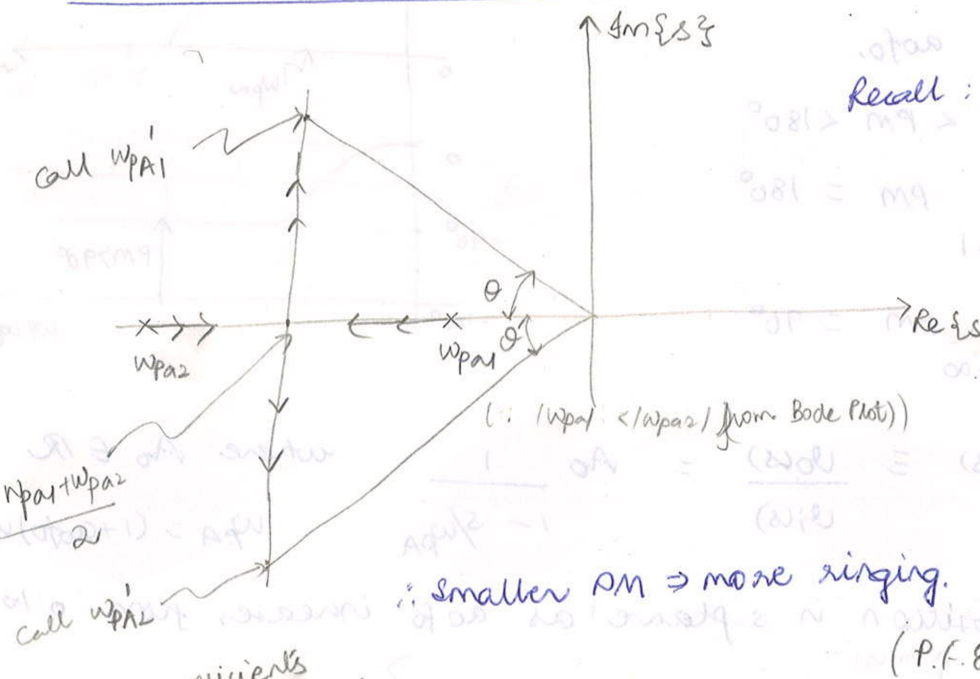
What abt GM?



⇒ $0^\circ < PM < 180^\circ$

⇒ always stable (at least barely)

Root Locus Plot: (Verify) (Plot of ω_{PA1} & ω_{PA2})



Recall: $\theta = \cos^{-1}\zeta$

where ζ = damping ratio

∴ $\zeta = \cos\theta$, so

$\zeta \to 0$ as $\theta \to 90^\circ$

(i.e. as $a_0 f_0 \to \infty$)

(i.e. as $PM \to 0$)

∴ Smaller PM ⇒ more ringing.

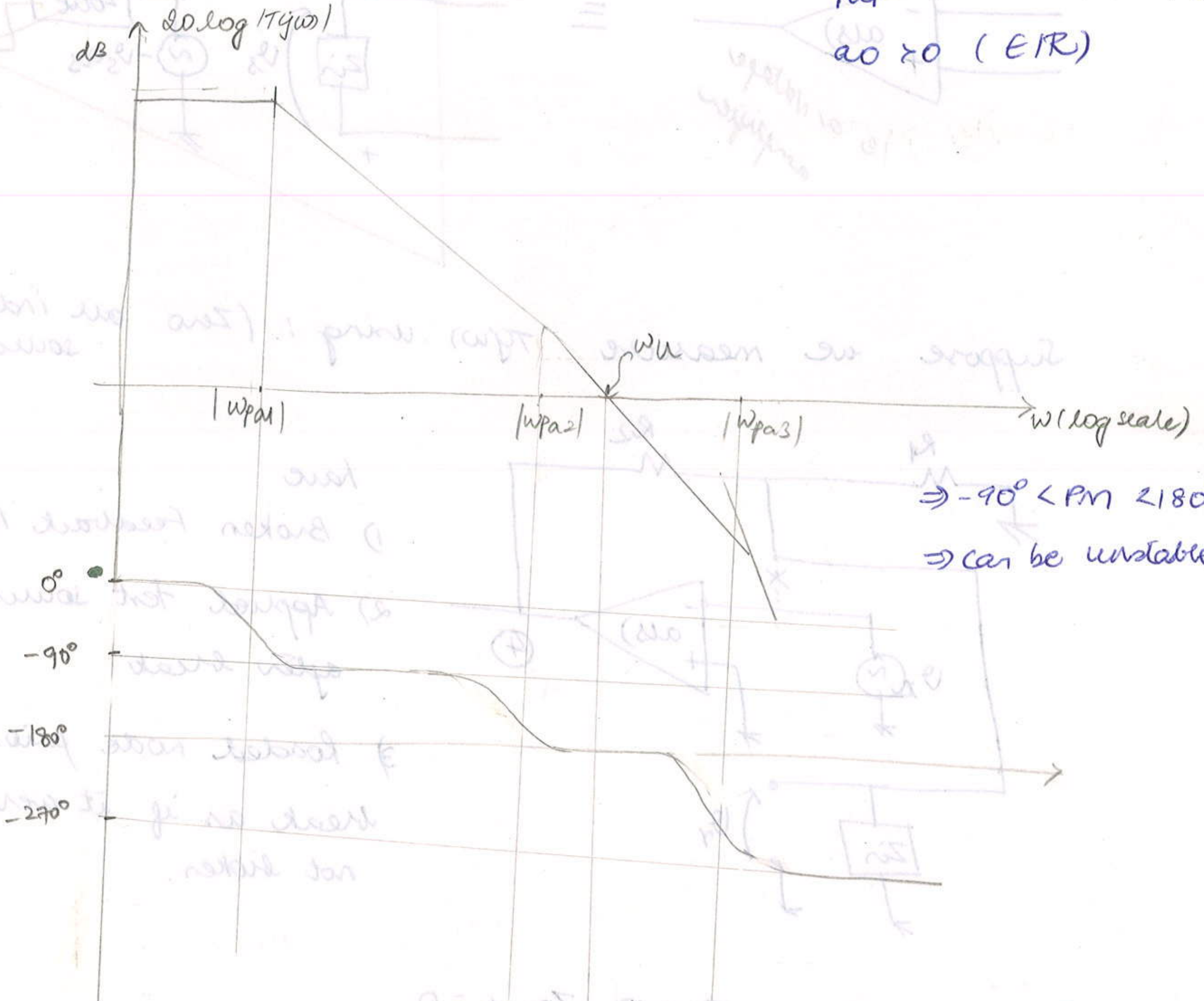
(P.F.E, general 2nd order system)

Real coefficients ⇒ conjugate symmetric.

Ex 3 (3) with $a(s) = \dfrac{a_0}{(1 - s/\omega_{pa1})(1 - s/\omega_{pa2})(1 - s/\omega_{pa3})}$

$\omega_{pai} < 0 \ (\in \mathbb{R}), \ i = 1,2,3$

$a_0 > 0 \ (\in \mathbb{R})$

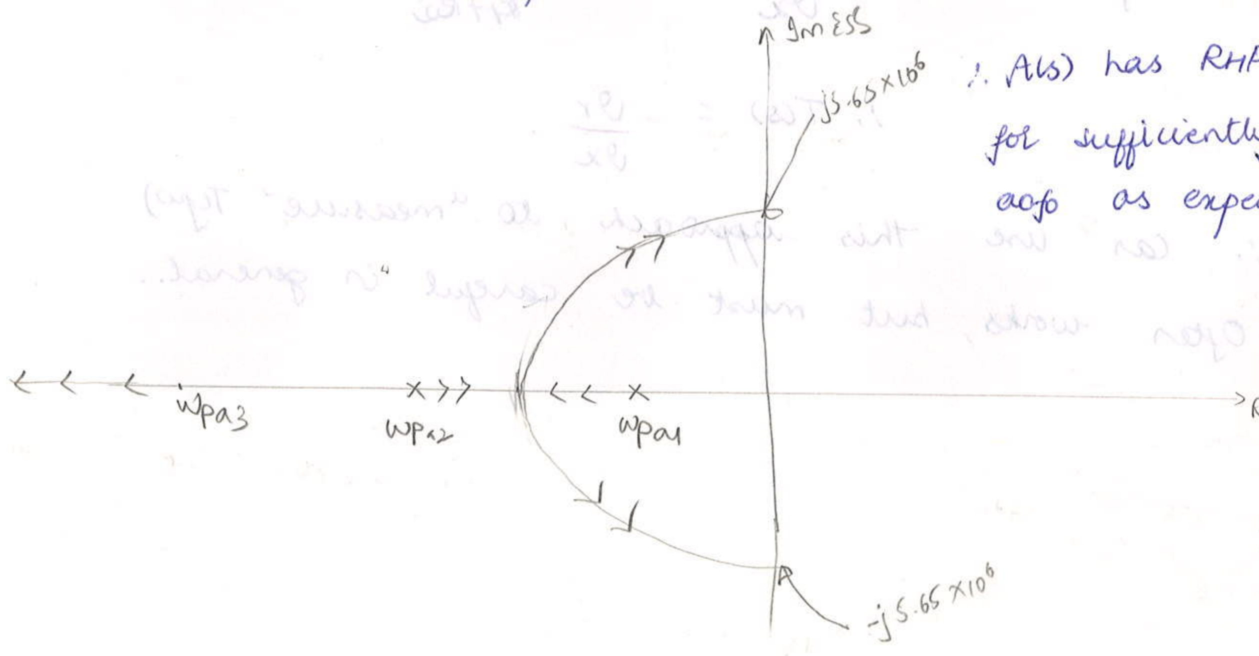


$\Rightarrow -90^\circ < PM < 180^\circ$

$\Rightarrow$ can be unstable

eg suppose $\omega_{pa1} = -10^6$ rad/s, $\omega_{pa2} = -2 \cdot 10^6$ rad/s, $\omega_{pa3} = -10^7$ rad/s

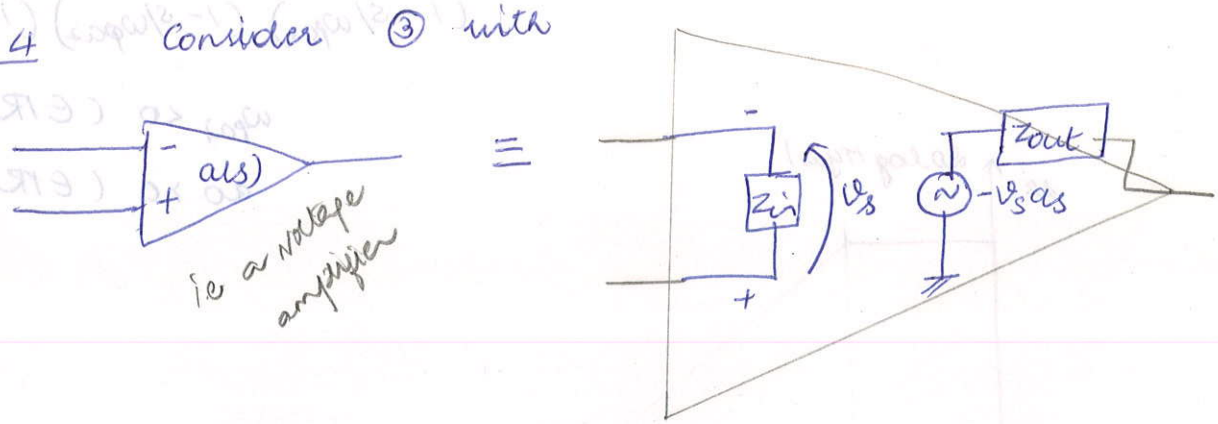
Then the root locus plot is



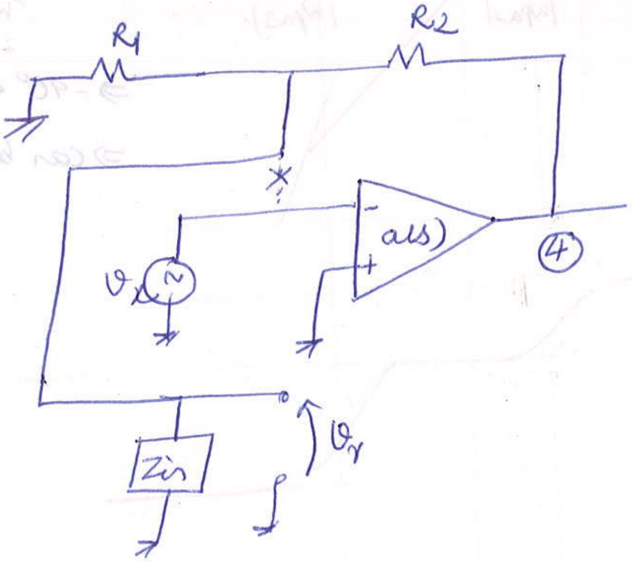
∴ A(s) has RHP poles for sufficiently large $a_0 f_0$ as expected

Using SPICE to Estimate $T(j\omega)$

Ex 4 Consider ③ with



Suppose we measure $T(j\omega)$ using: (Zero all indep. sources)



have

1) Broken feedback loop
2) Applied test source after break
3) Loaded node prior to break as if it were not broken.

First suppose $Z_{in} = \infty$, $Z_{out} = 0$

Inspection $\Rightarrow \dfrac{v_r}{v_x} = -a(s)\dfrac{R_1}{R_1+R_2} = -a(s)f_0 = -T(s)$

$$\therefore T(s) = -\frac{v_r}{v_x}$$

∴ Can use this approach to "measure" $T(j\omega)$

Often works, but must be careful in general.

Suppose $Z_{in} \neq \infty$, $Z_{out} \neq 0$

A.G.R. $\Rightarrow A(s) = A_\infty \frac{T}{1+T} + A_0 \cdot \frac{1}{1+T}$

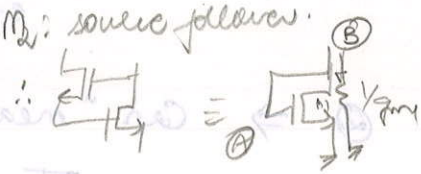
Before $A_0 = 0$ because $Z_{out} = 0$

Now $A_0 \neq 0$

We still get $T(\omega)$ using ④ but analysis will miss any poles contributed by $A_0(s)$.

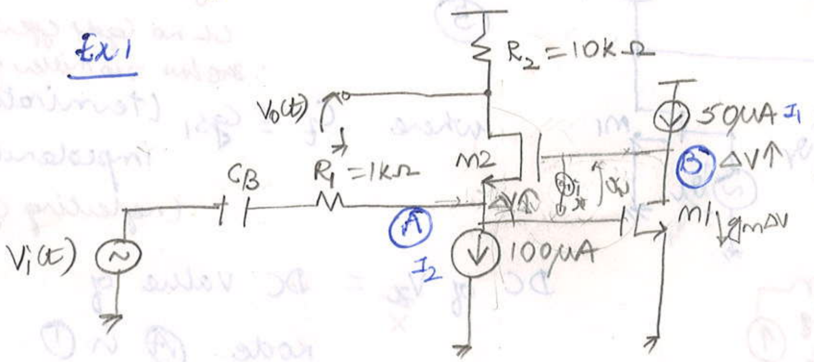
$\boxed{\text{LEC \#14 FEB 28 '08}}$

M_2: source follower.



Using SPICE to estimate $T(\omega)$:

Ex 1



M1: $W/L = 15\mu m / 1\mu m$

M2: $W/L = 20\mu m / 1\mu m$

C_B = large DC blocking cap. (assume $C_B \cong \infty$)

Observations

(i) Ⓐ is a low impedance node:

. inside loop bandwidth $\Rightarrow$ Ⓐ = almost virtual gnd. (H/W)

(freqs low enough that loop can respond)

. even w/ feedback disabled, impedance of Ⓐ $\cong 1k\Omega \| \frac{1}{g_{m2}}$

(ii) similar reasoning $\Rightarrow$ driving point resistance b/w Ⓐ & Ⓑ is relatively small. ($\cong 1/g_{m1}$)

(v_x/i_x relatively small $\sim 1/g_{m1}$)

(ii) $\Rightarrow$ Reasonable to neglect C_{gd1}, C_{gs2} ② (Pole contributed by C_{gd1}, C_{gs2} non-dominant)

Apply A.G.R. w/ $g_{m1} v_{gs1} \cong$ ref. source.

$\therefore A(s) = A_\infty(s) \frac{T(s)}{1+T(s)} + \underbrace{A_0(s)}_{\text{gain of the C.G. amp. formed by } M_2} \frac{1}{1+T(s)}$ (but w/ high imp. gain correction)

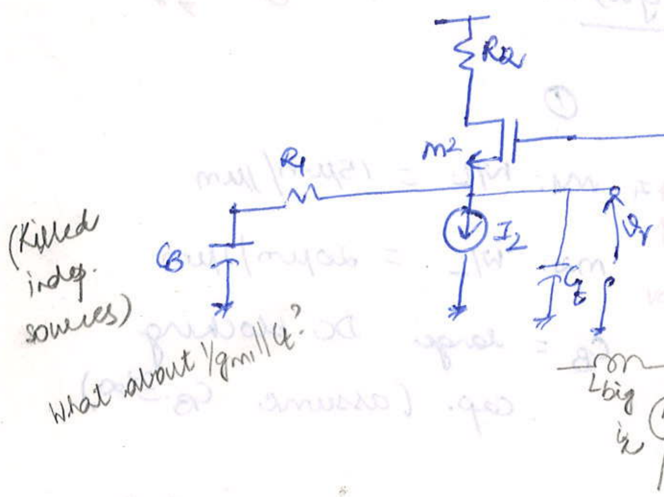
$1/g_{m2} \cong 1k\Omega$, so $A_0(0) \cong 5$ (resistive divider b/w R_1 & $1/g_{m2}$; all i/p current flows through R_2)

The point: $T(s)$ does not give whole stability "picture" because $A_0(s)$ might have marginal stability.

Good Approach: 1) Measure $T(s)$ to estimate degree of stability

2) If simulations show more ringing than expected $A_0(s)$, consider $A_0(s)$

② ⇒ Can "measure" $T(j\omega)$ using:



③ where $C_t = C_{gs1}$ (termination impedance) (neglecting C_{gd1})

C_{gd1} needed in ②
C_L no C_{gd1} effect
'cause b/w → no Miller effect

DC of V_x = DC value of node Ⓐ in ①

⇒ have "broken" feedback loop & applied test signal

$$③ \Rightarrow \underbrace{T(j\omega)}_{} = \frac{-v_r(j\omega)}{v_x(j\omega)}$$

(Same $T(j\omega)$ as in A.G.R. if ② holds - verify)

⇒ simple test to find PM, GM or Nyquist plot.

Simulations ⇒ PM $\cong 90°$ (We have shorted F.B. loop; 1 xtr for amp, 1 xtr for feedback)

In general, this always works but must:

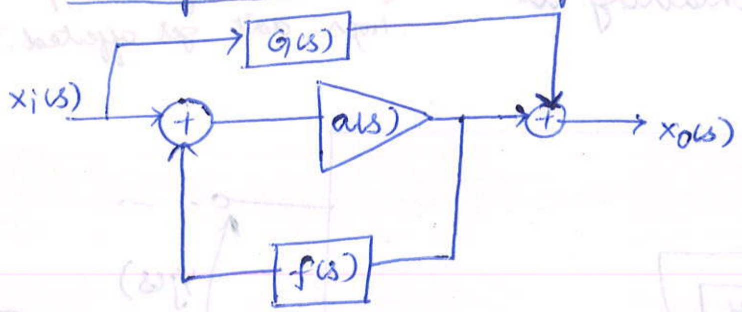
i) Bias the i/p node (following break point) as in closed loop ckt.

ii) Terminate o/p node prior to break point as in closed loop ckt.

inject a current pulse & can analyze stability

HW 6 ⇒ Method of avoiding need to terminate o/p node.

<u>Theory Behind Breaking Feedback loops:</u> (G why two loops?) (A.G.R: ckt → BD tool)



(4)

$$H(s) \equiv \frac{x_o(s)}{x_i(s)} = \underbrace{\frac{a(s)}{1+a(s)f(s)}}_{\text{call } A(s)} + G(s)$$

But $G(s)$ and $A(s)$ { • have poles in (general) • affect stability

In circuits, often { • $A(s)$ ⇒ desired behavior (dominant behavior) • $G(s)$ ⇒ parasitic signal path (indicated by A.G.R.) (non-dominant)

∴ Feedback loop in (4) ⇒ $A(s)$

Knowing $A(s)$ ⇏ insight into how $a(s)f(s)$ can be modified to improve performance.

<u>But</u>

Knowing $T(j\omega)$ ⇒ " ______________ "

(via Nyq. criterion, G.M/PM etc.)

Can "break" any point of feedback loop in (4) and "measure" $T(j\omega)$ via simulation. (In ckt. design usually have ckts not BDs)

The Problem: Usually have circuit, not a B.D. to analyze.

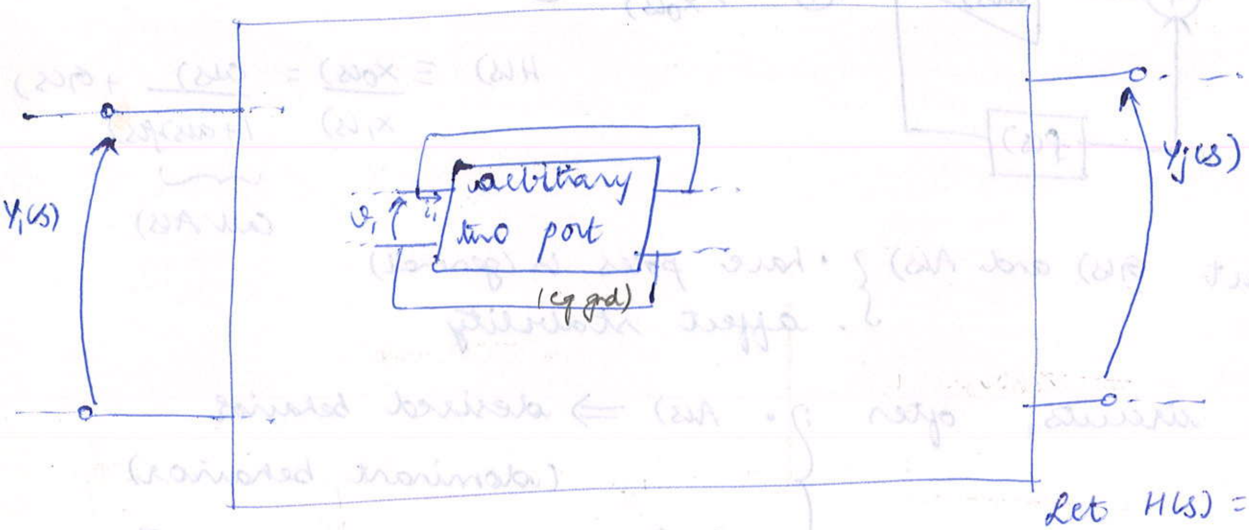
Previously have used AGR for specific ckts. to justify measuring $T(j\omega)$ directly from circuit → which ckt.?

(i.e. without first converting into a B.D.)

Does this work in general?

(If we have multiple loops, how do we make sure that by just breaking one loop, other loops don't get affected?)

Arbitrary LTI circuit containing a feedback loop:

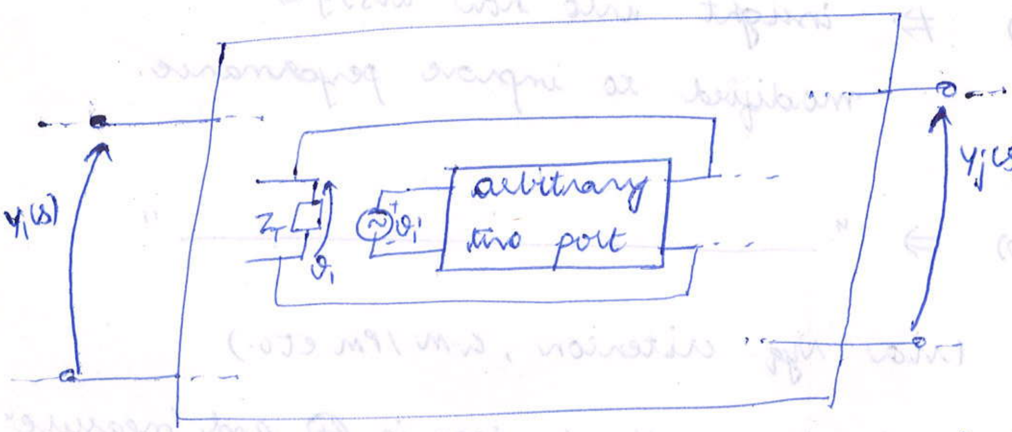


(5)

Let $H(s) = \dfrac{Y_j(s)}{Y_i(s)}$

Note: Can draw (5) such that $Y_i(s)$ = current and/or $Y_j(s)$ = current

Can redraw (5) as:

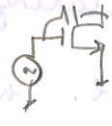


(6)

where $v_1' = \alpha v_1$ w/ $\alpha \equiv 1$, $Z_t(s) = \dfrac{v_1(s)}{i_1(s)}$ (i/p imp. of 2-port; depends on loading of $Y_i(s)$ & $Y_j(s)$ ports)

Now suppose $v_1'(s)$ is replaced by an indep. source v_x

LTI system $\Rightarrow$ $v_1 = C(s)\,Y_i(s) + D(s)\,v_x(s)$

Feedback loop "enabled" $\Longleftrightarrow$ $\underbrace{D(s) \not\equiv 0}_{(7)}$ (~ to  No Cgd effect)

(eg, $v_1 \equiv Y_i$ $\Rightarrow$ (7) violated)

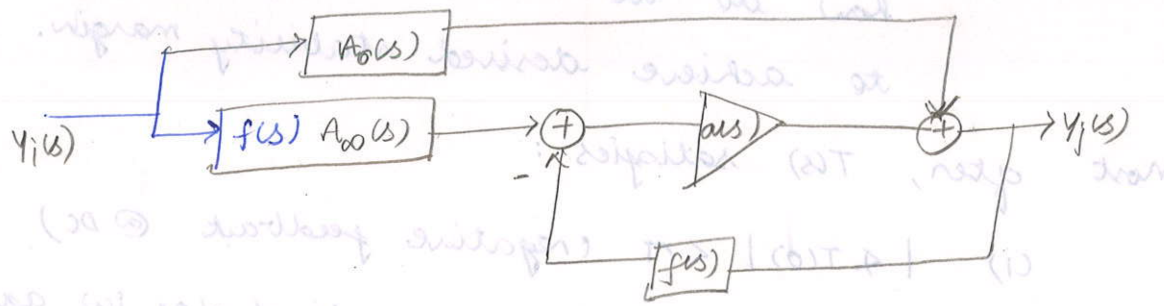
Provided ⑦ holds, can apply A.G.R.

Recall A.G.R. $\Rightarrow$ $H(s) = A_\infty(s)\dfrac{T(s)}{1+T(s)} + A_0(s)\dfrac{1}{1+T(s)}$ ⑧

where $T(s) = -\dfrac{v_1(s)}{v_x(s)}$ for $y_i(s) = 0$ & $v_1'(s)$ replaced by $v_x(s)$

$A_0(s) = \dfrac{y_j(s)}{y_i(s)}$ w/ $\alpha = 0$, $A_\infty(s) = \lim_{\alpha\to\infty}\dfrac{y_j(s)}{y_i(s)}$

∴ Block Diagram of ⑥:



⑨

where $a(s), f(s)$ are any functions such that $a(s)f(s) = T(s)$

<u>Observations</u>:

1) $T(j\omega)$ in ⑧ same as that found by breaking feedback loop in ⑤, terminating breaking w/ $Z_T(s)$, and injecting v_x (or i_x) @ i/p of broken loop.

$\Rightarrow$ Method always works provided ⑦ holds.

2) Can use Nyquist criterion (or PM, GM) to assess relative stability of FB loop in ⑧.

(If F.B. loop has marginal stability, ⑨ i.e. ⑤, also has marginal stability)

LEC #15 MAR 4 '08

Note 1: For the version of the Nyquist plot & criterion presented previously, must have at least as many poles as zeros. (In DSP, Nyquist plot is not directly used).

Compensation: (Act of making more stable)

The problem: Given an amplifier and feedback network, how do we add or adjust components to achieve desired stability margin.

Most often, $T(s)$ satisfies:

(i) $|\measuredangle T(0)| < \pi$ (negative feedback @ DC)

(ii) $T(s)$ has no R.H.P. poles (includes $j\omega$ axis) ①

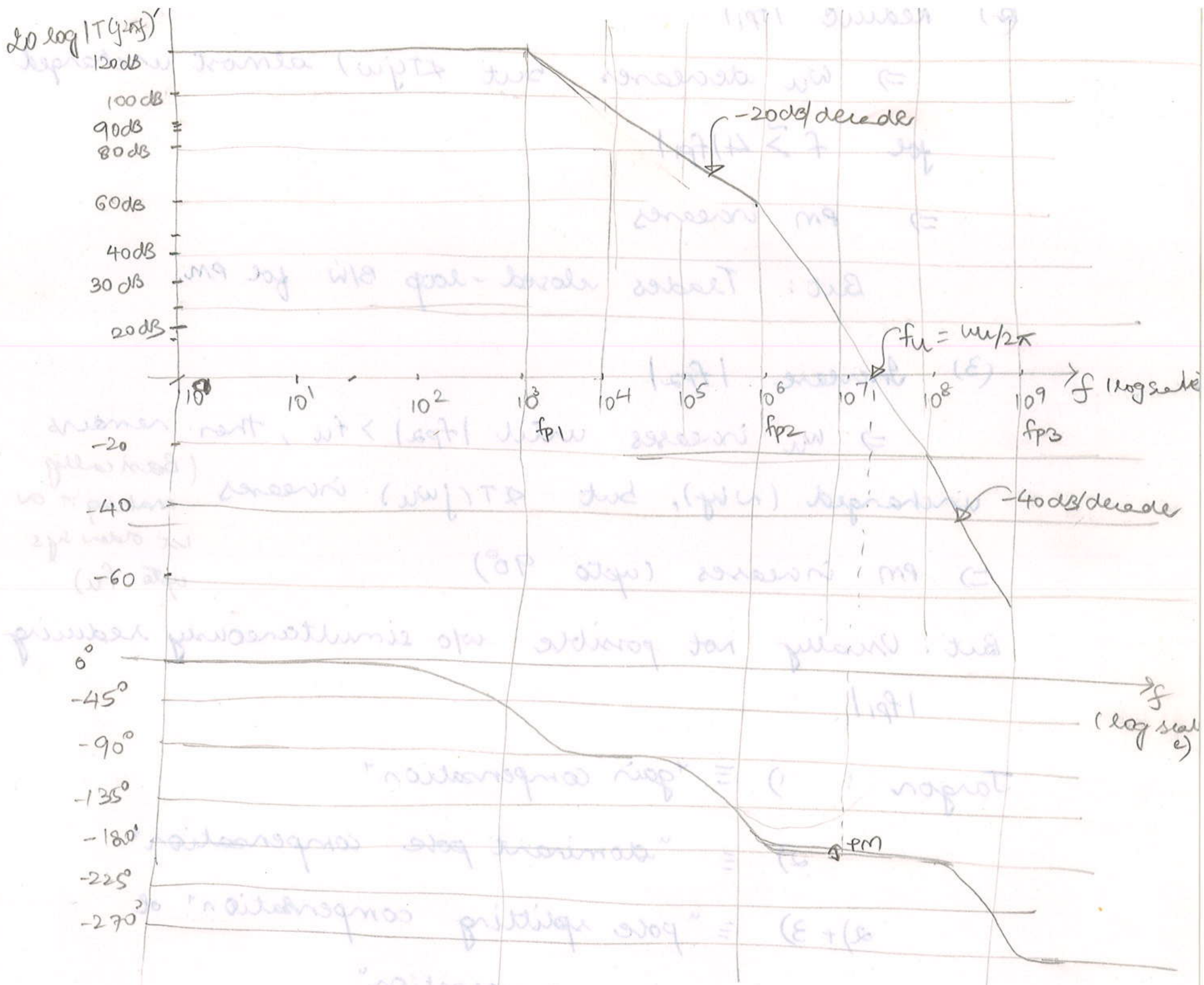
(iii) $|T(j\omega)| = 1$ has one positive solution, $\omega = \omega_u$ (PLL doesn't satisfy above)

① ⇒ PM, GM valid indicators of stability margin.

Ex1 $T(s) = T_0 \dfrac{1}{(1 - s/\omega_{p1})(1 - s/\omega_{p2})(1 - s/\omega_{p3})}$ ②

w/ $T_0 = 10^6$, $f_{p1} = \dfrac{\omega_{p1}}{2\pi} = -10^3\ Hz$

$f_{p2} = -10^6\ Hz$; $f_{p3} = -10^9\ Hz$



Observations:

(i) Both $\angle T(j\omega)$ and $|T(j\omega)|$ decrease monotonically.

(ii) $PM \geq 40°$ requires $|f_{p2}| > f_u$

1. Only 3 ways (compensation methods) to increase PM given $T(s)$ has form of ②.

(1) Reduce T_0 (moves f_u to left)

$\Rightarrow$ ω_u decreases, but $\angle T(j\omega)$ unchanged $\Rightarrow$ PM increases.

But: Trades closed loop accuracy for PM

(feedback benefits require a large T_0)

(2) Reduce $|f_{p1}|$

$\Rightarrow$ ω_u decreases, but $\measuredangle T(j\omega)$ almost unchanged for $f \gtrsim 4|f_{p1}|$

$\Rightarrow$ PM increases

But: Trades closed-loop B/W for PM.

(3) Increase $|f_{p2}|$

$\Rightarrow$ ω_u increases until $|f_{p2}| > f_u$, then remains unchanged ($\sim$ly), but $\measuredangle T(j\omega_u)$ increases (Basically making it a 1st order sys. upto f_u)

$\Rightarrow$ PM increases (upto 90°)

But: Usually not possible w/o simultaneously reducing $|f_{p1}|$.

Jargon: 1) $\equiv$ "gain compensation"

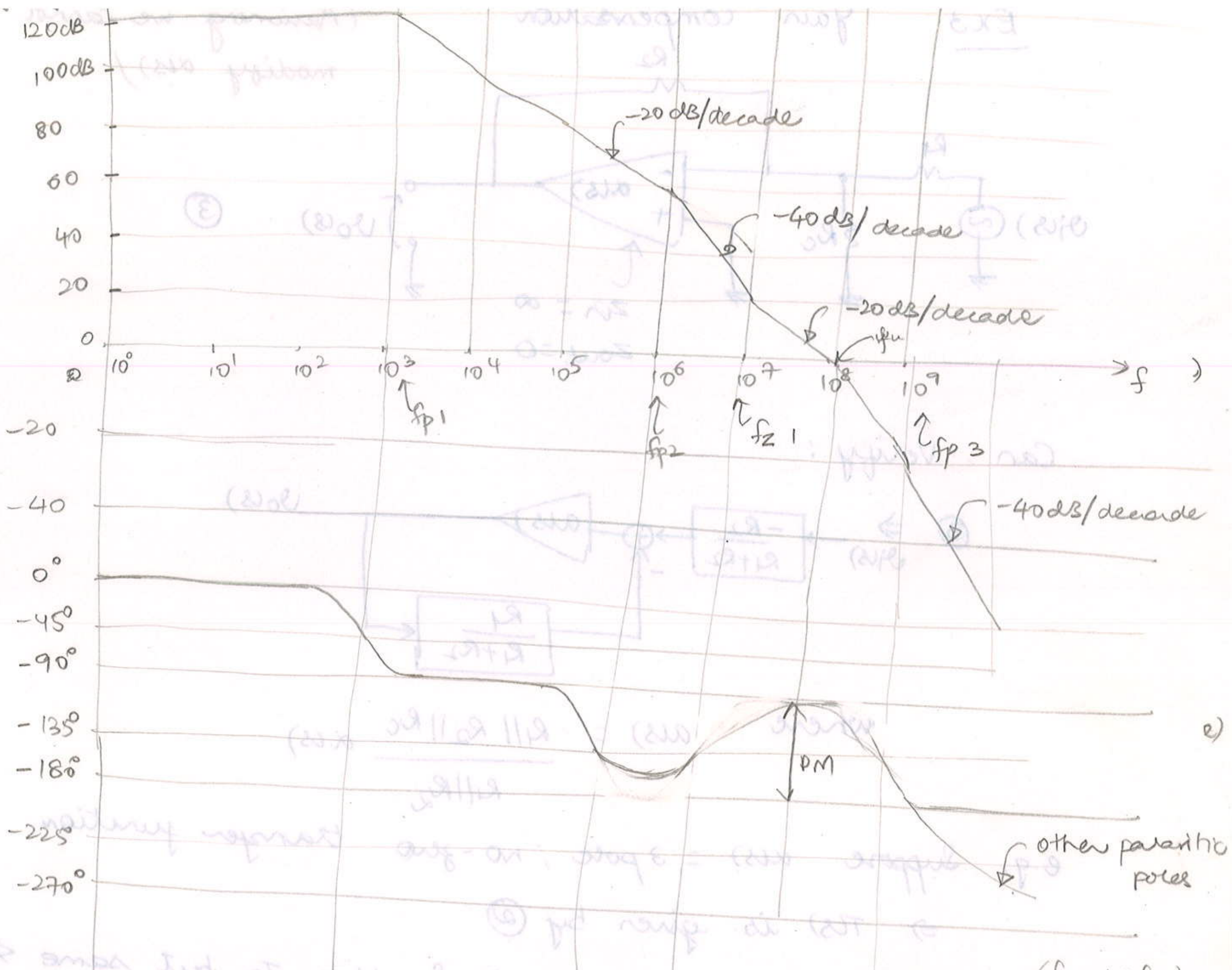
2) $\equiv$ "dominant pole compensation"

2) + 3) $\equiv$ "pole splitting compensation" or "miller compensation"

<u>Ex2</u> Suppose we can add a LHP zero to the $T(s)$ given by ②

e.g. let $T'(s) = \underbrace{T(s)}_{\text{same } T(s) \text{ as in ex1}} (1 - s/\omega_{z1})$

$$f_{z1} = \frac{\omega_{z1}}{2\pi} = -10^7 \text{ Hz}$$



$(f_z = 1.2 f_{p1})$

Observations:

(i) LHP zero added positive phase shift w/o significantly changing f_u

⇒ Increased PM

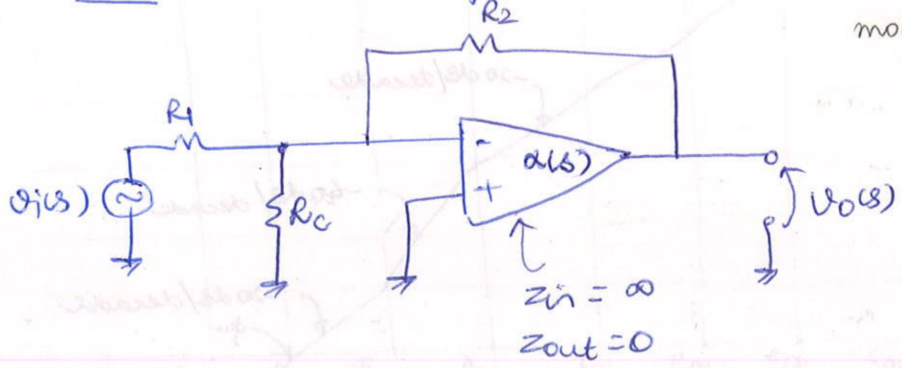
⇒ Adding a LHP zero ≡ compensation strategy.
called "lead compensation"

(ii) A RHP zero would have decreased PM

lead compensation used in 2-stage CMOS op-amps (soon)

Ex 3 Gain compensation

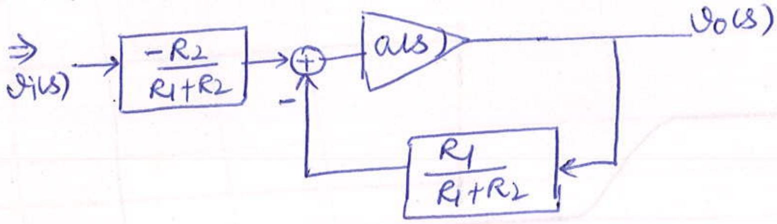
(Assuming we cannot modify α(s))



(3)

Can Verify:

(3) ⇒



where $a(s) = \dfrac{R_1 || R_2 || R_C}{R_1 || R_2} \alpha(s)$

e.g. Suppose $\alpha(s)$ = 3 pole, no-zero transfer function.

$\Rightarrow$ $T(s)$ is given by (2)

$\Rightarrow$ Decreasing R_C $\Rightarrow$ Smaller T_0 but same $\angle T(j\omega)$ as before.

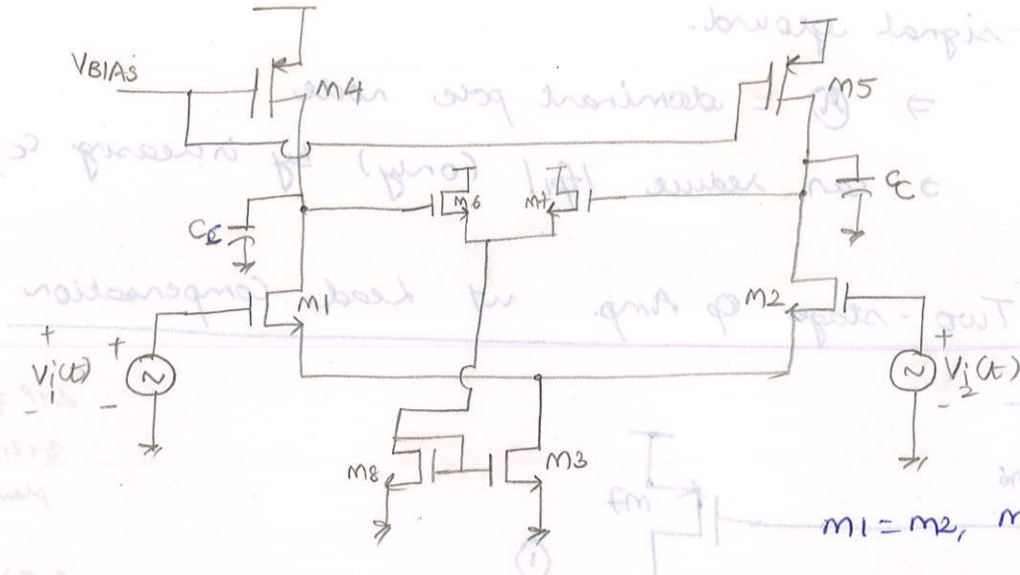
e.g. suppose w/ $R_C = \infty$, $T(s)$ = same as Ex 1

Then $PM \cong 4°$ (Very poor relative stability)

e.g. suppose $R_1 = R_2$, and $R_C = \frac{R_1}{1000}$. Then,

$PM \cong 35°$. But went from $T_0 = 72$ dB to $T_0 = 66$ dB

Ex 4 Dominant pole compensation of CMFB.



Need $I_{ds4} = I_{ds5}$
so that
$V_{DS1} = V_{DS2}$

$M1 = M2, \; M3 = M8$

$M4 = M5, \; M_6 = M_7.$

Exercise: Consider how C_c implements dominant pole comp.

LEC #16 MAR 6'08

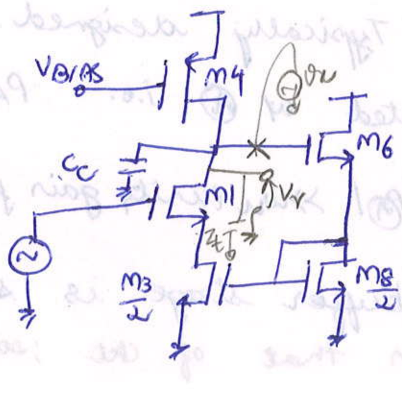
Recall: diff pair w/ CMFB diagram

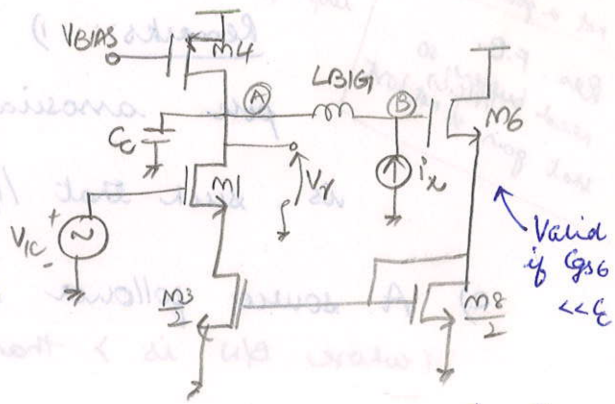
CM ½ ckt. and $T(\omega)$ measurement config.:-

C.M. large signal ½ ckt.:-

DC of (A) = DC of (B)

(- C_c may be cap. @ o/p of amp
- In this ckt, freq. of op is lowered by C_c)





(L_{big} ⇒ DC voltage at two terminals of L_{big} are the same but L_{big} blocks non-zero freq. components)

Note: transimpedance from node (A) to all other nodes is small.

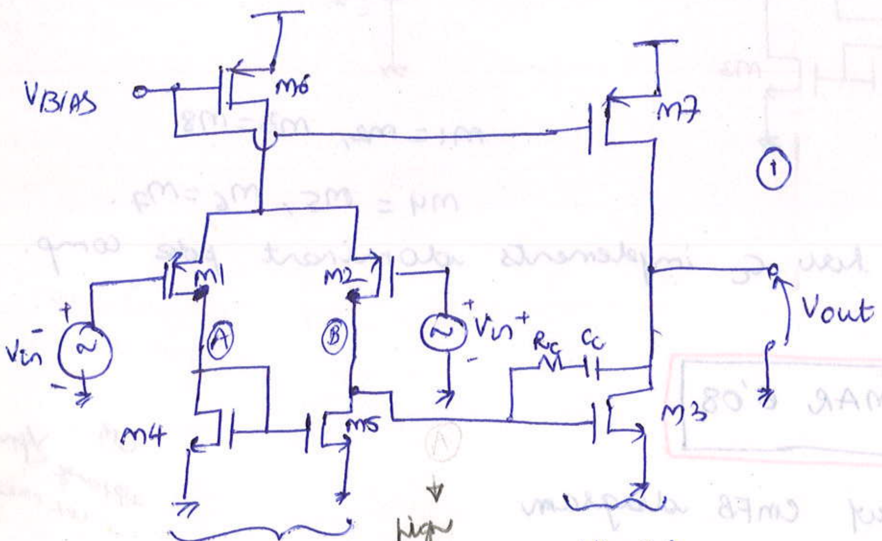
$\Rightarrow \dfrac{-1}{C_c R_{(A)}}$ = pole of $T(s)$ (C_c has little effect on other poles of $T(s)$)

where $R_{(A)}$ is small-signal resistance from node (A) to small-signal ground.

$\Rightarrow$ (A) = dominant pole node

$\Rightarrow$ can reduce $|f_{p1}|$ (only) by increasing C_c)

Ex Two-stage Op Amp. w/ Lead Compensation



LHP zero

$(s+z_1)$ $z_1>0$

plane > 0

$(s-z_1)$ $z_1>0$

plane ~ to LHP pole

high imp. node

compensation within loop b/w.

stage 2 $\Downarrow$ two xtors $\Rightarrow$ good tuning $\Rightarrow$ good SNR.

Compensating w.r.t. initial gnd not a good idea

Res. F.B. need buffer so $V(C_iC_2)$ that gain is not lost

Remarks: 1) Typically designed such that the pole associated w/ (A) i.e. $P_A \cong \frac{-g_{m1}}{C_{(A)}}$ is such that $|P_{(A)}| \gg \omega_u$ (unity gain freq.)

$C_{(A)}$ → cap. from (A) to small signal gnd.

2) A source follower buffer stage is sometimes added to (1) (whose B/W is > than that of ckt. ; so doesn't influence)

3) Could replace PMOSTs by NMOSTs & vice versa but PMOSTs have better 1/f noise performance, so (1) as shown has better 1/f noise (Why?)

~~Gain~~ (Noise should be lower in 1st stage).

First consider w/ $R_C = 0$

Using previous results & remark ①:

① has 2 significant poles p_1 & p_2 and 1 significant zero z_1, given by

$$p_1 \cong \frac{-1}{g_{m3} R_1 R_2 C_C} \quad ② \quad \text{where } R_1 = r_{ds2} \| r_{ds5}$$

$$R_2 = r_{ds3} \| r_{ds7}$$

and $C_C \gg C_{gd}$ is assumed.

$$p_2 \cong - \frac{g_{m3} C_C}{C_{d3} C_{gs3}' + C_C (C_{d3} + C_{gs3}')} \quad ③$$

where C_{d3} = total cap. from o/p to ground ~~ground~~ from M3, M7 & any load

$$\cong \frac{-g_{m3}}{C_{d3}} \quad \text{(often)}$$

$$z_1 \cong \frac{g_{m3}}{C_C} \quad ④$$

$\therefore$ p_1 = dominant pole (sets open-loop B/W)

p_2 = non-dominant pole (w/ p_1 sets open-loop phase shift @ unity gain freq.)

z_1 = RHP zero (adds negative phase shift just like a LHP pole)

e.g. $g_{m3} = 1.10^{-3}\ \Omega^{-1}$, $R_1 = R_2 = 100k\Omega$, $A_{v0_1} = 70$ (= DC gain of stage 1)

$C_{d3} = C_{gs3}' = 350fF$, $C_C = 1pF$ ⑤

② - ⑤ $\Rightarrow$ $p_1 = -100 \times 10^3$ rad/s (−16 kHz)

$p_2 = -1.2 \times 10^9$ rad/s (−191 MHz)

$z_1 = 10^9$ rad/s (+159 MHz)

A_{vo2} ($\equiv$ DC gain of stage 2) $= -g_{m3}R_2 = -100$

$\therefore A_v(s) \cong 7000 \dfrac{(1 - s/10^9)}{(1+s/10^5)(1+s/1.2\times10^9)}$

$\therefore \omega_u \cong 7.1\times10^8$ rad/s (113 MHz) (Verify)

$\measuredangle A_v(j\omega_u) = \tan^{-1}\left(-\dfrac{7.1\times10^8}{10^9}\right) - \tan^{-1}\left(\dfrac{7.1\times10^8}{10^5}\right) - \tan^{-1}\left(\dfrac{7.1\times10^8}{1.2\times10^9}\right)$

$\cong \underbrace{-35.4^\circ}_{\text{Zero}} \quad \underbrace{-90^\circ}_{\text{dom. pole.}} \quad \underbrace{-30.6^\circ}_{\text{non-dom pole}} = -156^\circ$

If $f(s) = 1$, then $T(s) = A_v(s)$, so PM $= 180^\circ - |-156^\circ| = 24^\circ$
(low Phase margin)

If $f(s) = 0.1$ say, then we may get good P.M.

Optimal Phase margin: ?

Note:- 1) Increasing C_c reduces negative phase shift from p_2 to ω_u
but increases " " " from z_1 to ω_u.
(because ω_u decreases but so does z_1)

2) Situation is actually worse than calculated because of pole @ (A).

Heuristics: 1) As ω increases, feed-forward path through C_c begins to dominate gain path through M_3. 2) Gain path through M_3 has 180° -ve polarity. phase shift.) but the feedforward path through C_c has phase $\to 0$ as $\omega\to\infty$.

(This prob is due to zero)

$\Rightarrow$ As $\omega\to\infty$, have positive feedback.

Now consider w/ $R_C > 0$: (What is range of R_C reqd? How/what is used by Lads?)

Can show, $R_C \neq 0 \Rightarrow \begin{cases} \text{have } z_1 \cong \dfrac{1}{C_C\left(\frac{1}{g_{m3}} - R_C\right)} & ⑥ \\ \text{have a 3rd pole, } p_3 \end{cases}$

Usually : 1) $|p_3| >> |p_1|, |p_2|, |z_1|$, so can ignore p_3

2) p_1, p_2 are hardly affected by R_C. ⑦

⑥ $\Rightarrow R_C > \frac{1}{g_{m3}} \Rightarrow z_1$ "moves" to LHP

$\Rightarrow z_1$ contributes positive phase shift

Fact: ⑦ breaks down for very large R_C

Typical rule of thumb: Choose R_C such that $|z_1| \cong 1.2\,\omega_u$ — ⑧

⑥ & ⑧ $\Rightarrow 1.2\,\omega_u \cong \left| \dfrac{1}{C_C\left(\frac{1}{g_{m3}} - R_C\right)} \right|$

$\therefore R_C = \dfrac{1}{g_{m3}} + \dfrac{1}{1.2\,\omega_u C_C}$ ⑨ (for $R_C > 1/g_{m3}$)

e.g. Using ⑤, ⑨ $\Rightarrow R_C = 2.17\,k\Omega$

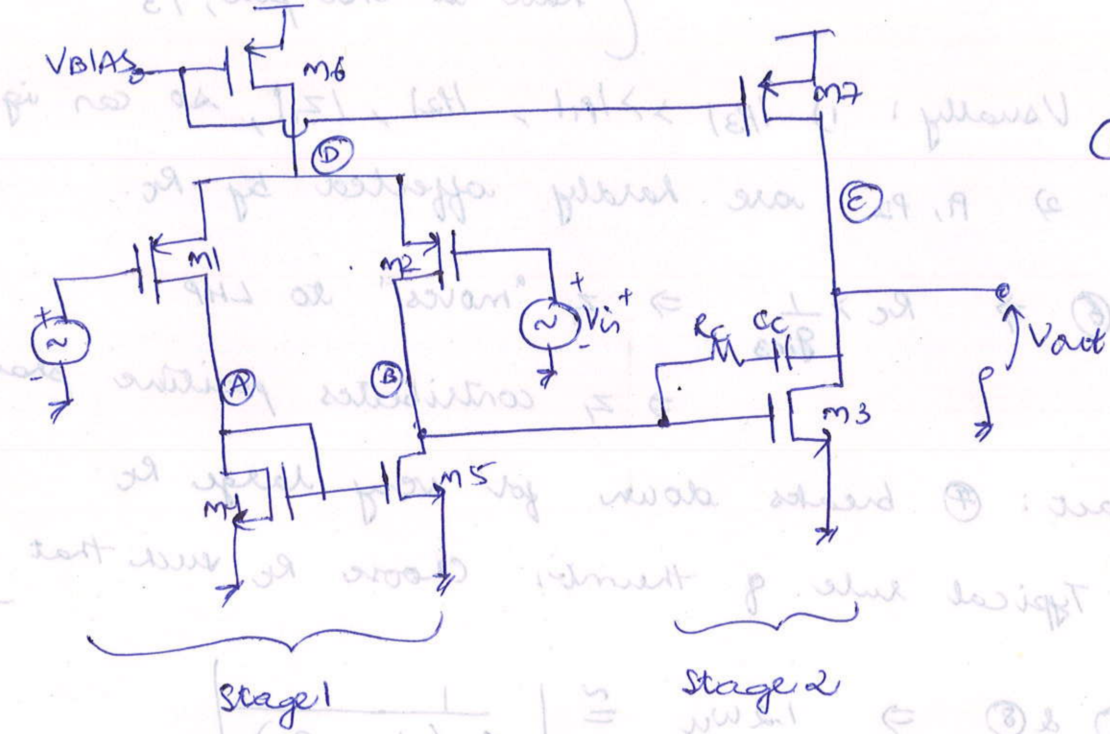
Now $\angle A_v(j\omega_u) = \tan^{-1}\left(\dfrac{\omega_u}{1.2\,\omega_u}\right) - 90^\circ - 30.6^\circ \cong -80.8^\circ$

$\therefore$ New PM for $f(s) = 1$: PM $\cong 99.2^\circ$

$\Rightarrow$ Can afford to reduce C_C ($\Rightarrow$ increases BW) to reduce PM (typ. want PM $> 65^\circ$)

Two stage op-amp. (contd.) :-

(back in vogue; high gain w/o stacking)



1

1) common-mode rejection of stage 1

$\Rightarrow$ usually can neglect pole associated w/ node D when considering DM behavior of op amp. (Non linearity of M1 & M2; r_o's of M1 & M2 cause deviation from this, whence D no longer acts as virtual gnd.)

2) Usually designed such that pole associated w/ A:

i.e. $p_A \cong -\frac{g_{m4}}{C_A}$

is S.T. $|p_A| \gg \omega_u$

2

Then M2, M5, M3, M7, Rc, Cc determine PM when 1 is used in F.B. system.

3) Previously found:

gain of stage 1 ↑   gain of stage 2 ↑

$$A_o(j\omega) = \frac{v_{out}(j\omega)}{v_{in}^{+}(j\omega) - v_{in}^{-}(j\omega)} \cong \frac{\underbrace{(g_{m1}R_1)}_{\text{stage 1}}\underbrace{(g_{m3}R_2)}_{\text{stage 2}}(1 - j\omega/z_1)}{(1 - j\omega/p_1)(1 - j\omega/p_2)} \quad \text{③}$$

where $R_1 = r_{ds2} \| r_{ds5}$, $R_2 = r_{ds3} \| r_{ds7}$

$$p_1 \cong \frac{-1}{g_{m3}R_1R_2C_c} \quad \text{(dominant pole)} \rightarrow \text{④}$$

Figure dominant pole using ZVTC

$$p_2 \cong \frac{-g_{m3}}{C_{ds3} + C'_{gs3}} \quad \text{(First non-dom pole)} \rightarrow \text{⑤}$$

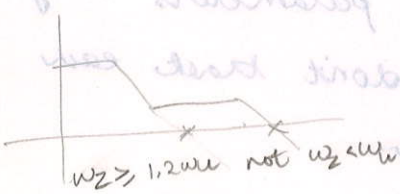
$$\& \quad z_1 \cong \frac{1}{C_c\left(\frac{1}{g_{m3}} - R_c\right)} \quad \text{(Dom zero)} \rightarrow \text{⑥}$$

Note: ④ - ⑥ hold for typical design parameters.

4) $p_1 \Rightarrow$ 3dB BW and introduces $-90°$ phase @ ω_u.

doesn't have effect on P.M.

$p_2, z_1 \Rightarrow$ PM (i.e. PM varies with p_2, z_1; not with p_1)



assumes $|p_2|, |z_1| \gtrsim 10|p_1|$

(generally true)

5) R_c chosen s.t. z_1 in LHP $\Rightarrow$ introduces positive phase shift @ ω_u.

Jargon: PM of an open-loop op-amp. (= PM of op-amp. w/ unity gain F.B. ⇒ worst case)

$$\equiv 180° + \angle A_o(j\omega_u)$$

Why? When connected as a voltage follower, i.e. $f=1$, (hardest non-attenuating config. to compensate) loop gain

$\equiv T(j\omega) = A_o(j\omega)$

Usually, ① designed such that $\underbrace{|P_2|, |Z_1|}$ > ω_u same order of magnitude.

$$\therefore ③ \Rightarrow |A_v(j\omega_u)| \cong \left| g_{m1} g_{m3} R_1 R_2 \frac{1}{1 - j\frac{\omega_u}{P_1}} \right|$$

$$\cong g_{m1} g_{m3} R_1 R_2 \left| \frac{P_1}{j\omega_u} \right|$$

$$\therefore ④ \Rightarrow \cong \left| \frac{g_{m1}}{C_c j\omega_u} \right|$$

$$\therefore |A_v(j\omega_u)| = 1 \Rightarrow \omega_u \cong \frac{g_{m1}}{C_c} \quad ⑦$$

$$PM = 180° - \tan^{-1}\frac{\omega_u}{Z_1} + \tan^{-1}\frac{\omega_u}{P_2} - \underbrace{90°}_{\text{contributed by } P_1} \quad ⑧$$

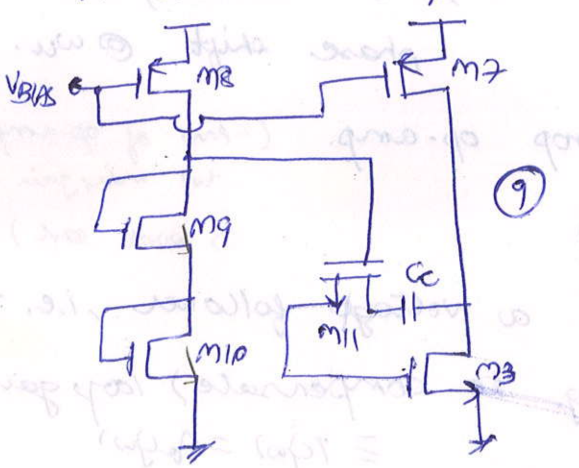
$\because Z_1$ is -ve $\Rightarrow -\tan^{-1}\frac{\omega_u}{Z_1}$ is +ve, she has to increase PM

Problem: How to maintain constant PM across temp. / process / supply voltage variations?

$$⑤ - ⑧ \Rightarrow PM \text{ depends on } \underbrace{g_{m1}, g_{m3}, C_{d3}, C_{gs3}, C_c, R_c}$$

These parameters vary and don't track each other.

A Solution: Replace stage 2 of ① by:



⑨

$$\text{w/ } \frac{W_7/L_7}{W_3/L_3} = \frac{W_8/L_8}{W_{10}/L_{10}} \quad ⑩$$

M_{11} biased by $M_8 - M_{10} \equiv R_C$

M_{11} has no $\underbrace{\text{DC current}}_{\text{b/c of } C_c}$

$\Rightarrow M_{11}$ is in triode

$$\Rightarrow R_c \cong \frac{1}{\mu_n C_{ox} (W_{11}/L_{11}) \underbrace{(V_{GS11} - V_{Tn})}_{\text{call } V_{eff11}}}$$

$$g_{m3} = \mu_n C_{ox} (W_3/L_3) V_{eff3}$$

$$\therefore \quad R_c g_{m3} = \frac{(W_3/L_3)}{(W_{11}/L_{11})} \frac{V_{eff3}}{V_{eff11}} \qquad (11)$$

$$(5) \Rightarrow \quad z_1 = \frac{g_{m3}}{C_c\left[1 - \left[\frac{W_3/L_3}{W_{11}/L_{11}} \frac{V_{eff3}}{V_{eff11}}\right]\right]} \qquad (12)$$

$$(7) \,\&\, (12) \Rightarrow \frac{w_u}{z_1} = \frac{g_{m1}}{g_{m3}} \left[1 - \frac{W_3/L_3}{W_{11}/L_{11}} \frac{V_{eff3}}{V_{eff11}}\right]$$

$$(5),\ \cancel{(6)},\ (7) \Rightarrow \frac{w_u}{p_2} = \frac{g_{m1}}{g_{m3}} \frac{C_{d3} + C_{gs3}}{C_c}$$

$$\frac{g_{m1}}{g_{m3}} = \sqrt{\frac{2\mu_p C_{ox} (W_1/L_1) I_{D1}}{2\mu_n C_{ox} (W_3/L_3) I_{D3}}}$$

Facts :

1) $\frac{\mu_p}{\mu_n} \cong$ const. for a given process

2) $\frac{I_{D1}}{I_{D3}} \cong$ const. because derived from a common bias network.

$\Rightarrow \frac{g_{m1}}{g_{m3}} \cong$ const.

3) $\frac{C_{d3} + C_{gs3}}{C_c} \neq$ const. - but doesnot vary much over process & temp because dominated by oxide cap.

$\therefore$ PM $\cong$ const. provided $V_{eff3}/V_{eff11} \cong$ const.

(10) $\Rightarrow$ $V_{eff10} = V_{eff3}$

$V_{eff9} = V_{eff11}$

$$\therefore \frac{V_{eff3}}{V_{eff11}} = \frac{V_{eff10}}{V_{eff9}} = \frac{\sqrt{\dfrac{2 I_{D10}}{\mu_n C_{ox} (W_{10}/L_{10})}}}{\sqrt{\dfrac{2 I_{D9}}{\mu_n C_{ox} (W_9/L_9)}}}$$

$$= \frac{1}{\sqrt{\dfrac{W_{10}/L_{10}}{W_9/L_9}}}$$ = ratio of like quantities.

$\therefore$ PM $\cong$ indep. of process & temp.