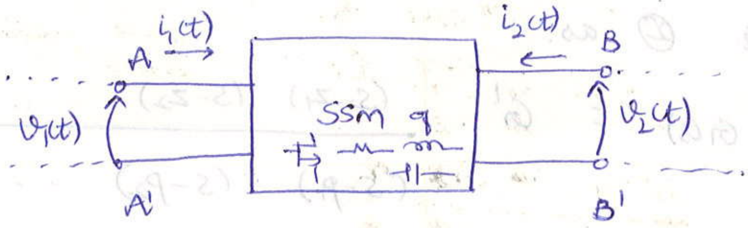
LEC #1

8 JAN '08

"It all boils down to amplification & matching."

Poles, Zeros & Freq Response (Mostly Review)

SSM of any ckt. containing transistors & RLC n/ws → SSM



(1)

A, A' ≡ any two nodes of ckt.

B, B' ≡ " ______ "

Defn: Let ~~$x(t) = 0 \;\forall\; t < 0$~~ $x(t)$ be any real signal

Then $\underbrace{\mathcal{L}\{x(t)\}}_{\text{Call } X(s)} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$ ≡ Bilateral Laplace transform

Let $G(s) = \dfrac{X(s)}{Y(s)}$ where $x(t) = v_1(t),\, i_1(t),\, v_2(t)$ or $i_2(t)$

& $y(t) =$ " ______ "

e.g. $G(s) = \cancel{\dfrac{V_2(s)}{V_1(s)}}\ \dfrac{v_2(s)}{v_1(s)}$ ≡ Laplace transform of gain from A-A' to B-B'

(1)'

or $G(s) = \dfrac{v_1(s)}{i_1(s)}$ ≡ Laplace transform of impedance @ A, A' terminals

etc.

Fact: $G(s)$ is always rational w/ real coefficients.

(Something true but not going to prove it)

i.e. $G(s) = \dfrac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}$ → (2) "rational"

where $a_k, b_k \in \mathbb{R}$ "real coeffs"

Q Do $\exists$ practical ckts. for which $G(s) \neq$ rational

there exists

A Yes! eg: delay element:

$$G(s) = e^{-s\tau} \equiv \tau \text{ second delay}$$

(But $e^{-s\tau}$ is an infinite power series expansion which can be truncated & hence its always fine to treat $G(s)$ as being rational)

can write ② as:

$$G(s) = G' \cdot \frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)} \quad ③$$

where $G', z_k, p_k \in \mathbb{C}$

$z_k, \ k = 1, 2, \ldots m \equiv$ zeros of $G(s)$

$p_k, \ k = 1, 2, \ldots n =$ poles of $G(s)$

Fact

$G(s)$, $s \in \mathbb{C}$

$G(j\omega)$, $\omega \in \mathbb{R}$ (if it exists)

$g(t) = \mathcal{L}^{-1}\{G(s)\}$

} Contain equivalent info about ①

"Transfer Function"

"Impulse Response"

"Freq Response"

(can obtain one fn. from other interchangeably)

- Goal: Find simple way to deduce "shape" of $G(j\omega)$ from poles and zeros of $G(s)$

Claim 1: Real coefficients $\iff$ (is equivalent to) if $s_0 = \alpha + j\beta$ (α, β are real no.s i.e. $\alpha, \beta \in \mathbb{R}$) is a pole (or zero) of $G(s)$

then $s_0^* = \alpha - j\beta$ is a pole (or zero) of $G(s)$

Proof: Exercise

Ex $$(s-s_0)(s-s_0^*) = s^2 - s(\underbrace{s_0+s_0^*}_{2\mathrm{Re}\{s_0\}}) + \underbrace{s_0 s_0^*}_{|s_0|^2}$$

$$= s^2 + s(2\zeta\omega_0) + \omega_0^2$$

ζ ज़ेटा (Zeta)

where $\omega_0 \equiv |s_0| \equiv$ "resonant freq"

$$\zeta = \frac{-\mathrm{Re}\{s_0\}}{\omega_0} \equiv \text{"damping ratio"}$$

(Claim 1 $\Rightarrow \zeta, \omega_0 \in \mathbb{R}$)

∴ Can group conjugate poles & zeros to get:

$$G(s) = G' \left[\prod_{\substack{\text{real}\\\text{zeros}}} (s-z_k)\right] \left[\prod_{\substack{\text{complex}\\\text{zero pairs}}} \left(s^2+s(2\zeta_i\omega_{0i})+\omega_{0i}^2\right)\right]$$

$$* \left[\prod_{\substack{\text{real}\\\text{poles}}} \frac{1}{(s-p_k')}\right] \left[\prod_{\substack{\text{complex}\\\text{pole pairs}}} \frac{1}{s^2+s(2\zeta_i'\omega_{0i}')+\omega_{0i}'^2}\right]$$

$\longrightarrow$ (4)

Let ω_{zi}, $i = 1, 2, \ldots, N \equiv$ the set of (non-zero) real zeros

ω_{pi}, $i = 1, 2, \ldots N' \equiv$ the set of (non-zero) real poles

Do complex #s always occur in pairs?

$$\therefore\ G(s) = G(0)\, s^{n_0} \left[\prod_{\substack{\text{real}\\\text{zeros}}} (1 - s/\omega_{zk})\right] \left[\prod_{\substack{\text{complex}\\\text{zero pairs}}} \left(\frac{s^2}{\omega_{0zi}^2} + \frac{2\zeta_{zi}}{\omega_{0zi}} s + 1\right)\right]$$

evaluated @ $s=0$

$$* \left[\prod_{\substack{\text{real}\\\text{poles}}} \frac{1}{(1 - s/\omega_{pk})}\right] \left[\prod_{\substack{\text{complex}\\\text{pole pairs}}} \frac{1}{\left(\frac{s^2}{\omega_{0pi}^2} + \frac{2\zeta_{pi}}{\omega_{0pi}} s + 1\right)}\right]$$

where $G(0) = \frac{a_0}{b_0}$

$\longrightarrow$ (5)

?? n_0 = # of DC zeros − # of DC poles

Usually interested in $\underbrace{10 \log_{10}(|G(j\omega)|^2)}_{\text{"gain in dB"}}$ & $\measuredangle G(j\omega) = \tan^{-1}\left(\frac{\text{Im}\{G(j\omega)\}}{\text{Re}\{G(j\omega)\}}\right)$

Gain (Assume $n_0 = 0$ for now)

$$10 \log |G(j\omega)|^2 = 10 \log |G(0)|^2 \quad (6)$$

$$+ \sum_{\substack{\text{real} \\ \text{zeros}}} 10 \log\left(1 + \omega^2/\omega_{zk}^2\right) \quad (7)$$

$$- \sum_{\substack{\text{real} \\ \text{poles}}} 10 \log\left(1 + \omega^2/\omega_{pk}^2\right) \quad (8)$$

$$+ \sum_{\substack{\text{complex} \\ \text{zero pairs}}} 10 \log\left(\left(1 - \frac{\omega^2}{\omega_{0zi}^2}\right)^2 + 4 \frac{\zeta_{zi}^2}{\omega_{0zi}^2} \omega^2\right) \quad (9)$$

$$- \sum_{\substack{\text{complex} \\ \text{pole pairs}}} 10 \log\left(\left(1 - \frac{\omega^2}{\omega_{0pi}^2}\right)^2 + 4 \frac{\zeta_{pi}^2}{\omega_{0pi}^2} \omega^2\right) \quad (10)$$

Exercise: What if $n_0 \neq 0$?

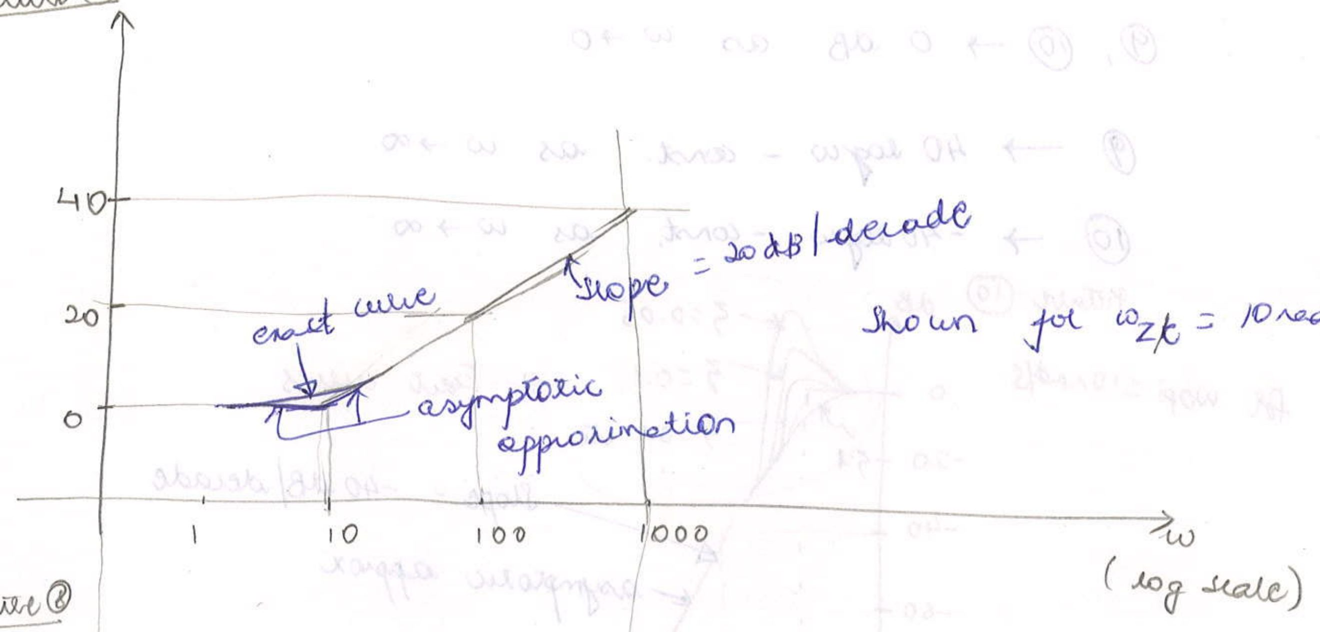
$\Rightarrow$ Only 2 "types" of curves summed together

(7), (8) $\to 0$ as $\omega \to 0$ ($\because \log 1 \to 0$)

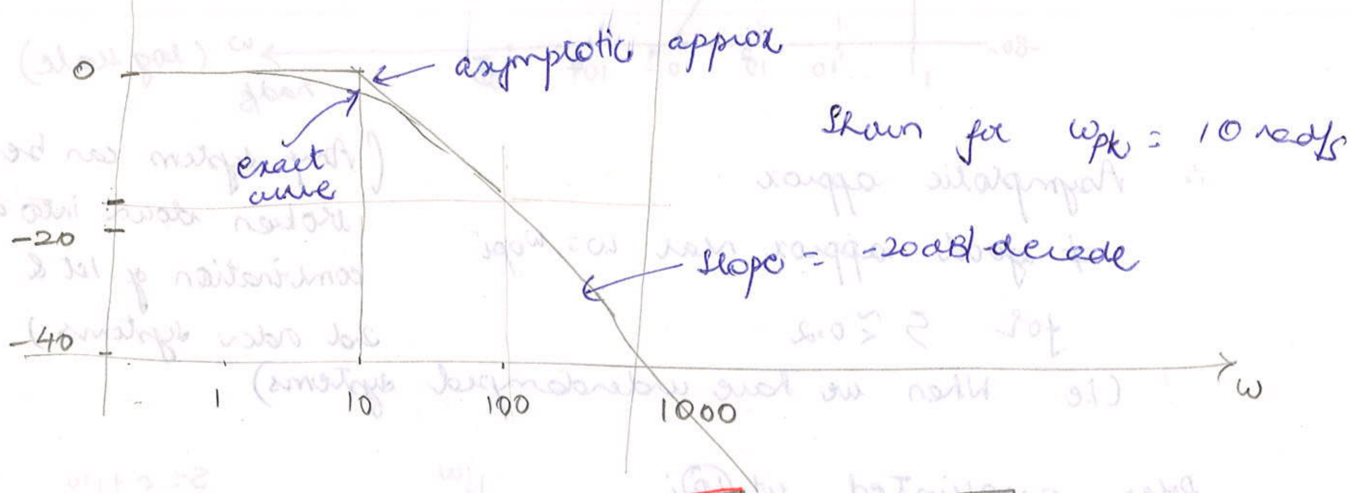
(7) $\to 20 \log \omega + \underbrace{\text{const.}}_{(-20 \log \omega_{zk})}$ (as $\omega \to \infty$)

(8) $\to -20 \log \omega - \underbrace{\text{const.}}_{(20 \log \omega_{pk})}$ (as $\omega \to \infty$)

Picture ⑦



Picture ⑧



REC#2 JAN 10'08

Recall ⑩ : $$-\sum_{\substack{\text{complex}\\\text{pole}\\\text{pairs}}} 10 \log \left[\left(1 - \frac{\omega^2}{\omega_{opi}^2}\right)^2 + 4 \frac{\zeta_{pi}^2}{\omega_{opi}^2} \omega^2 \right]$$

Claim 2: Max departure of ~~real~~ exact curve for real pole & real zeros factors from asymptotic approx. is factor of 0.707 in both ⑦ & ⑧.

Proof: Exercise

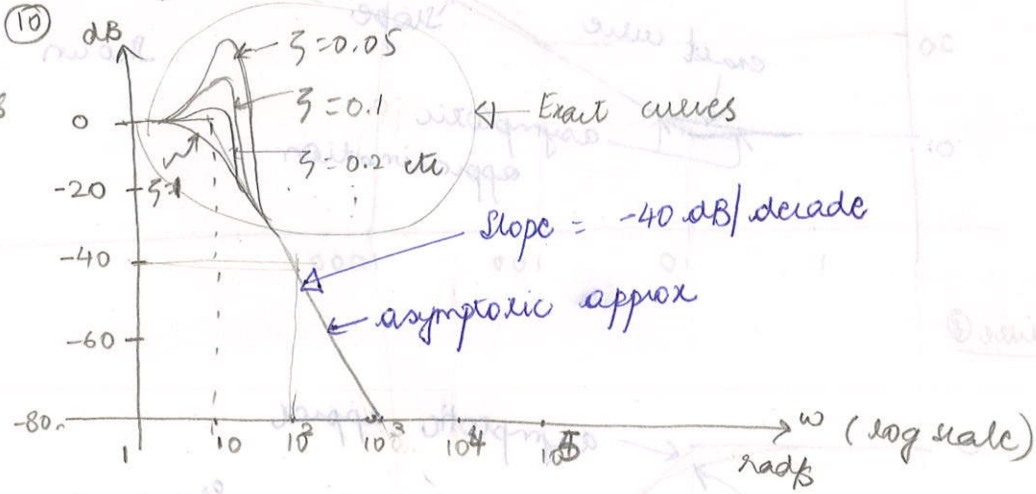
⑨, ⑩ $\rightarrow$ 0 dB as $\omega \rightarrow 0$

⑨ $\longrightarrow$ $40 \log \omega$ − const. as $\omega \rightarrow \infty$

⑩ $\rightarrow$ $-40 \log \omega$ − const. as $\omega \rightarrow \infty$

Picture ⑩

for $\omega_{opi} = 10$ rad/s



∴ Asymptotic approx
$\neq$ good approx near $\omega = \omega_{opi}$
for $\zeta \lesssim 0.2$
(i.e. When we have underdamped systems)

(Any system can be broken down into a combination of 1st & 2nd order systems)

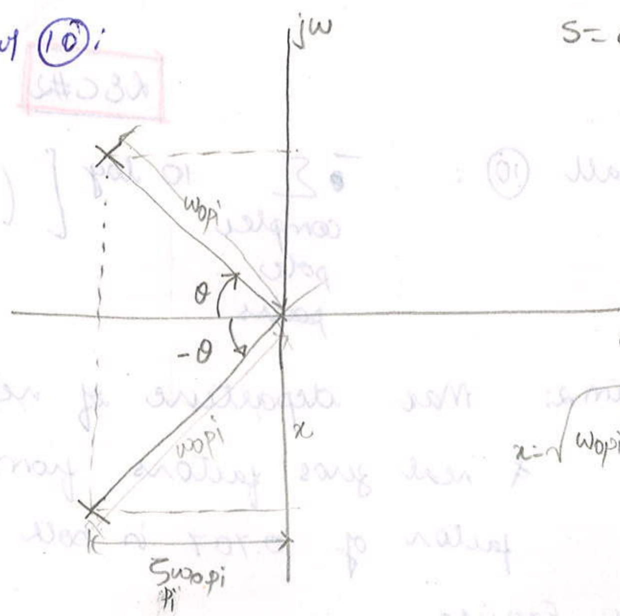
Poles associated w/ ⑩:
(Verify)

$s = \sigma + j\omega$

$$\theta = \tan^{-1}\sqrt{\frac{1}{\zeta_{pi}^2} - 1}$$

$$= \cos^{-1}(\zeta_{pi})$$

$(\zeta_{pi} < 1)$



$x = \sqrt{\omega_{opi}^2 - \zeta_{pi}^2 \omega_{opi}^2}$

$\boxed{\text{Start of new lecture (new \#s for equations)}}$

$$G(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0} \quad ①$$

where $a_k, b_k \in \mathbb{R}$ & $s \in \mathbb{C}$

② Let

$\{w_{zk} : k = 1, 2, \dots N\}$ = non-zero, real zeros of ① if any

$\{w_{pk}, k = 1, 2, \dots N'\}$ = non-zero, real poles of ① if any

$\{s_{zk}, s_{zk}^* : k = 1, \dots N''\}$ = non-real, conjugate zeros of ① if any

$\{s_{pk}, s_{pk}^* : k = 1, \dots N'''\}$ = non-real conjugate poles of ① if any

$w_{0x_k} = |s_{x_k}|$ $(x \equiv z \text{ or } p)$ ($\equiv$ "natural freq" when $x = p$) (doesn't have any physical meaning for the case of zero)

$\zeta_{x_k} = \frac{-Re\{s_{xk}\}}{w_{0x_k}}$ $(x \equiv z \text{ or } p)$ ($\equiv$ "damping ratio" when $x = p$)

Using ②, ① becomes

③
$$G(s) = G(0)\, s^{n_0} \left[\prod_{k=1}^{N} (1 - s/w_{zk})\right] \left[\prod_{k=1}^{N'} \frac{1}{w_{0_{z_k}}^2} (s^2 + 2\zeta_{z_k} w_{0_{z_k}} s + w_{0_{z_k}}^2)\right]$$
$$* \left[\prod_{k=1}^{N''} \left(1 - \frac{s}{w_{pk}}\right)^{-1}\right] \left[\prod_{k=1}^{N'''} w_{0_{pk}}^2 (s^2 + 2\zeta_{pk} w_{0_{pk}} s + w_{0_{pk}}^2)^{-1}\right]$$

where $G(0) = \frac{a_0}{b_0}$, n_0 = (# of zero-freq zeros − # of zero-freq poles)

$$\boxed{G(jw) = \underbrace{|G(jw)|}_{\text{Magnitude}}\, e^{j \underbrace{\angle G(jw)}_{\text{Phase}}}}$$

(Mag. response indicates how I/P P.S.D. is transferred @ O/P)

(Phase response matters when for e.g. we build feedback systems)

Phase response:

Exercise: Verify $\measuredangle G(j\omega) = \sum \measuredangle \text{(each factor in ③)}\Big|_{s=j\omega}$

factor in ③		$\measuredangle$ factor in ③$\Big\|_{s=j\omega}$
$G(0) = \dfrac{a_0}{b_0}$	$\longrightarrow$	$\begin{cases} 0 & \text{if } G(0) > 0 \\ \pi & \text{if } G(0) < 0 \end{cases}$
s^{n_0}	$\longrightarrow$	$\dfrac{\pi}{2} n_0$
$\left(1 - \dfrac{s}{\omega_{xk}}\right)^{\pm 1}$	$\longrightarrow$	$\mp \tan^{-1}\left(\dfrac{\omega}{\omega_{xk}}\right)$ $x = p$ or z
$\left[\dfrac{1}{\omega_{0x_k}^2}\left(s^2 + 2\zeta_{x_k}\omega_{0x_k} s + \omega_{0x_k}^2\right)\right]^{\pm 1}$	$\longrightarrow$	$\pm \tan^{-1}\left(\dfrac{2\zeta_{xk}\omega_{0xk}\omega}{\omega_{0xk}^2 - \omega^2}\right)$ $x = p$ or z

$(\alpha + j\omega_0)^n$

$\dfrac{j\omega}{\omega_k}$

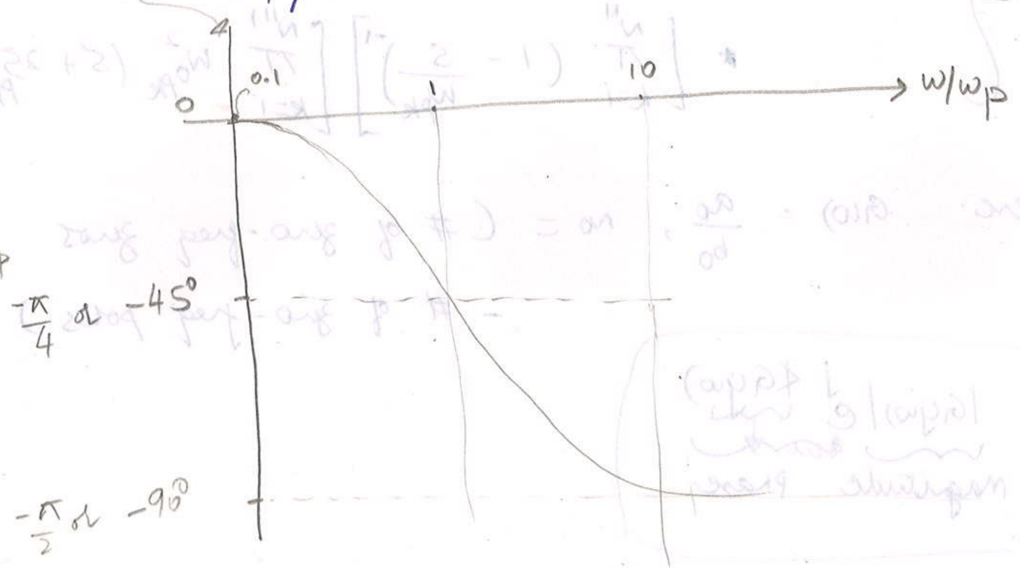
Picture (For LHP poles, only sign changes for LHP zeros, same for RHP zeros)

Is this $\measuredangle\left(\cancel{\dfrac{1}{1+j\omega/\omega_p}}\right)$ $\measuredangle\left(\dfrac{1}{1 - j\omega/\omega_p}\right)$, $\omega_p < 0$ $\dfrac{1}{1 + j\frac{\omega}{2}}$

$\dfrac{1}{\omega_p - j\omega}$

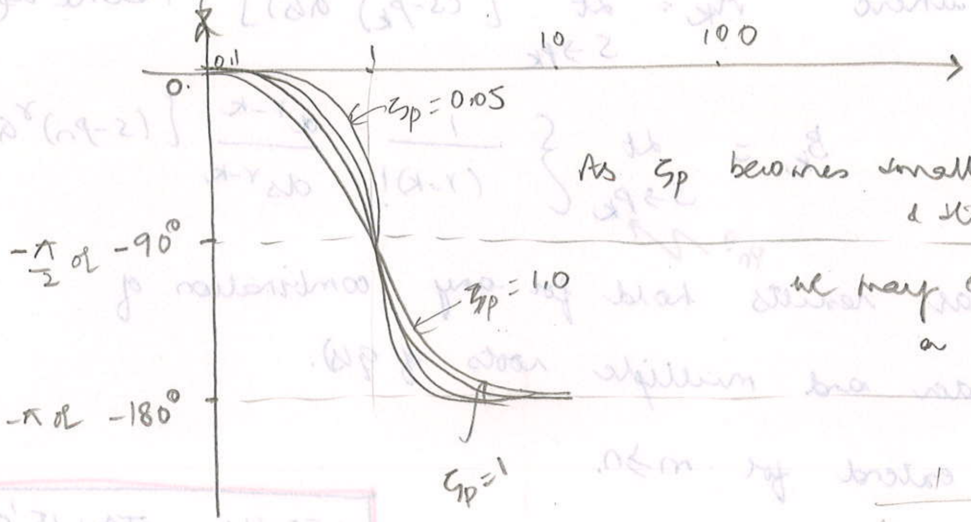
$\dfrac{d}{d\omega}$

$\tan^{-1} \omega/\omega_p$



$$K\left(\frac{w_{op}^2}{s^2+2w_{op}\zeta_p s+w_{op}^2}\right)\Bigg|_{s=j\omega}$$

$Re\{S_p\} < 0$
($\Rightarrow$ In the LHP.)



As ζ_p becomes small, we get steeper & steeper curve & we may end up getting a step function

Poles, Zeros & time response

Let $p(s) = a_m s^m + a_{m-1}s^{m-1} + \ldots + a_0$

$q(s) = b_n s^n + b_{n-1}s^{n-1} + \ldots + b_0$

$$\Rightarrow G(s) = \frac{p(s)}{q(s)} = \frac{p(s)}{b_1(s-p_1)(s-p_2)\ldots(s-p_n)}$$

Recall "Partial Fraction Expansion" (PFE)

Ex1 $m < n$ (i.e. more poles than zeros) $\underbrace{p_i \neq p_j \text{ for } i \neq j}$

$\Leftrightarrow$ Roots of $q(s)$ are all first order roots

Then $G(s) = \sum_{k=1}^{n} \frac{A_k}{(s-p_k)}$ where $A_k \in \mathbb{R}$

where $\underbrace{A_k = \lim_{s\to p_k}\left[(s-p_k)G(s)\right]}_{4}$

Ex2 $m<n$, $p_1, p_2, \ldots p_{n'}$ = 1st order roots of $q(s)$
where $n' \leq n-2$

and $\underbrace{p_{n'+1} = p_{n'+2} = \ldots = p_n}$
r "multiple roots" of $q(s)$ where $r = n-n'$

Then $$G(s) = \sum_{k=1}^{n'} \left(\frac{A_k}{s-p_k}\right) + \sum_{k=1}^{r} \frac{B_k}{(s-p_n)^k}$$

where $A_k = \lim_{s \to p_k} \left[(s-p_k)\, G(s) \right]$ [(same as ④)]

$$B_k = \lim_{s \to p_k} \left\{ \frac{1}{(r-k)!} \frac{d^{r-k}}{ds^{r-k}} \left[(s-p_n)^r G(s) \right] \right\} \quad ⑤$$

p_n?

Similar results hold for any combination of 1st-order and multiple roots of $q(s)$.

Can extend for $m \geq n$.

$\boxed{\text{LEC \#3 \quad JAN 15 '08}}$

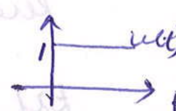
<u>Continuing from last time</u> (i.e. retaining sequence of #s for eqs.)

Let $g(t) = \mathcal{L}^{-1}\{G(s)\}$

PFE $\Rightarrow$ $g(t)$ = Weighted sum of terms of types:

$$\left\{ \begin{array}{l} \mathcal{L}^{-1}\left\{\frac{1}{s-p_k}\right\} = e^{p_k t} u(t) \quad ⑥ \\ \mathcal{L}^{-1}\left\{\frac{1}{(s-p_k)^r}\right\} = \frac{t^{r-1}}{(r-1)!} e^{p_k t} u(t) \quad ⑦ \end{array} \right.$$

step fn.



Note: $⑦ = \underbrace{⑥ * ⑥ * ⑥ \cdots * ⑥}_{\text{Convolution, } r \text{ times}}$

(Multiplication in freq. domain = convolution in time domain)

In circuits, rarely have multiple roots of $Q(s)$

(i.e. Rarely have multiple equal poles)

$\Rightarrow$ Only concerned w/ ⑥ usually

[Very often in DSP correlation of analog ckts., we can have multiple identical terms]

For real non-zero poles,

$$⑥ \quad \alpha \quad \mathcal{L}^{-1}\left\{\frac{1}{1-s/\omega_{pk}}\right\}$$ (ω_{pk} is real pole from last time)

For non-real poles, can group pairs for which ~~terms~~
⑥ results in terms of form

$$\mathcal{L}^{-1}\left\{\frac{\omega_{opk}^2}{s^2+s(2\zeta_{pk}\,\omega_{opk})+\omega_{opk}^2}\right\}$$

(Impulse response difficult to stimulate in real world or in lab)

<u>Step Response</u> (is more convenient)

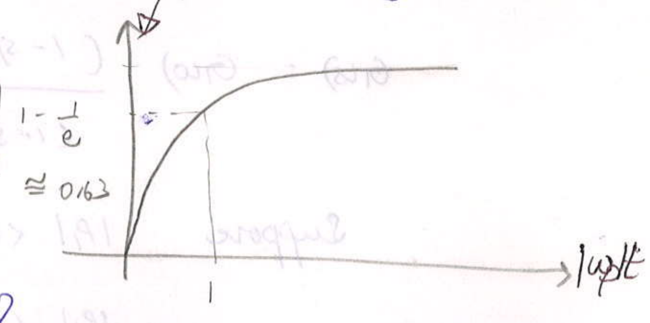
$$\mathcal{L}\{u(t)\} = \frac{1}{s} \qquad \left[\int_{-\infty}^{\infty} e^{-st} f(t)\,dt = \int_{0}^{\infty} e^{-st}\,dt = \frac{e^{-st}}{-s}\Big|_0^\infty = \frac{1}{s}\right]$$

Response of "output" w/ "input" = $u(t)$ ≡ "step response"

$$\therefore g_{step}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}(G(s))\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(1-s/\omega_p)}\right\} = u(t)\left[1-e^{\omega_p t}\right]$$

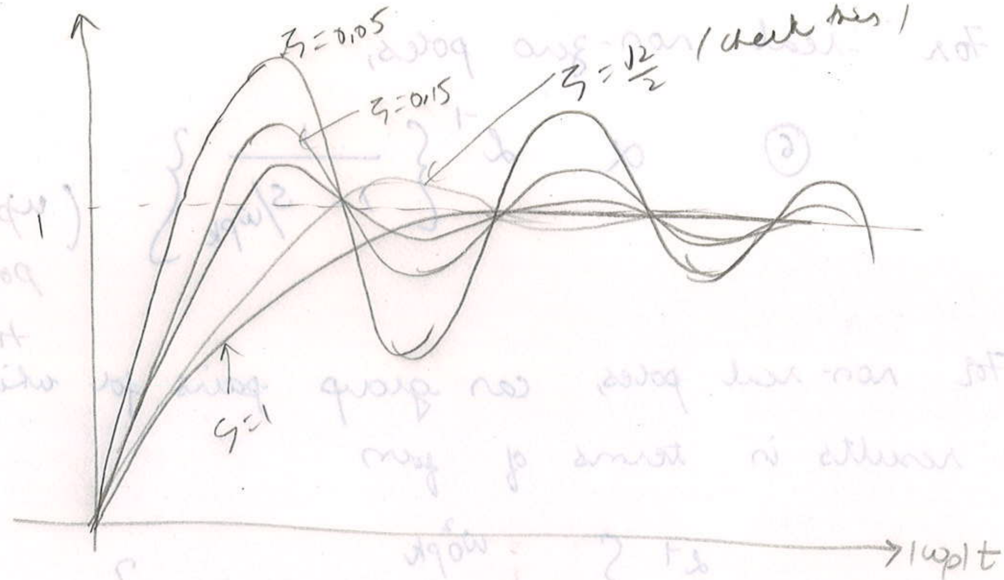
[ω_p is -ve for real systems]



$$\mathcal{L}^{-1}\left\{\frac{1}{s}\;\frac{\omega_{op}^2}{s^2+s(2\zeta_p\omega_{op})+\omega_{op}^2}\right\}$$

$$= u(t)\left\{1 - \frac{1}{\sqrt{1-\zeta_p^2}}\, e^{-\zeta_p \omega_{op} t} \sin\left(\omega_{op}\sqrt{1-\zeta_p^2} + \theta\right)\right\}$$

where $\theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta_p^2}}{\zeta_p}\right)$



For $\zeta_p = 0$, poles on imaginary axis $\Rightarrow$ oscillation

For $\zeta_p < 0$, poles in RHP $\Rightarrow$ oscillation w/ exponentially increasing amplitude

$$s = \frac{-2\zeta_p \omega_{op} \pm \sqrt{4\zeta_p^2 \omega_{op}^2 - 4\omega_{op}^2}}{2}$$

For $0 < \zeta_p < 1$, "overshoots" but settles to 1

$$-\zeta_p \omega_{op} \pm \omega_{op}\sqrt{\zeta_p^2 - 1}$$

(Ringing is bad.
Process instabilities may lead to oscillation)

$$-\zeta_p \omega_{op} \pm j\omega_{op}\sqrt{1 - \zeta_p^2}$$

<u>Dominant Pole Approximation</u>

Provided have no DC poles or zeros

$$G(s) = G(0) \frac{(1 - s/z_1)(1 - s/z_2) \cdots (1 - s/z_m)}{(1 - s/p_1)(1 - s/p_2) \cdots (1 - s/p_n)}$$

Suppose $|p_1| << |p_k|$, $2 \le k \le n$ $\Rightarrow$ ($p_1 \in \mathbb{R}$,

$|p_1| << |z_k|$, $1 \le k \le m$; Complex ~~pair~~ poles come in pairs)

Thus $$|G(j\omega)| \simeq \frac{G(0)}{\sqrt{1 + \frac{\omega^2}{|p_1|^2}}}$$

$\therefore \quad |G(j\omega_{-3dB})| = \frac{G(0)}{\sqrt{2}}$

$\Rightarrow \omega_{-3dB} = |P_1|$ ↳ Called "3 dB Bandwidth"

$\boxed{\text{Reset } \# \text{ System}}$

Common Source Amplifier's Freq. Response:

(Can understand 90% of op-amps w/ C.S.A.)

Aside: Course convention:

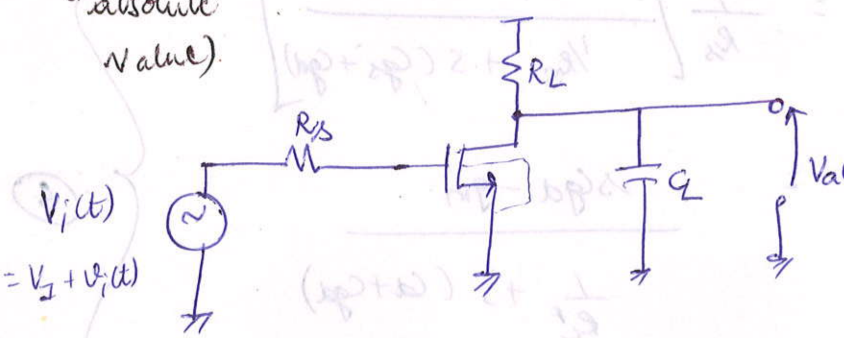
$I_d = I_D + i_d$

$V_{gs} = V_{GS} + v_{gs}$ etc

v_{gs}: time-varying term (this may have DC part too!) (i.e. variations abt. constant bias)

V_{gs}: Total gate-source voltage ("total" or "absolute" value)

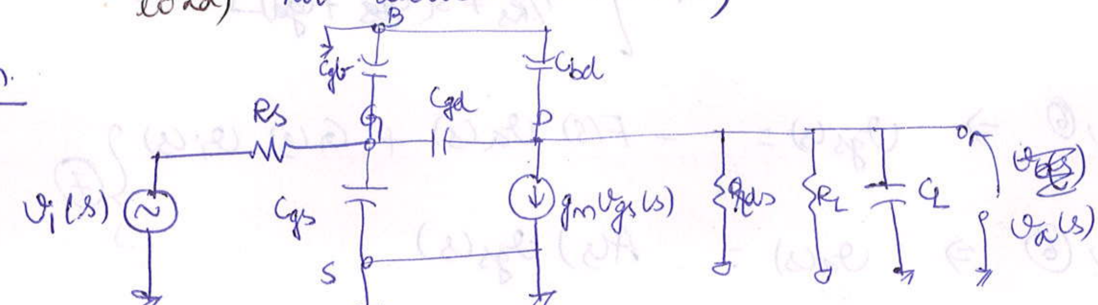
V_{GS}: DC or fixed bias voltage (or constant bias value)



①

(R_S & R_L from active ckts (eg. R_L can be current source load) not actual resistors)

S.S.M.



②

C_{gs} & C_{gb} are in ||.

Let
$$\begin{cases} C_{gs}' = C_{gs} + C_{gb} & (\simeq C_{gs} \text{ usually}) \\ C_d = C_{bd} + C_L \\ R_L' = R_L \,||\, r_{ds} \end{cases}$$

eg. For $I_D = 200\mu A$, $\frac{W}{L} = 50$, $r_{ds} = 500k\Omega$

$C_{gs}' = 90fF$, $C_{gd} = 20fF$

$C_d = 80fF$, $g_m = 1.7 \times 10^{-3}\ \Omega^{-1}$ ③

Note: (2) also SSM of diff. pair DM 1/2 ckt,

KCL @ G:

$$\frac{V_i(s) - v_{gs}(s)}{R_s} - v_{gs}(s)\, sC_{gs}' - \left[v_{gs}(s) - v_a(s)\right] sC_{gd} = 0$$

$$\therefore \quad v_{gs}(s) = \left[\frac{1}{\frac{1}{R_s} + s(C_{gs}' + C_{gd})}\right]\left(v_a(s)\, sC_{gd} + v_i(s)\frac{1}{R_s}\right) \quad (4)$$

KCL @ D:

$$\left[v_{gs}(s) - v_a(s)\right] sC_{gd} - g_m v_{gs}(s) - v_a(s)\left(\frac{1}{R_L'} + sC_L\right) = 0$$

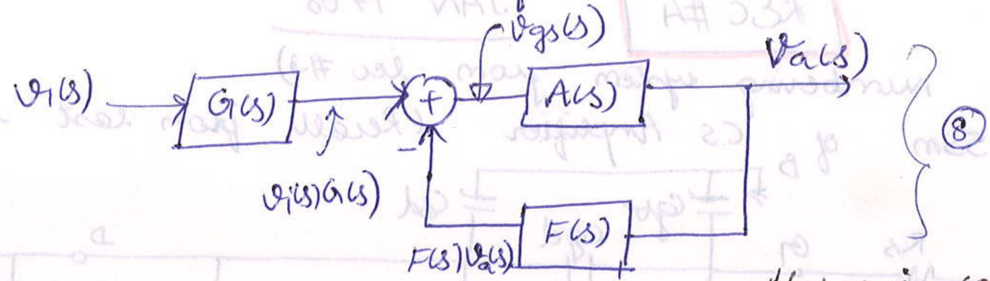
$$\therefore \quad v_a(s) = \left[\frac{1}{\frac{1}{R_L'} + s(C_L + C_{gd})}\right]\left(sC_{gd} - g_m\right) v_{gs}(s) \quad (5)$$

Let:

$$\left.\begin{aligned} G(s) &= \frac{1}{R_s}\left[\frac{1}{1/R_s + s(C_{gs}' + C_{gd})}\right] \\ A(s) &= \frac{sC_{gd} - g_m}{\frac{1}{R_L'} + s(C_L + C_{gd})} \\ F(s) &= -sC_{gd}\left[\frac{1}{1/R_s + s(C_{gs}' + C_{gd})}\right] \end{aligned}\right\} (6)$$

$$\left.\begin{aligned} (4),(6) &\Rightarrow v_{gs}(s) = -F(s)\, v_a(s) + G(s)\, v_i(s) \\ (5),(6) &\Rightarrow v_a(s) = A(s)\, v_{gs}(s) \end{aligned}\right\} (7)$$

⑦ ⇒ "Block Diagram" of ②.



- Helps in conceptually understanding ckts.
- eg. Gain Margin & Phase margin tools were inherently developed using block diagrams. (Need to know how those tools were developed in order to know their limitations. eg. −ve P.M. may be stable & +ve PM may be unstable)
- Block Diagrams work well in discrete time too.
- Feedback is clearly seen in block diagrams eg. Cgd which is a feedback + feedforward cap. is difficult to understand just from ckts.

⑧ using Mason's gain Formula (SSM)

(or)

⑦ using algebra

$$\Rightarrow A_v(s) = \frac{G(s) \cdot A(s)}{1 + A(s)F(s)} \quad ⑨$$

Block Diagrams

SSM of ② ⟺ system of equations in 's' (eq. ④ & ⑤)

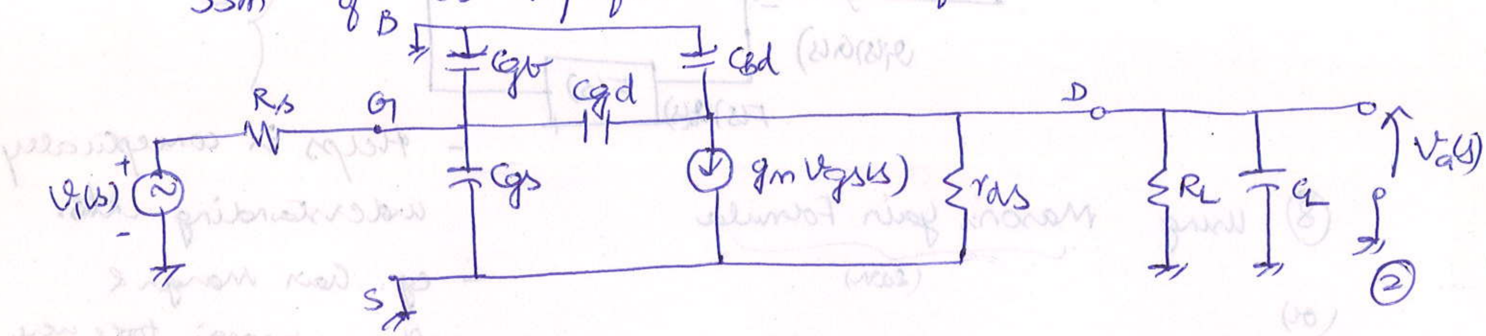
⟺ block diagram (eg. ⑧)

- SSM is symbolic ⇒ good for insight
  - but components are bi-directional ⇒ bad for insight
- Can't go directly from a S.S.M. to gains & impedances. (need system of equations first)
- Block diagrams provide insight because components are unidirectional ⇒ Feedback paths are obvious
- Can go directly from block diagrams to gains & impedances (using Mason's gain formula)

REC #4 JAN 17 08

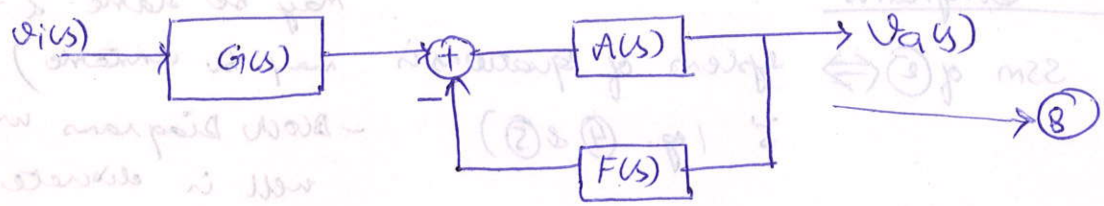
(Still numbering system from lec #3)

SSM of CS Amplifier (Recall from last time)



Last time found,

$$\left.\begin{aligned} v_{gs}(s) &= -F(s)\, v_a(s) + G(s)\, v_i(s) \\ v_a(s) &= A(s)\, v_{gs}(s) \end{aligned}\right\} \quad (7)$$



$$\frac{v_a(s)}{v_i(s)} \equiv A_v(s) = \frac{G(s)\, A(s)}{1 + A(s) F(s)} \longrightarrow (9)$$

Recall: stability (for a causal system, which is usually the case in ckt. design)

$\Leftrightarrow$ no poles in RHP.

(9) $\Rightarrow$ "stability" $\Leftrightarrow$ $A(s_0) F(s_0) \neq -1$ for any s_0 w/ $Re\{s_0\} \geq 0$ → i.e. on imaginary axis or R.H.P.

$s = -2 + j\omega$

(assumes $F(s) \neq 0$ and no poles of $A(s)$ is a zero of $F(s)$)

($A(s) F(s)$ can be -1 for s in L.H.P. but not on imaginary axis or R.H.P)

First consider $A_{v0} = A_v(j\omega)\Big|_{\omega=0}$ "DC gain"

$$A_{v0} = \frac{G(j\omega)\ A(j\omega)}{1 + A(j\omega)F(j\omega)}\Bigg|_{\omega=0}$$

$$F(j\omega)\Big|_{\omega=0} = 0$$

($Z_{Cgd} = 0$ @ loo freq.)

(makes sense because $F(s)$ represents feedback through C_{gd})

$$\Rightarrow \quad A_{v0} = -g_m R_L' \quad \text{— (10)}$$

Now consider poles & zeros: let $H(j\omega) = \dfrac{A_v(j\omega)}{A_{v0}}$

$$\therefore A_v(j\omega) = A_{v0} H(j\omega)$$

(normalised dc gain)

(6) & (9) $\Rightarrow$ $H(s) = \dfrac{1 - s/z_1}{1 + a_1 s + a_2 s^2}$

(where $z_1 = \dfrac{g_m}{C_{gd}}$; $a_1 = R_S\left[C_{gs}' + C_{gd}(1 + g_m R_L')\right] + R_L'(C_{gd} + C_d)$

$$a_2 = R_S R_L'\left[C_d C_{gs}' + C_d C_{gd} + C_{gs}' C_{gd}\right]$$

$$\therefore \quad A_v(s) = A_v(0)\left[\frac{1 - s/z_1}{(1 - s/p_1)(1 - s/p_2)}\right]$$

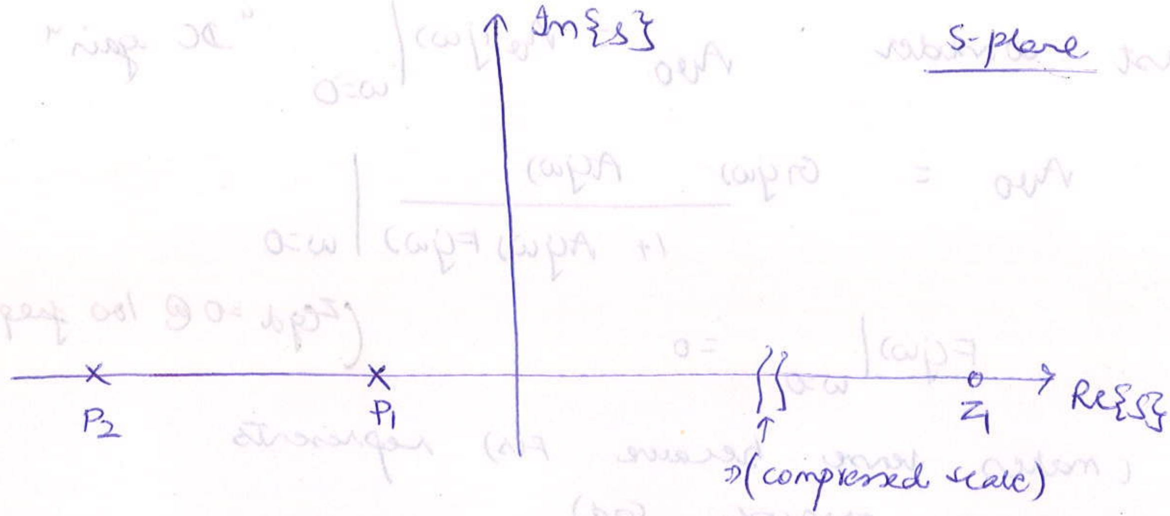
where
$$p_1 = \frac{-a_1 + \sqrt{a_1^2 - 4a_2}}{2a_2}$$
$$p_2 = \frac{-a_1 - \sqrt{a_1^2 - 4a_2}}{2a_2}$$
— (11)

(Always check units)

Usually, device parameters $\Rightarrow a_1^2 > 4a_2$ for C.S. amp.

$\Rightarrow p_1$ & p_2 are real.

(i.e. C.S.A by itself won't generally ring)



eg Using ③ (from last line) w/ $R_L = 10k\Omega$, $R_S = 5k\Omega$,

gives $A_{v0} = -15.5$, $f_{Z1} = \frac{Z_1}{2\pi} = 13.5\,GHz$

$f_{P1} = \left|\frac{P_1}{2\pi}\right| = 56\,MHz$, $f_{P2} = \left|\frac{P_2}{2\pi}\right| = 860\,MHz$.

(Easier to calculate in rad/s, easier to think in cycles/s or Hz)

$$\left.\begin{aligned}|P_1| &<< |P_2| \\ |P_1| &<< |Z_1|\end{aligned}\right\} \Rightarrow P_1 = \text{dominant pole.}$$

$$\Rightarrow |A_v(j\omega)| \simeq A_{v0}\left(\frac{1}{1 + j\omega/(2\pi \times 56M)}\right)$$

valid for $\omega \leq f_{P2} \cdot 2\pi$

$\Rightarrow$ 3dB BW = 56MHz

Now consider ① w/ capacitor C_c connected between D and G.

⑪ $\Rightarrow$ C_c in parallel w/ C_{gd} in ②

$\Rightarrow$ All equations so far hold if C_{gd} is replaced by $C' = C_{gd} + C_c$. ⑫

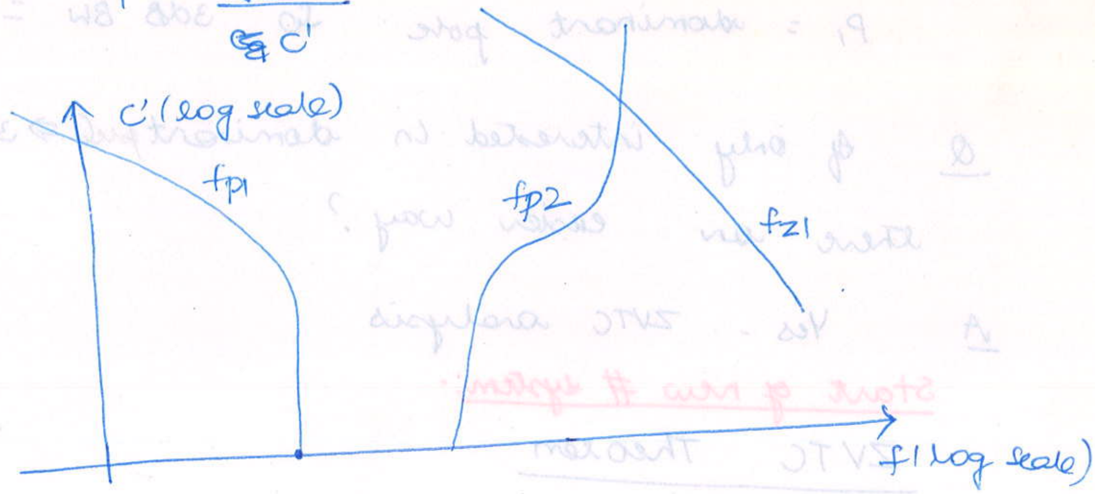
Can show using ⑪ and ⑫ (exercise)

$$P_1 \simeq \frac{-1}{(C_d + C')R_L' + (C_{gs}' + C')R_S + g_m R_S R_L' C'} \quad ⑬$$

$$\simeq \frac{-1}{g_m R_S R_L' C'} \quad \text{for large } R_S \text{ \& } R_L$$

$$P_2 \cong \frac{-g_m C'}{C_d C_{gs}' + C'(C_d + C_{gs}')} \qquad \frac{-g_m}{C_d + C_{gs}'} \text{ if } C' \gg C_d$$

$$Z_1 = \frac{g_m}{C'}$$

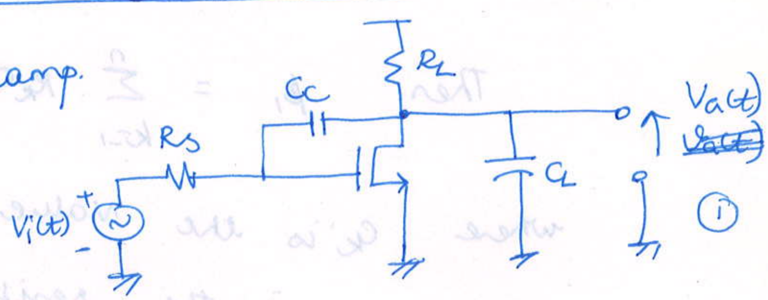


∴ Increasing C' splits poles but moves zero close to origin

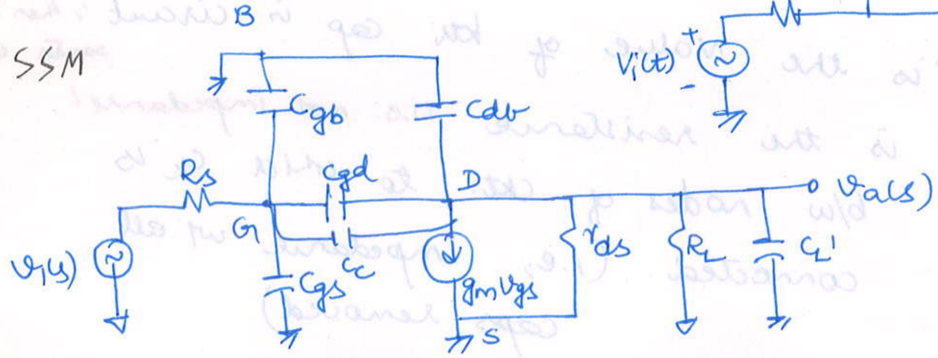
$\boxed{\text{Reset Number System}}$

Zero Value Time Constant Analysis :-

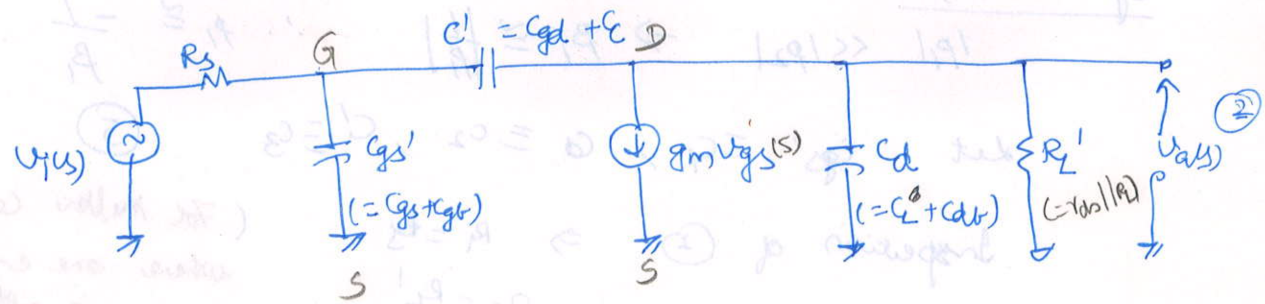
Ex Recall C.S. amp.



SSM



≡



Last time, hard work $\Rightarrow p_1 \simeq \dfrac{-1}{(C_d+C')R_L + (C_{gs}'+C')R_s + g_m R_s R_L' C'}$ ③

p_1 = dominant pole so 3dB BW = $\dfrac{|p_1|}{2\pi}$

Q If only interested in dominant pole ($\Rightarrow$ 3dB BW) is there an easier way?

A Yes - ZVTC analysis

Start of new # system:

ZVTC Theorem

Consider a circuit that contains only sources, resistors and capacitors (i.e. no inductors) and poles $p_1, p_2, \cdots, p_n$ where $p_k \neq 0$

Let $\beta_1 = -\sum_{k=1}^{n} \frac{1}{p_k}$

Then $\beta_1 = \sum_{k=1}^{n} R_k C_k$ ④ (no -ve sign here ∵ poles are -ve)

even imaginary poles are +ve in circuit ∵ then imaginary parts cancel

where C_k is the value of kth cap

R_k is the resistance (i.e. not impedance) b/w nodes of ckt. to which C_k is connected (i.e., impedance w/ all caps removed)

eg (contd.)

$|p_1| \ll |p_2| \Rightarrow \beta_1 \cong \left|\frac{1}{p_1}\right| \quad \therefore p_1 \cong \frac{-1}{\beta_1}$

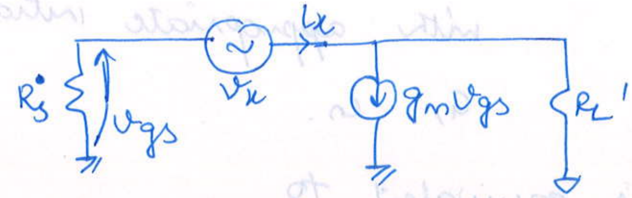
Let $C_{gs}' \equiv C_1$, $C_d \equiv C_2$, $C' \equiv C_3$ ⑤

Inspection of ② $\Rightarrow R_1 = R_s$

$R_2 = R_L'$

(For Miller cap, where one end of cap is not grounded then need to modify ~~theorem~~ terms)

To find R_3, use:



$\Rightarrow \quad v_{gs} = -i_x R_S$

$v_x = (i_x - g_m v_{gs}) R_L' - v_{gs}$

$\Rightarrow \quad R_3' = R_L' + R_S + g_m R_L' R_S$

(4) $\Rightarrow \quad p_1 \cong \dfrac{-1}{R_s c_{gs}' + R_L' C_d + \underbrace{(R_L' + R_S + g_m R_L' R_S)}\, C'}$

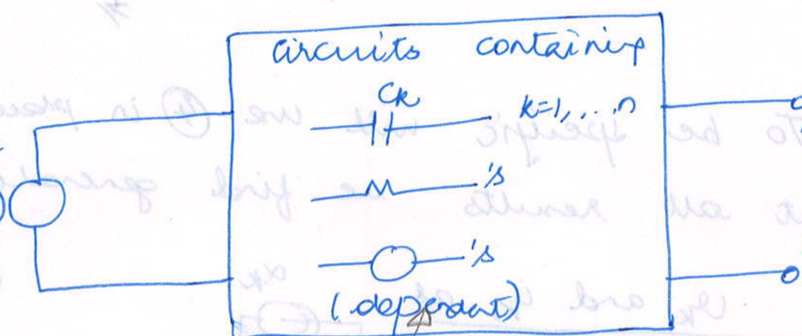
$=$ (3) from last time.

$\boxed{\text{LEC \#5}}$ $\boxed{\text{JAN 22 '08}}$

ZVTC Theorem (contd.) (same # system)

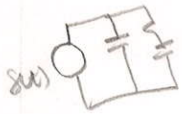
PROOF:-

Have:
Impulse:
$v_i(t) = \delta(t)$
$(v_i(s) = 1)$



Impulse Response
$v_a(t) = g(t)$
$(v_a(s) = G(s))$
(2)

∵ For calculating Rs & Cs we need to turn off independent sources

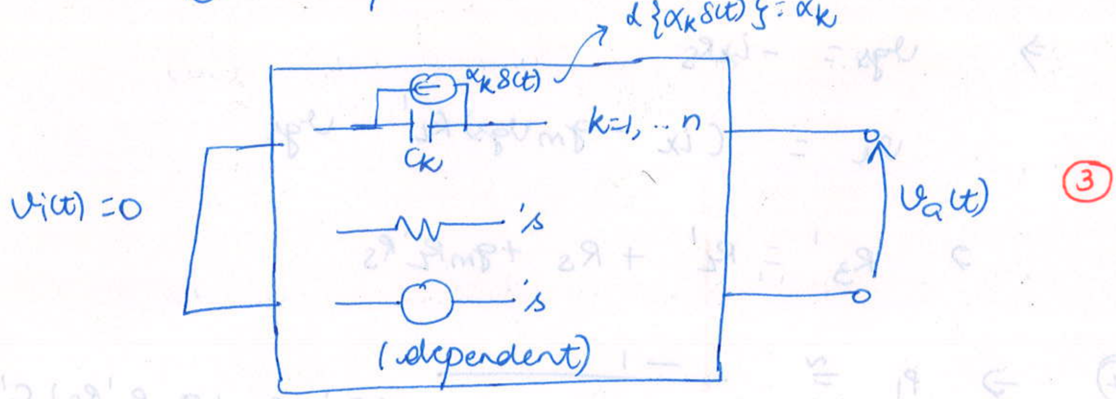


where $G(s) = A_0 \dfrac{P(s)}{(1-s/p_1)(1-s/p_2)\dots(1-s/p_n)}$ ← some polynomial

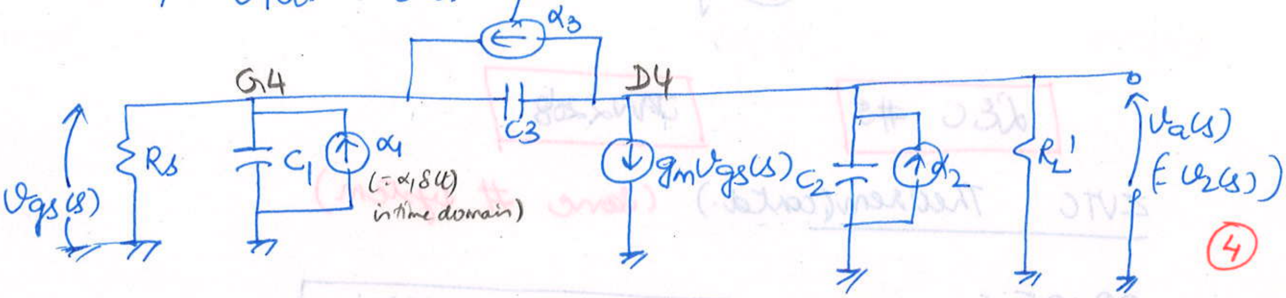
Note: In (2), $v_i(t) = \delta(t)$ sets initial condition of each capacitor C_k $k=1..n$. After $t=0$, $v_i(t)=0$
(Atleast one cap will get charged)

⇒ $g(t)$ = transient of ② w/ $v_i(t) = 0$ but with appropriate initial conditions on $C_1, \dots C_n$.

∴ ② is equivalent to

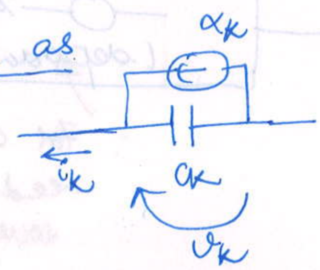


e.g. S.S.M. of C.S. amplifier (last time) w/ $v_i(t) = \delta(t)$ produces same $v_a(t)$ as



NOTE! To be specific will use ④ in place of ③ for today. But all results we find generalize to ③.

Define v_k and i_k as



∴ $i_k(s) = \alpha_k - v_k(s)\, sC_k$ — ⑤

KCL @ any node ⇒ equation of the form:

$$d_1 i_1(s) + d_2 i_2(s) + d_3 i_3(s) + g_1 v_1(s) + g_2 v_2(s) + g_3 v_3(s) = 0$$

where $d_k = 1, 0,$ or -1 (corresponding to current entering, not entering/leaving & leaving a node) and g_k = some conductance

e.g. KCL @ gate of (4):

$$-i_1(s) - i_3(s) + \frac{1}{R_S} v_i(s) = 0$$ where is $v_i(s)$? have only $v_{gs}(s)$

Can solve all the equations of this form to get:

$$\left.\begin{aligned} i_1(s) &= a_{11} v_1(s) + a_{12} v_2(s) + a_{13} v_3(s) \\ i_2(s) &= a_{21} v_1(s) + a_{22} v_2(s) + a_{23} v_3(s) \\ i_3(s) &= a_{31} v_1(s) + a_{32} v_2(s) + a_{33} v_3(s) \end{aligned}\right\}$$ (6) where a_{jk} = some conductance

Note: To force (6) to be linearly independent, may need to add tiny resistors in series w/ some of the caps (add conceptually, not physically)

("tiny" $\Leftrightarrow$ small enough to have negligible effect on ckt.)

In matrix notation, (6) is $\underline{i}(s) = \underline{A}\, \underline{v}(s)$

$$\underline{i}(s) = \underbrace{\begin{bmatrix} i_1(s) \\ i_2(s) \\ i_3(s) \end{bmatrix}}_{3\times1}, \quad \underline{A} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{3\times3} \quad \& \quad \underline{v}(s) = \underbrace{\begin{bmatrix} v_1(s) \\ v_2(s) \\ v_3(s) \end{bmatrix}}_{3\times1}$$

Using (5),:

$$\underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}}_{\text{Call } \underline{\alpha}} = \underbrace{\begin{bmatrix} (a_{11}+sC_1) & a_{12} & a_{13} \\ a_{21} & (a_{22}+sC_2) & a_{23} \\ a_{31} & a_{32} & (a_{33}+sC_3) \end{bmatrix}}_{\text{Call } \underline{B}} \underline{v}(s) \qquad (7)$$

(Real life: CSA, 3 caps, 2 poles)

Recall "Cramer's Rule"

$$\Rightarrow v_k(s) = \frac{\Delta_k(s)}{\Delta(s)}$$ where $\Delta(s) = \det(\underline{B})$ (= polynomial in s)

$\Delta_k(s) = \det(\underline{B}$ with kth column replaced by $\underline{\alpha})$

Note: $v_a(s) = v_2(s)$ in (4), so $G(s) = \underbrace{\dfrac{\Delta_2(s)}{\Delta(s)}}_{8}$

(i.e. we can calculate impulse response from cramer's rule)

We know denominator of $G(s)$ has form : $b_0 + b_1 s + b_2 s^2$

$= b_0 (1 + \beta_1 s + \beta_2 s^2)$

(Because of pole-zero cancellation, we have 2nd order polynomial in real life rather than 3rd order polynomial)

$= b_0 (1 - s/p_1)(1 - s/p_2)$

Algebra $\Rightarrow \underbrace{\beta_1 = -\sum_{k=1}^{n} 1/p_k,}_{\text{true for any } n \geq 1} \quad n = 2$

$$\left.\begin{aligned} &⑦, ⑧, \text{ defn. of } \det(B) \Rightarrow b_0 = \Delta(s)\Big|_{c_1=c_2=c_3=0} = \Delta(0) \\ &\& \quad b_0\beta_1 s \equiv b_1 s = h_1 s c_1 + h_2 s c_2 + h_3 s c_3 \end{aligned}\right\} 9$$

where h_1, h_2, h_3 are real #s, i.e. $\in \mathbb{R}$

Let $\Delta_{ij} = (-1)^{i+j} \det(\underline{B}_{ij})$ where $\underline{B}_{ij} = \underline{B}$ with ith row & jth column deleted.

Can expand $\det(\underline{B})$ as

$\Delta(s) = \underbrace{(a_{11} + sc_1)\Delta_{11}} + a_{21}\Delta_{21} + a_{31}\Delta_{31}$

inspection of ⑦ $\Rightarrow c_1$ only occurs in 1st term

$\therefore h_1 = \Delta_{11}\Big|_{c_2 = c_3 = 0} = \Delta_1'(0)$

Similarly, $\boxed{h_2 = \Delta_{22}(0) \quad , h_3 = \Delta_{33}(0)}$

$\therefore$ (9) $\Rightarrow \beta_1 = \frac{\Delta_{11}(0)}{\Delta(0)} \cdot C_1 + \frac{\Delta_{22}(0)}{\Delta(0)} \cdot C_2 + \frac{\Delta_{33}(0)}{\Delta(0)} C_3$

Now set $C_1 = C_2 = C_3 = 0$

& $i_2 = i_3 = 0$ to get

$$\begin{bmatrix} i_1 \\ 0 \\ 0 \end{bmatrix} = \underline{A} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Cramer's Rule $\Rightarrow$ $$v_1 = \frac{i_1 \det \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}}{\det(\underline{A})}$$

$$= i_1 \frac{\Delta_{11}(0)}{\Delta(0)} \quad \text{by defn.}$$

$$\therefore \left.\frac{v_1}{i_1}\right|_{\substack{C_1 = C_2 = C_3 = 0 \\ i_2 = i_3 = 0}} = R_1 = \frac{\Delta_{11}(0)}{\Delta(0)}$$

Similarly for R_2 and R_3

$\therefore \beta_1 = \boxed{R_1 C_1} + R_2 C_2 + R_3 C_3$

1) One RC should dominate

2) One pole should dominate.

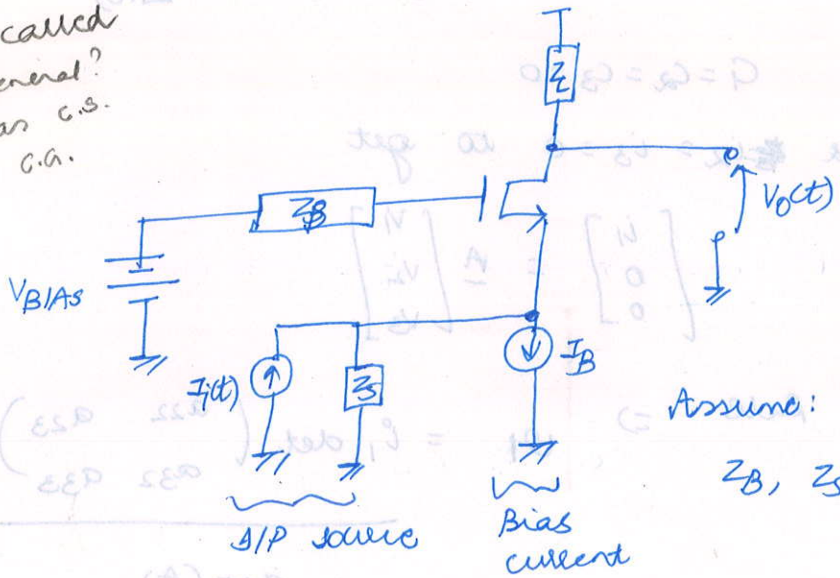
$\beta_1 \simeq R_1 C_1$

β_1

LEC #6 JAN 24 '08

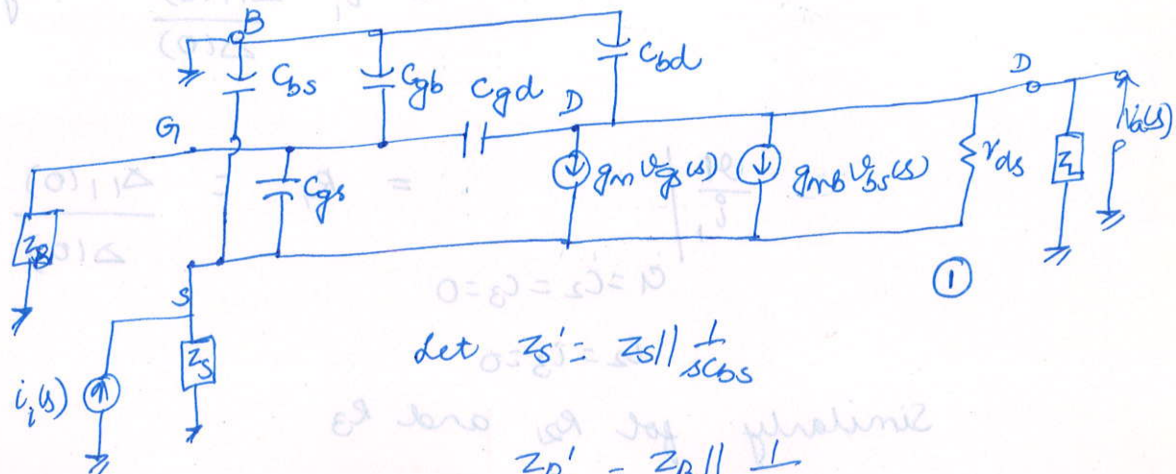
Freq. Response of Common Gate (Cascode) stage

Is C.G. called cascode in general? Thought it was C.S. followed by C.G.



Assume: $Z_B, Z_S, Z_L \equiv$

SSM:



Let $Z_S' = Z_S \| \frac{1}{sC_{bs}}$

$$Z_B' = Z_B \| \frac{1}{sC_{gb}}$$

$$Z_L' = Z_L \| \frac{1}{sC_{db}}$$

KCL @ G:

$$\frac{v_g}{Z_B'} + (v_g - v_d)\, sC_{gd} + (v_g - v_s)\, sC_{gs} = 0$$

$$\Rightarrow v_g = \underbrace{\left[\frac{1}{Z_B'} + s(C_{gd} + C_{gs})\right]^{-1}}_{\text{Call } G(s)} \left[sC_{gs}v_s + sC_{gd}v_d\right] \quad \text{— } 2$$

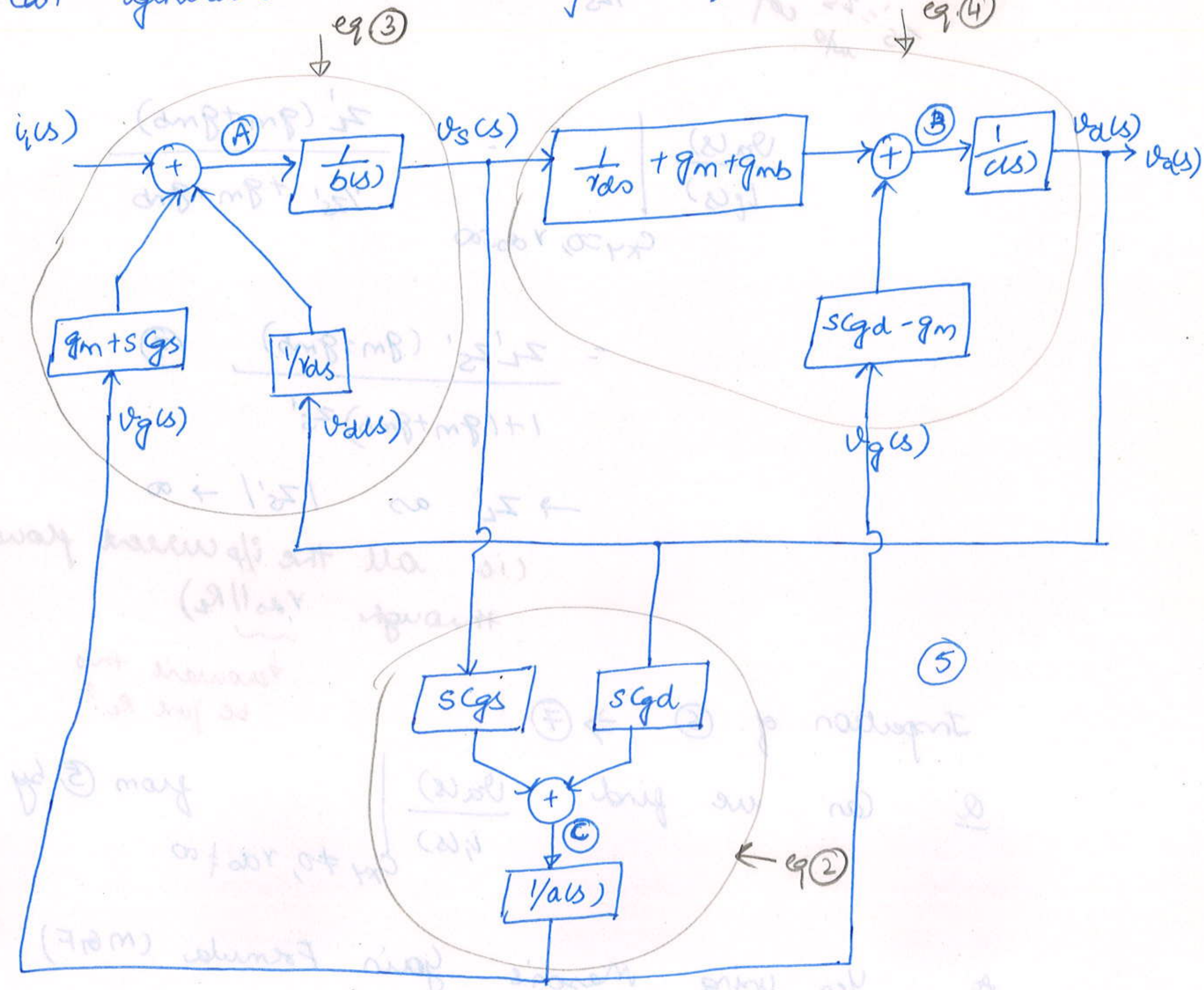
KCL @ S:

$$v_S = \underbrace{\left[\frac{1}{Z_S'} + g_m + g_{mb} + \frac{1}{r_{ds}} + sC_{gs}\right]}_{\text{Call } B(s)}{}^{-1}\left[i_i + (g_m + sC_{gs})v_g + \frac{1}{r_{ds}}v_d\right] \rightarrow ③$$

KCL @ D:

$$v_d = \underbrace{\left[\frac{1}{Z_L'} + \frac{1}{r_{ds}} + sC_{gd}\right]}_{\text{Call } C(s)}{}^{-1}\left[\left(\frac{1}{r_{ds}} + g_m + g_{mb}\right)v_s + (sC_{gd} - g_m)v_g\right] \rightarrow ④$$

Can generate "block diagram" from ②-④:



Observations:-

1) All blocks are unidirectional (in direction of arrows)

2) All feedback paths are from parasitic elements

(i.e., if $C_{XY}=0$ & $r_{ds}=\infty$, then no feedback loops)

With $C_{XY}=0$, $r_{ds}=\infty$, (5) reduces to

$$i_i(s) \longrightarrow \underbrace{\boxed{\frac{1}{b(s)}}}_{\equiv \frac{1}{1/Z_S' + g_m + g_{mb}}} \longrightarrow \boxed{g_m + g_{mb}} \longrightarrow \underbrace{\boxed{\frac{1}{c(s)}}}_{\equiv Z_L'} \longrightarrow v_a(s) \qquad (6)$$

$Z_S' = Z_S$ w/o caps

$$\left.\frac{v_a(s)}{i_i(s)}\right|_{C_{XY}=0,\ r_{ds}=\infty} = \frac{Z_L'(g_m+g_{mb})}{1/Z_S' + g_m + g_{mb}}$$

$$= \frac{Z_L' Z_S' (g_m+g_{mb})}{1+(g_m+g_{mb})Z_S'} \qquad (7)$$

$\rightarrow Z_L'$ as $|Z_S'| \rightarrow \infty$

(i.e. all the i/p current flows through $\underbrace{r_{ds} \| R_L}$)

shouldn't this be just R_L?

Inspection of (6) $\Rightarrow$ (7)

Q Can we find $\left.\frac{v_a(s)}{i_i(s)}\right|_{C_{XY}\neq 0,\ r_{ds}\neq\infty}$ from (5) by inspection

A Yes, using Mason's Gain Formula (MGF)

(It applies to both digital & analog systems)

Application of MGF to (5):

(7.B) "Loops": 1: A → B → A

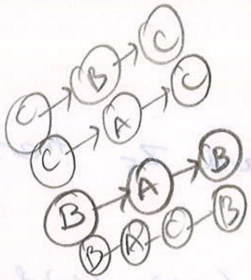
2: A → C → A

3: A → C → B → A

4: A → B → C → A

5: B → C → B

Note: All loops touch each other (i.e. they share a common node)



"Forward Paths": 1: A → B → v_d

2: i_i → A → C → B → v_d

Note: deleting either path breaks all loops.

"Loop gains":

$$L_1 = \left(\frac{1}{r_{ds}} + g_m + g_{mb}\right)\frac{1}{r_{ds}\, b(s)\, c(s)}$$

$$L_2 = sC_{gs}\,(g_m + sC_{gs})\,\frac{1}{a(s)\, b(s)}$$

$$L_3 = sC_{gs}\,(sC_{gd} - g_m)\,\frac{1}{r_{ds}\, a(s)\, b(s)\, c(s)}$$

$$L_4 = sC_{gd}\,(g_m + sC_{gs})\left(\frac{1}{r_{ds}} + g_m + g_{mb}\right)\frac{1}{a(s)\, b(s)\, c(s)}$$

$$L_5 = sC_{gd}\,(sC_{gd} - g_m)\,\frac{1}{a(s)\, c(s)}$$

(8)

"Path gains"

$$P_1 = \left(\frac{1}{r_{ds}} + g_m + g_{mb}\right)\frac{1}{b(s)\, c(s)}$$

$$P_2 = sC_{gs}\,(sC_{gd} - g_m)\,\frac{1}{a(s)\, b(s)\, c(s)}$$

(9)

Fact : MGF (this case) $\Rightarrow \dfrac{v_a(s)}{i_i(s)} = \dfrac{P_1 + P_2}{1 - L_1 - L_2 - L_3 - L_4 - L_5}$ (10)

$\therefore$ Can easily find $G(s) = \dfrac{v_a(s)}{i_i(s)}$ but it's really messy to look at.

For negligible capacitance in Z_L, Z_{13} and Z_5 then

$G(s) \cong \dfrac{\text{second-order poly}}{\text{Third-order poly}}$ ($\because a(s)$, $b(s)$ & $c(s)$ are first order polynomials)

$\Downarrow$

(Call Full version of $G(s)$ (11)) have 3rd-order system w/ 2 zeros & 3 poles.

The good news : (11) = "exact" expression

The bad news : "Exact" expression is too complicated to give insight.

Q So what now?

A Plug #s (i.e., frequencies and component values) into boxes in (5) and eliminate highly attenuated paths

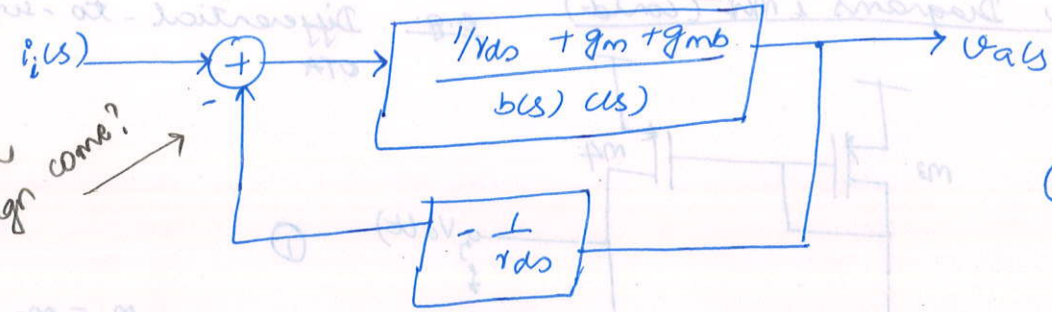
eg Z_B usually arises from parasitics

- often $Z_B \neq 0$ causes stability problems in feedback applications

$V_{BIAS} = DC$, so can use bypass cap to reduce $|Z_B|$

$\therefore$ Suppose $|Z_B| \cong 0$. Then $\left|\dfrac{1}{a(s)}\right| \cong 0$

and (5) becomes



How did the -ve sign come?

(12')

Now, MGF gives

$$G(s)\Big|_{Z_B=0} = \frac{\dfrac{1/r_{ds} + g_m + g_{mb}}{b(s)\, c(s)}}{\left[1 - \dfrac{1}{r_{ds}}\left(\dfrac{1/r_{ds} + g_m + g_{mb}}{b(s)\, c(s)}\right)\right]}$$

Let $g_m' = g_m + g_{mb}$ ($\cong 1.2 g_m$), assume $g_m \gg \frac{1}{r_{ds}}$

Then

$$b(s) = \frac{1}{Z_s'} + g_m' + sC_{gs}$$

$$c(s) = \frac{1}{Z_L'} + \frac{1}{r_{ds}} + sC_{gd}$$

$$\therefore G(s)\Big|_{Z_B=0} = \frac{g_m'}{\left(\frac{1}{Z_s'} + g_m' + sC_{gs}\right)\left(\frac{1}{Z_L'} + \frac{1}{r_{ds}} + sC_{gd}\right) - g_m'/r_{ds}} \quad (13)$$

Sanity checks: 1) Units = $\frac{V}{I}$

2) $A(j0)\Big|_{r_{ds}, Z_s = \infty} = Z_L'$

(13) $\Rightarrow$ 2nd order behavior, 2 poles, no zeros

$\Rightarrow$ always stable, denominator of (13) has form

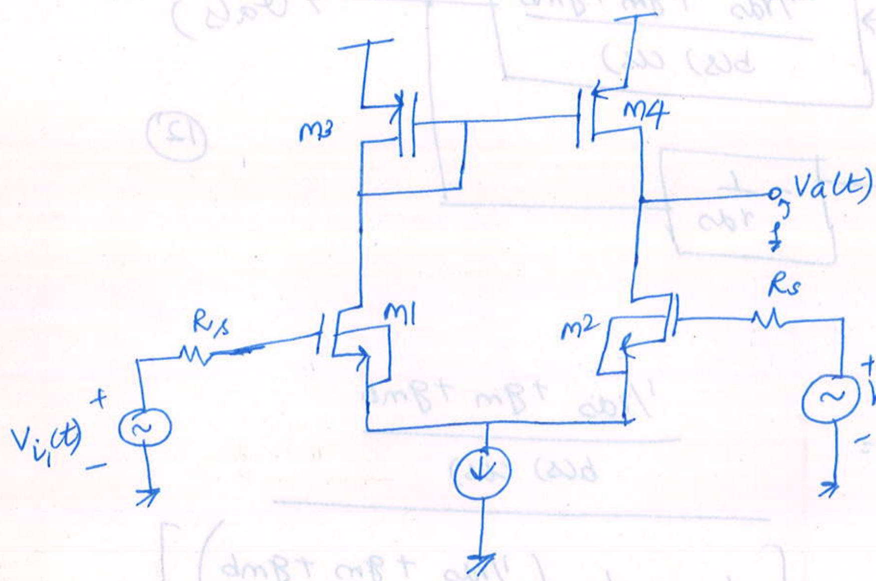
$\alpha_0 + \alpha_1 s + \alpha_2 s^2$ w/ poles $= \dfrac{-\alpha_1}{2\alpha_2} \pm \dfrac{1}{2\alpha_2}\underbrace{\sqrt{\alpha_1^2 - 4\alpha_0\alpha_2}}_{<\alpha_1}$

$\therefore$ Re{Poles} < 0

LEC #7 JAN 29'08

Block Diagrams & MGF (contd)

e.g. Differential-to-single ended OTA

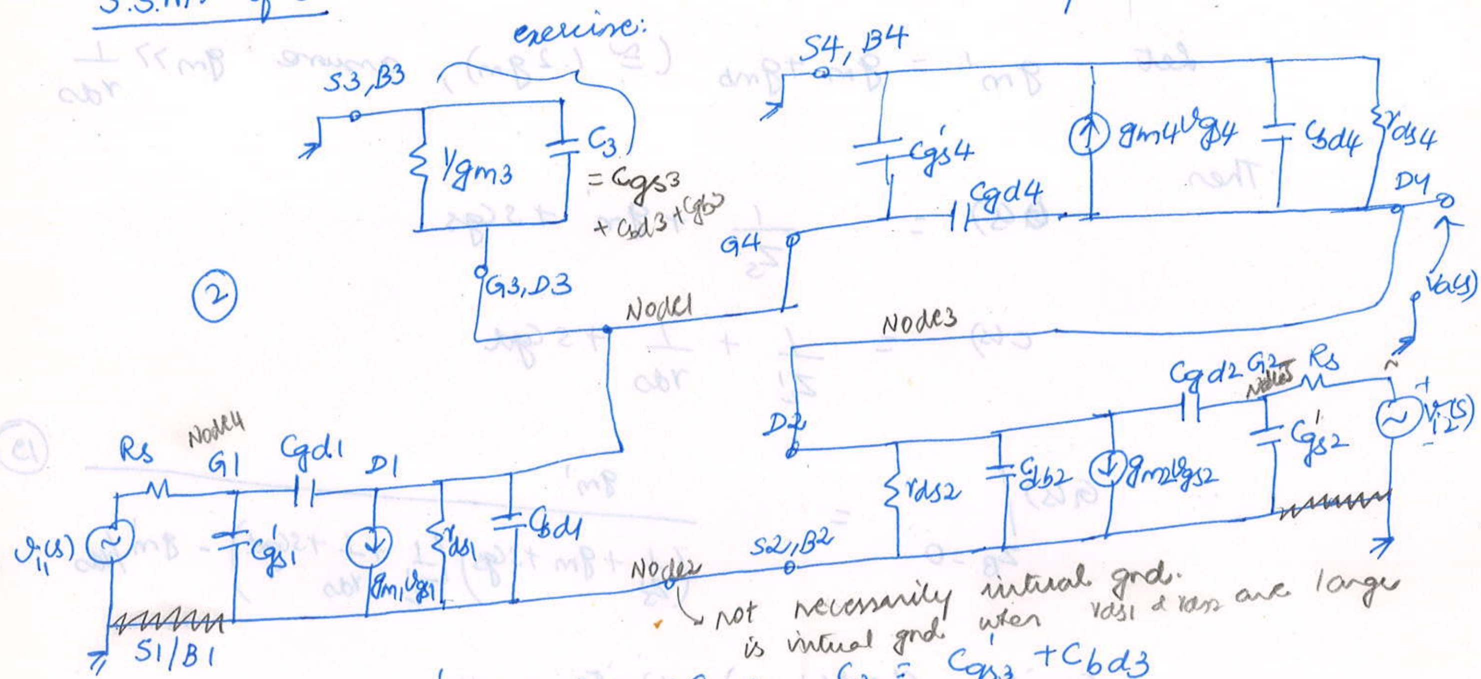


$M_1 \equiv M_2$

$M_3 \equiv M_4$

(OTA generally means that o/p is from a high impedance node)

S.S.M. of ①:



where $C'_{gsi} = C_{gsi} + C_{gbi}$, $C_3 = C_{gs3} + C_{bd3}$

Want to analyze differential mode (DM) operation:

$$v_{i1} = \frac{v_{id}}{2}, \quad v_{i2} = -\frac{v_{id}}{2}$$

Assume R_s = negligible for analysis (based on application this has to be determined)

Want block diagram (BD) containing nodes:

$$v_{id},\ v_{d1},\ v_s \equiv v_{s1} = v_{s2},\ v_a$$

KCL @ D1:-

$$v_{d1}\left[g_{m3} + s\,(C_3 + C'_{gs4})\right] + \frac{v_{d1} - v_s}{r_{ds1}\,\|\,C_{bd1}} + (v_{d1} - v_a)\,sC_{gd4}$$

$$+ \left(v_{d1} - \frac{v_{id}}{2}\right) sC_{gd1} + g_{m1}\left(\frac{v_{id}}{2} - v_s\right) = 0$$

where $"r_x \| C_y" = \left(\frac{1}{r_x} + sC_y\right)^{-1}$ $\quad r_x \| \frac{1}{sC_y} \quad \frac{\frac{r_x}{sC_y}}{r_x + \frac{1}{sC_y}} = \frac{r_x}{1 + r_x s C_y}$

What happened to r_{ds1} in g_{d1}? $\because g_{m3} \gg \frac{1}{r_{ds1}}$ $\Rightarrow g_{m3} + \frac{1}{r_{ds1}} \sim g_{m3}$

$$\Rightarrow v_{d1}\underbrace{\left[g_{m3} + \cancel{\frac{1}{r_{ds1}}} + s(C_3 + C'_{gs4} + C_{bd1} + C_{gd4} + C_{gd1})\right]}_{\text{call } a(s)}$$

$$- v_s\underbrace{\left[g_{m1} + \frac{1}{r_{ds1}\,\|\,C_{bd1}}\right]}_{\cong\ g_{m1} + sC_{bd1}} - v_a\, sC_{gd4} + \frac{1}{2} v_{id}\,(g_{m1} - sC_{gd1}) = 0 \qquad \frac{1}{\frac{1}{r_x} + sC_y}$$

$$\therefore\ v_{d1} = \frac{1}{a(s)}\left[v_s(g_{m1} + sC_{bd1}) + v_a\, sC_{gd4} - \frac{1}{2} v_{id}\,(g_{m1} - sC_{gd1})\right] \quad (3)$$

KCL @ D2:

...

$$\therefore\ v_a = \frac{1}{b(s)}\left[v_s\,(g_{m2} + sC_{bd2}) + v_{d1}\,(sC_{gd4} - g_{m4}) + \frac{1}{2} v_{id}\,(g_{m2} - sC_{gd2})\right] \quad (4)$$

$$\text{where } b(s) = \frac{1}{r_{ds4}\,\|\,r_{ds2}} + s\,(C_{bd4} + C_{bd2} + C_{gd1} + C_{gd2})$$

Isn't this C_{gd4}? How is coming C_{gd1} in?

(Used $g_{m2} \gg \frac{1}{r_{ds2}}$)

KCL @ S1, S2:-

...

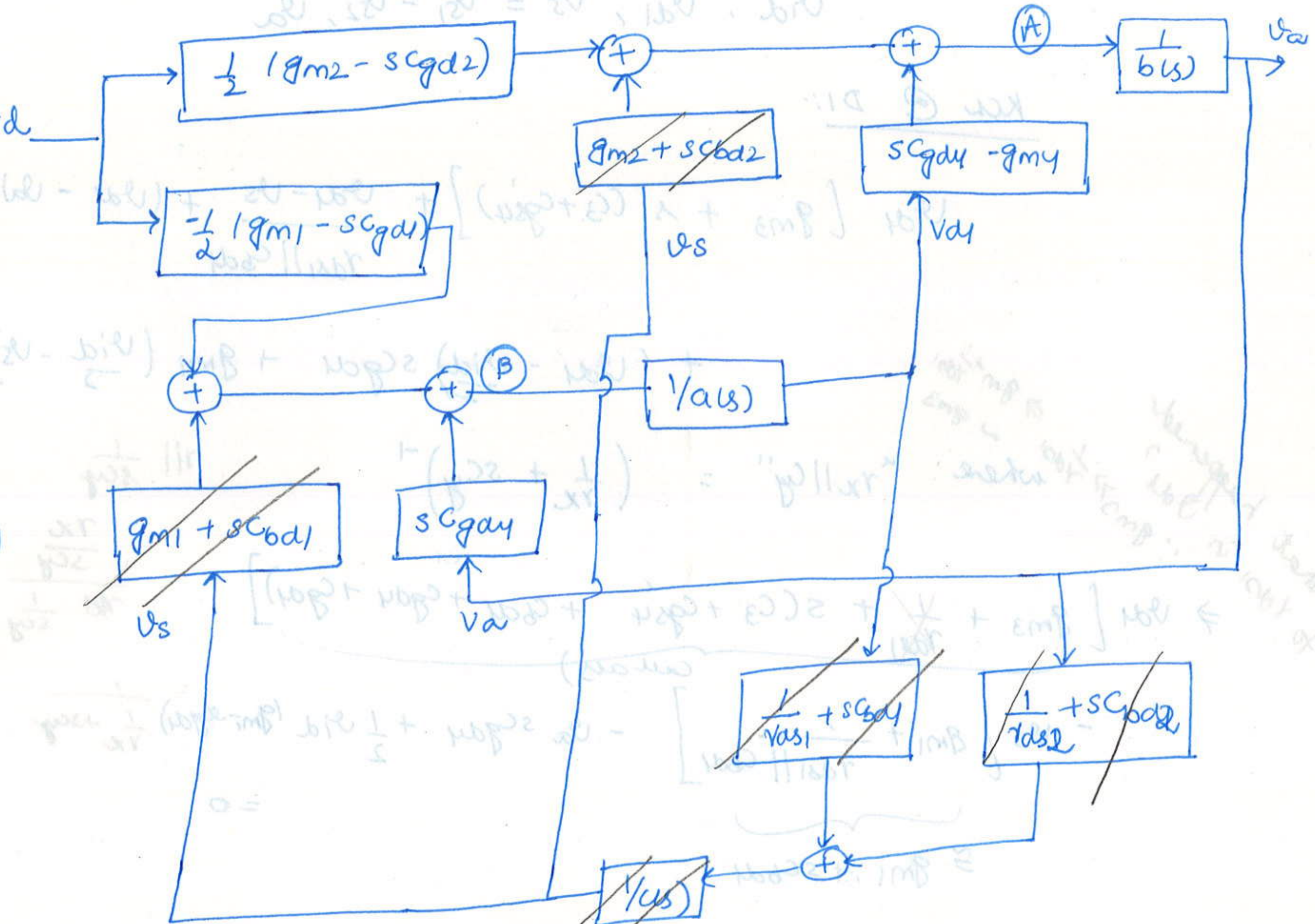
$$v_s = \frac{1}{c(s)}\left[\frac{v_{d1}}{r_{ds1}\,\|\,C_{bd1}} + \frac{v_a}{r_{ds2}\,\|\,C_{bd2}}\right] \quad (5)$$

$$\text{where } c(s) = g_{m1} + g_{m2} + s(C'_{gs1} + C'_{gs2})$$

used $M_1 = M_2$ & $M_3 = M_4$ — where is this used?

What happened to C_{bd1} & C_{bd2}?

(3) - (5) ⇒ block diagram



(6)

Let $p_1, p_2, \ldots$ be the poles of $A_v(s) = \frac{v_a(s)}{v_{id}(s)}$ w/ $|p_1| \le |p_2| \le \ldots$

Provided $\left| \frac{1}{r_{ds1,2}} + j\omega C_{bd1,2} \right|$ is sufficiently small that

$|v_s(j\omega)| << |v_a(j\omega)|$, ~~$v_a(j\omega)$~~ for $|\omega| \le |p_2|$ (7)

can eliminate the feedback paths in (6).

(Asymmetry ⇒ $v_s \neq 0$)

We'll assume (7) holds for now, can later check assumption by testing with resulting value at p_2.

Sanity check: Find $A_{v0} = A_v(j\omega)\big|_{\omega=0}$ $\omega = 0 \Rightarrow s = 0$

∴ $a(0) = g_{m3}$ $b(0) = \frac{1}{r_{ds2} \| r_{ds1}}$ r_{ds4}

∴ (6) ⇒ $v_a = \left(\frac{1}{2} g_{m2}\, r_{ds2} \| r_{ds1} - \frac{1}{2} g_{m1} \frac{-g_{m4}}{g_{m3}} r_{ds2} \| r_{ds4} \right) v_{id}$

r_{ds4}

$A_{v0} = g_{m1}\, r_{ds2} \| r_{ds4}$

Loop: Ⓐ → Ⓑ → Ⓐ   Forward paths: 1) v_{id} → Ⓐ → v_o

2) v_{id} → Ⓑ → Ⓐ → v_o

⑥ ⇒

$$P_1 = \frac{g_{m2} - sC_{gd2}}{2\,b(s)}$$

$$P_2 = \frac{(g_{m1} - sC_{gd1})(-sC_{gd4} + g_{m4})}{2\,a(s)\,b(s)}$$

$$L = \frac{sC_{gd4}(sC_{gd4} - g_{m4})}{a(s)\,b(s)}$$

MGF ⇒ $A_v(s) = \dfrac{P_1 + P_2}{1 - L}$

$$= \frac{1}{2}\,\frac{a(s)(g_{m2} - sC_{gd2}) + (g_{m1} - sC_{gd1})(g_{m4} - sC_{gd4})}{a(s)\,b(s) - sC_{gd4}(sC_{gd4} - g_{m4})} \quad (8)$$

⇒ 2 zeros, 2 poles

2nd-order ⇒ Can easily find poles & zeros but results give little insight

Instead, note: feedback depends on sC_{gd4} (no Miller effect, why?)

∵ gain of M4 is low

If $R_s \approx 0$ then no Miller effect on M1 & M2

Usually $C_{gd4} \ll C_{gsi}, C_{bdi}$

∴ Taking $sC_{gd4} \approx 0$ in (8) = reasonable approx.

Now MGF ⇒ $A_v(s) = P_1 + P_2 \quad (L \approx 0) = \dfrac{\text{2nd order poly}}{a(s)\,b(s)}$

$$= g_{m1} (r_{ds2} || r_{ds4}) \frac{(1 - s/z_1)(1 - s/z_2)}{(1 - s/p_1)(1 - s/p_2)}$$

where $p_1 = \frac{-1}{(r_{ds2} || r_{ds4}) C_2}$, $p_2 = \frac{-g_{m3}}{C_1}$

where $C_1 = C_{gs3}' + C_{bd3} + C_{gs4}' + C_{bd1} + C_{gd1}$

$C_2 = C_{bd2} + C_{bd4} + C_{gd2} + \cancel{C_{gd4}}^{\to 0}$

C_1 and C_2 have same order of magnitude

$r_{ds2} || r_{ds4} \gg \frac{1}{g_{m3}}$ (typ.) $\Rightarrow$ p_1 = dominant pole

p_2 = non-dominant pole

(this may not be true for sub-90nm processes)

Observation:-

$1/g_{m3}$ = resistance to s.s. grd. @ D_1 in (2)

C_1 = cap to " " " "

$(r_{ds2} || r_{ds4})$ = resistance to s.s. grd @ D_2 in (2)

C_2 = cap. to " " "

∴ In (2) could have found p_1 & p_2 by calculating R_i, C_i where R_i = res. to s.s. ground @ node i

C_i = cap to s.s " " "

Then $p_1 = \frac{-1}{\text{largest}\{R_i C_i\}}$, $p_2 = \frac{-1}{\text{next largest}\{R_i C_i\}}$

Q: Coincidence?

A: No, This method is reasonable approx in many cases (where there is a unilateral condition).

Slang: Because of this, people often say that dominant pole occurs @ node D_2 and the non-dominant pole occurs @ node D_1.

Full Version of MGF:

Block diagram defs:

1) "path": route through BD (in direction of arrows) connecting a pair of nodes.

2) "path gain": product of block gains along the path

3) "loop": path which starts and ends @ same node w/ no node along path encountered more than once.

4) "loop gain": path gain of loop.

5) "determinant" Δ: 1 − Σ(all loop gains)

+ Σ (product of loop gains of all loop pairs w/ no common nodes)

− Σ (product of loop gains of all loop triples w/ no common nodes)

+ ⋮

6) "Forward Path": path from i/p to o/p containing no full loops

Let $H = \dfrac{X_{out}}{X_{in}}$ where X_{in} = i/p of B.D.

X_{out} = o/p of B.D.

Then $H = \dfrac{1}{\Delta} \sum_{k=1}^{L} P_k \Delta_k$ (M.G.F.)

Δ = determinant

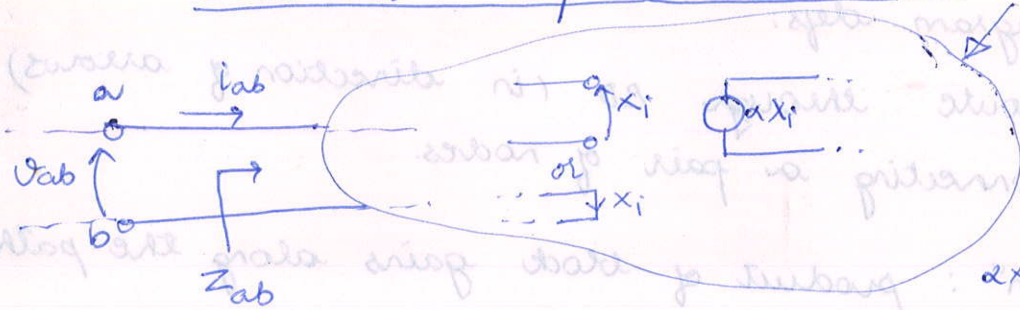
L = # of forward paths

P_k = kth forward path

Δ_k = determinant of B.D. that remains after deleting kth path, P_k

LEC #8 JAN 31 '08

Blackman's Impedance Relation



arbitrary s.s.m. ckt. containing ≥ 1 controlled source.

x_i = voltage or current

αx_i = " "

(eg. $x_i = v_{gs}$, $\alpha = g_m$

$\therefore \alpha x_i$ = current)

$$Z_{ab} = Z_{ab}^{0} \frac{1+T_{sc}}{1+T_{oc}} \quad (1)$$

$$\left(\equiv \frac{v_{ab}(s)}{i_{ab}(s)} \right)$$

where $Z_{ab}^{0} \equiv$ impedance between a & b w/ controlled source removed ($\alpha = 0$)

LEC #8 JAN 31 '08

$$Z_{ab} = Z_{ab}^{\circ} \, \frac{1 + T_{sc}}{1 + T_{oc}} \quad (1)$$

$\left(\equiv \frac{v_{ab}(s)}{i_{ab}(s)} \right)$

where $Z_{ab}^{\circ} \equiv$ Impedance b/w a & b w/ controlled source removed ($\alpha = 0$)

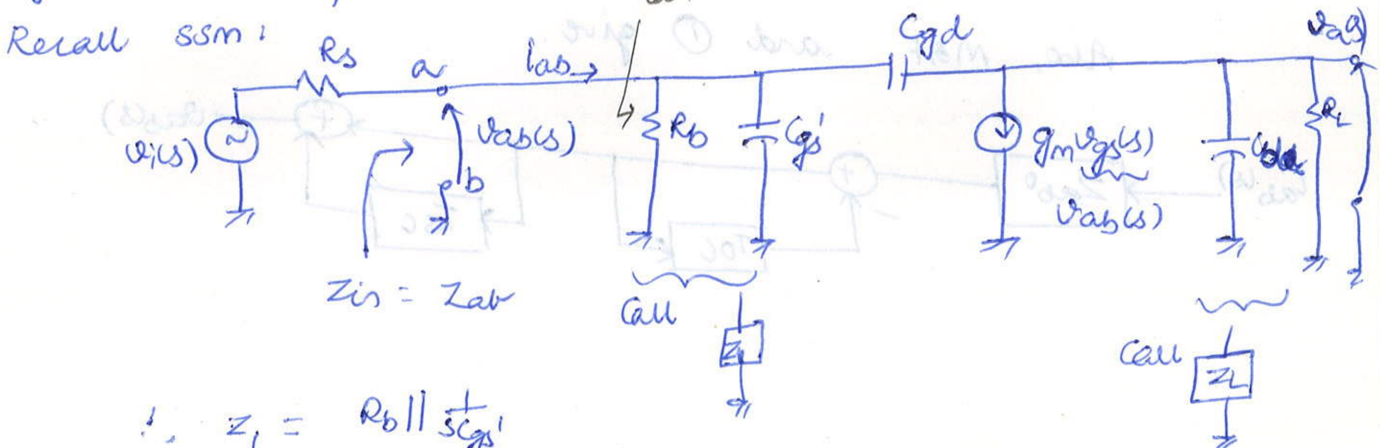
$$T_{sc} = -\alpha \frac{x_i}{x_x}$$ when $v_{ab} = 0$ (a, b shorted) and αx_i is replaced by an "independent test source", x_x

$T_{oc} \equiv$ same as T_{sc} except w/ $i_{ab} = 0$ (a, b open circuited)

Note: If have ≥ 1 controlled sources, pick any one (call it the reference source)

eg 1 CS Amp.

Recall SSM:



$\therefore \; z_1 = R_b \,||\, \frac{1}{sC_{gs}'}$

$z_L = R_L' \,||\, \frac{1}{sC...}$

BIR: $\alpha = g_m$, $x_i = v_{ab}$, $x_x = i_x$

$$Z_{ab}^{0} = Z_1 \,||\, \left(\frac{1}{sC_{gd}} + Z_L\right)$$

$\cancel{1 + sC_{gd} Z_L}$

$Z_1 (1 + Z_L s C_{gd}$

$T_{sc} = 0$ (because $x_i = v_{ab} = 0$)

$$T_{oc} = -g_m \cdot \frac{1}{i_x}\Big[\underbrace{-i_x \left(Z_L || \left(Z_1 + \frac{1}{sC_{gd}}\right)\right) \cdot \frac{Z_1}{Z_1 + \frac{1}{sC_{gd}}}}_{v_{ab})}\Big]$$

$$= \frac{g_m Z_1 Z_L}{Z_L + Z_1 + \frac{1}{sC_{gd}}}$$

| How is $T_{oc} = 0$ when $C_{gd} = 0$?)

(note: $\to 0$ as $C_{gd} \to 0$)

$\therefore$ (1) $\Rightarrow$ $Z_{in} = Z_{ab}^{0} \cdot \dfrac{1}{1 + T_{OC}}$

$$V = Z_1 \frac{1 + sC_{gd} R_L}{1 + [\underbrace{Z_L + Z_1 (1 + g_m Z_L)}_{\text{Miller Effect}}] sC_{gd}} \quad (2)$$

Z_L?

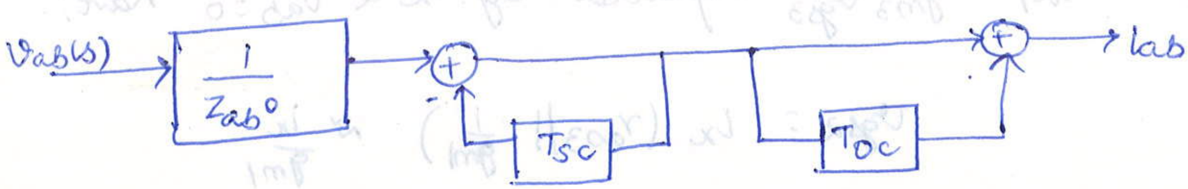
Observations:

1) T_{OC} represents feedback around reference source

Why? def of TOC $\Rightarrow$ if o/p of ref. source has no connection to its controlling voltage (or current) then $T_{oc} = 0$

Also, MGF and (1) give:

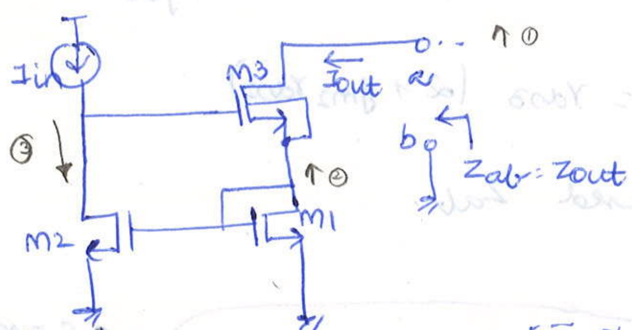
(a)



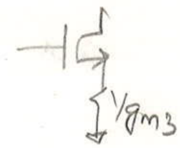
$\therefore$ T_{sc} also arises from a feedback path within ckt.

2) Feedback can either increase or decrease Z_{ab}
(In case of C.S.A, fb reduced Z_{in} by miller effect of C_{gd})

eg 2 Wilson current mirror (low freq. analysis)



If $V_{gs3} = V_{G3}$



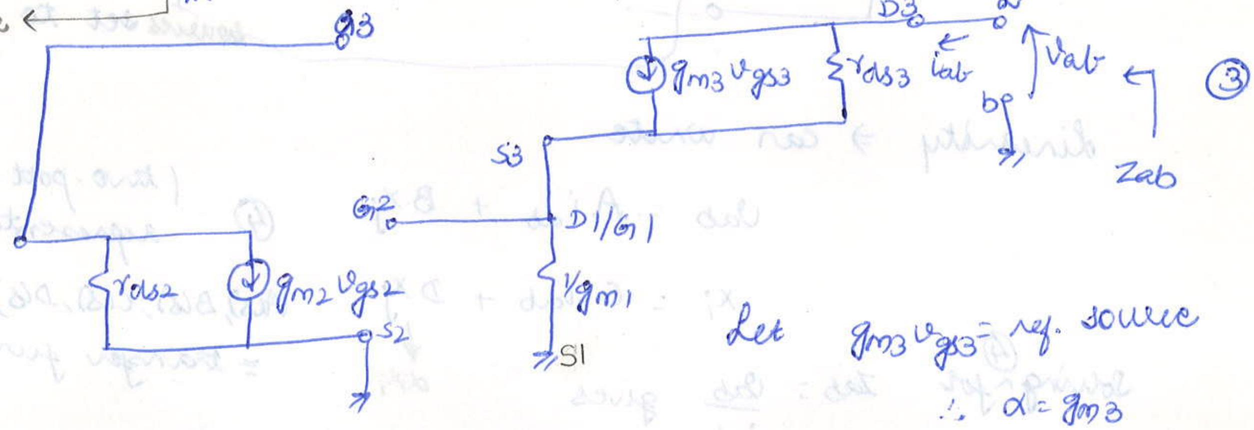
If $V_{d3} \uparrow$, then $I_{ds3} \uparrow$
$\Rightarrow V_{ds1} \uparrow \Rightarrow V_{g2} \uparrow$
$\Rightarrow V_{d2} \downarrow \Rightarrow V_{g3} \downarrow$
$\Rightarrow I_{ds3} \sim$ const.

Low freq. s.s.m. (using

)

(Set $i_{in} = 0$ to find Z_{out})

the indep current source ←



Let $g_{m3} v_{gs3}$ = ref. source
$\therefore \alpha = g_{m3}$
$x_i = v_{gs3}$

Inspection of ③ $\Rightarrow Z_{ab}^0 = r_{ds3} + \frac{1}{g_{m1}}$
$\cong r_{ds3}$

Inspection of ③ $\Rightarrow$ If $i_{ab} = 0 \Rightarrow v_{gs2} = 0 \Rightarrow v_{gs3} = 0$
$\Rightarrow x_i = 0$ $\therefore T_{oc} = 0$

$T_{sc} = ?$

T_{sc} :- w/ $g_{m3} v_{gs3}$ replaced by i_x & $v_{ab} = 0$ have;

$$v_{gs2} = i_x \left(r_{ds3} \| \frac{1}{g_{m1}} \right) \approx \frac{i_x}{g_{m1}}$$

$$\text{KVL} \Rightarrow v_{gs2} + v_{gs3} = -g_{m2} v_{gs2} r_{ds2}$$

$$\Rightarrow v_{gs3} = \frac{-i_x}{g_{m1}} (1 + g_{m2} r_{ds2})$$

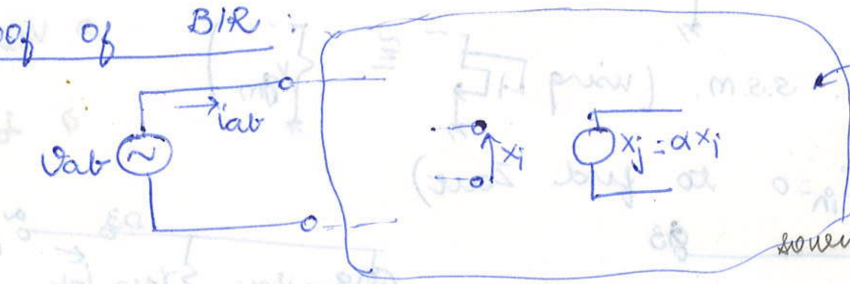
$$\therefore T_{sc} = \frac{g_{m3}}{g_{m1}} (g_{m2} r_{ds2} + 1) = g_{m2} r_{ds2} + 1$$

(for $m1 = m2 = m3$)

$$\therefore ① \Rightarrow Z_{ab} = r_{ds3} (2 + g_{m2} r_{ds2})$$

$\therefore$ Feedback increased Z_{ab}.

Proof of BIR :



S.S.M. ckt. as before. (all independent sources set to zero)

linearity $\Rightarrow$ can write

$$v_{ab} = A i_{ab} + B x_j \quad ④$$

(two-port representation)

$$x_i = C i_{ab} + D x_j$$

$x_j \to \alpha x_i$

$A(s), B(s), C(s), D(s)$ = transfer functions.

solving ④ for $Z_{ab} = \frac{v_{ab}}{i_{ab}}$ gives

$$Z_{ab} = A \frac{1 - \frac{\alpha (AD - BC)}{A}}{1 - \alpha D} \quad ⑤$$

$$④ \quad A = \left. \frac{v_{ab}}{i_{ab}} \right|_{x_j = 0} = \text{def. of } Z_{ab}^0 \quad ⑥$$

Now suppose $v_{ab} = 0$ and $x_j = X_x$ (independent of x_i):

Then (4) $\Rightarrow$ $i_{ab} = \frac{-B}{A} X_x$

$$x_i = C i_{ab} + D X_x$$

$$\Rightarrow \underbrace{-\alpha \frac{x_i}{X_x}}_{\equiv T_{sc}} = \frac{-\alpha (AD - BC)}{A} \quad (7)$$

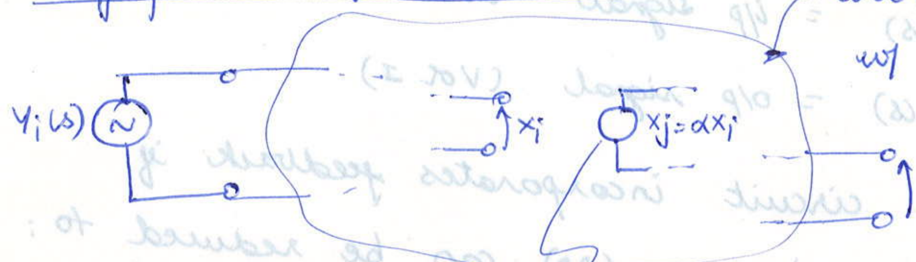
Now suppose $i_{ab} = 0$ and $x_j = X_x$ (indep. of x_i):

Then (4) similarly $\Rightarrow$ $T_{oc} = -\alpha D$ (8)

$\therefore$ (5) - (8) $\Rightarrow$ $Z_{ab} = Z_{ab0} \frac{1 + T_{sc}}{1 + T_{oc}}$

□ Asymptotic Gain Relation (A.G.R.)

arbitrary s.s.m. ckt. w/ ≥ 1 controlled source



(9)

$x_i \neq y_i$ reqd

(if $x_i = y_i$ $\Rightarrow$ no F.B.

$\Rightarrow$ result incorrect

no F.B. $\because$ that node of G_i in C.S.A. is pegged @ $Y_i(s)$)

any controlled source in ckt. ($\equiv$ "ref source")

$\left.\begin{matrix} x_i, x_j \\ y_i, y_j \end{matrix}\right\}$ = voltages and/or currents

A.G.R. $\quad A(s) \left(= \frac{Y_j(s)}{Y_i(s)} \right) = A_\infty \frac{T}{1+T} + A_0 \frac{1}{1+T}$ (10)

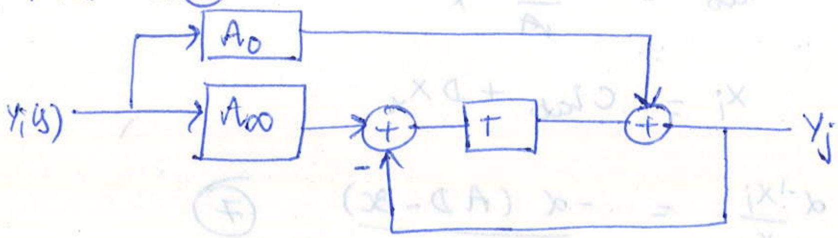
where $\underbrace{T \equiv -\frac{\alpha x_i}{X_x}}_{\text{loop gain}}$ where $Y_i = 0$ & x_j replaced by an "independent test source" X_x

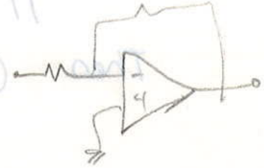
$A_\infty(s) \equiv \left. \frac{Y_j(s)}{Y_i(s)} \right|_{\alpha \to \infty}$ $\equiv$ "asymptotic gain"

$A_0(s) = \left. \frac{Y_j(s)}{Y_i(s)} \right|_{\alpha = 0}$ $\equiv$ "direct tran~~sfer~~smission term"

Observations: ~~A_0~~

MGF & (10) $\Rightarrow$ can redraw (9) as





$\Rightarrow$ large T $\Leftrightarrow$ feedback dominates behavior

$\Rightarrow A(s) \cong A_\infty(s)$

$A_0(s)$ arises from forward path through feedback n/w.

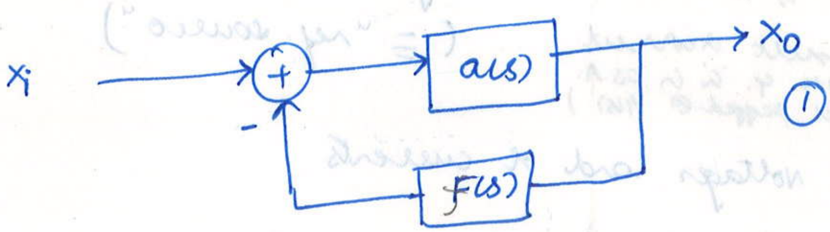
$\boxed{\text{LEC \#9 FEB 12'08}}$

FEEDBACK

Let $x_i(s)$ = i/p signal (V or I)

$x_o(s)$ = o/p signal (V or I)

Then a circuit incorporates feedback if its block diagram (BD) can be reduced to:



(1)

MGF + (1) $\Rightarrow$ $A(s) = \left(\dfrac{a(s)}{1+a(s)F(s)}\right)$ (2)

where $A(s) \triangleq \dfrac{x_o(s)}{x_i(s)}$

"Ex1" "Passive Feedback"

⇒

∴ $a(s) = 1, \quad f(s) = sRC$

(More generally, poles ⇒ feedback)

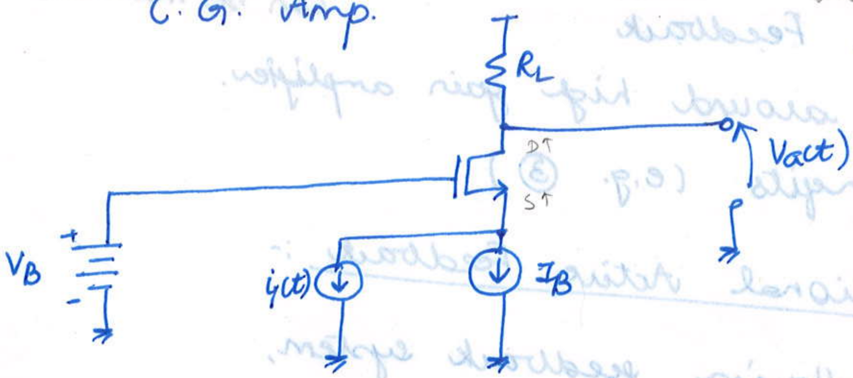
(Dig world; poles ⇏ F.B. ∵ poles & zeros cancel)

"Ex 2" "Parasitic Feedback"

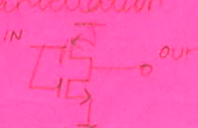
→ ∵ r_{ds} is due to C.L.M. & undesired

C.G. Amp.

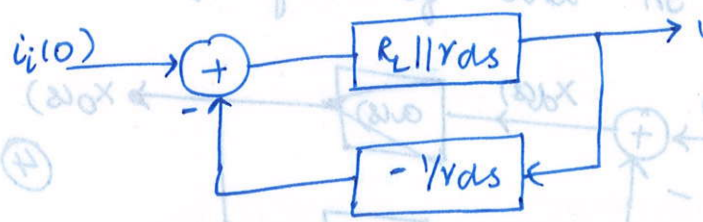
(a(s) may have poles ∴ BD ⇏ F.B. But F.B. loop in B.D ⇒ F.B.)



- pole-zero cancellation in dig. ckts.



⇒ (found previously) (low freq. analysis)

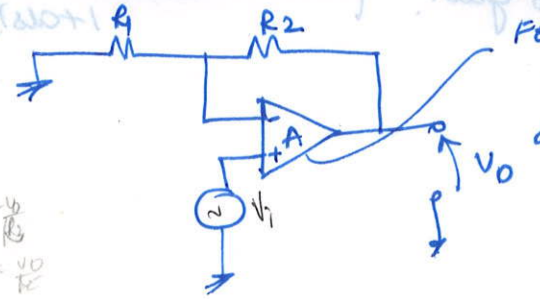


(+ve F.B.)

<u>Note:-</u> In Ex 1, feedback reduces the magnitude of x_o ⇒ "negative feedback"

In Ex 2, feedback increases the magnitude of x_o ⇒ "positive feedback".

"Ex 3" "Intentional Active Feedback" (Will focus mostly on -ve F.B.)



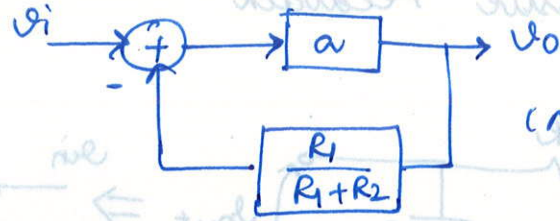
For now assume ∞ i/p imp. & 0 o/p imp.

$$v_o = a\left[v_i - \frac{R_1}{R_1+R_2} v_o\right]$$

$\frac{-V_a + 0}{R_1} = \frac{V_a - V_o}{R_2}$

$\frac{+V_a}{R_1} + \frac{V_a}{R_2} = \frac{V_o}{R_2}$

$\therefore V_a\left[\frac{R_1+R_2}{R_1R_2}\right] = \frac{V_o}{R_2}$



(neg. feedback)

$$\therefore A = \frac{a}{1 + a\left/\frac{R_1}{R_1+R_2}\right)} \rightarrow 1 + \frac{R_2}{R_1} \quad \text{as } a \to \infty$$

$\therefore$ $\underbrace{\text{large 'a'}}$ $\Rightarrow$ A depends only on the ③ $\underbrace{\text{ratio of R's.}}$

easy in IC's (atleast @ low freq.)

accurate in IC's

(abs R's may have Temp. but relative R's can be matched)

Benefits in today's ICs

Can use DSP to correct non linearity

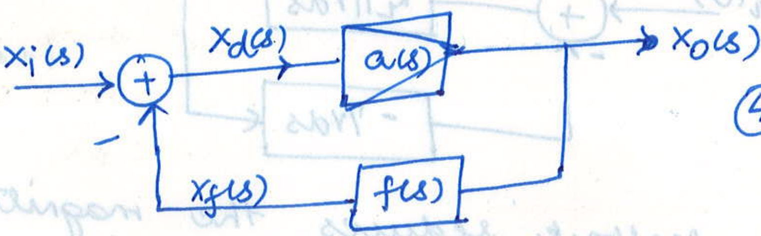
Intentional Active Feedback

$\Rightarrow$ Feedback around high gain amplifier.

$\Rightarrow$ many benefits (e.g. ③)

Negative Intentional Active Feedback :-

Defs:- In the following feedback system,



④

$a(s) \equiv$ "open loop gain".

$f(s) \equiv$ "feedback factor"

$T(s) \equiv a(s)f(s) \equiv$ "loop gain".

$A(s) \equiv$ "closed loop gain". $\left(\text{Recall } A(s) = \frac{a(s)}{1+a(s)f(s)}\right)$

Let $A_{ideal} = \lim_{a \to \infty} A = \frac{1}{f}$ (Dropping 's' for ease)

(eg. $A_{ideal} = 1 + \frac{R_2}{R_1}$ in Ex 3)

$\therefore$ ② $\Rightarrow A = A_{ideal}\left(1 - \underbrace{\frac{1}{1+T}}\right)$

fractional deviation of A from A_{ideal}.

Also, MGF & ④

$\Rightarrow x_d = \frac{1}{1+T} x_i \to 0$ as $T \to \infty$ $(a \to \infty)$

↳ Typically $a \to \infty$ & not $f \to \infty$

$x_f = \frac{1}{1+1/T} x_i \to x_i$ as $T \to \infty$ $(a \to \infty)$

$\therefore x_f$ "tracks" x_i w/ "error signal" x_d

Benefits: (i) gain desensitivity (basically $|a|$ must be high, need not be accurate)

(ii) non-linear distortion reduction.

(iii) in-loop noise suppression. (eg. VCO in PLL has bad loo freq. noise performance)

(iv) broadbanding (high gain $\Rightarrow$ typ. low b/w $\because$ a big cap (high imp) to gnd)

(v) input/output impedance adjustment

Drawbacks: (i) $A < a$

(Potential) (ii) excessive phase shift can cause instability.

(1) Gain Desensitivity:-

② $\Rightarrow \frac{dA}{da} = \frac{1}{(1+af)^2} \Rightarrow \frac{\Delta A}{\Delta a} \cong \frac{1}{(1+af)^2}$ (small Δa)

$\therefore \frac{\Delta A}{A} \cong \left(\frac{1}{1+T}\right)\frac{\Delta a}{a}$; large variation in 'a' $\Rightarrow$ small " " 'A'

$\Rightarrow A$ is "stable" w.r.t. variations in a.

↳ w.r.t. same value i.e. not variable rather than w.r.t. BIBO stability

② $\Rightarrow \quad \dfrac{dA}{df} = -A^2$

$$\therefore \quad \frac{\Delta A}{\Delta f} \cong -A \cdot \frac{1}{f} \cdot \frac{T}{1+T}$$

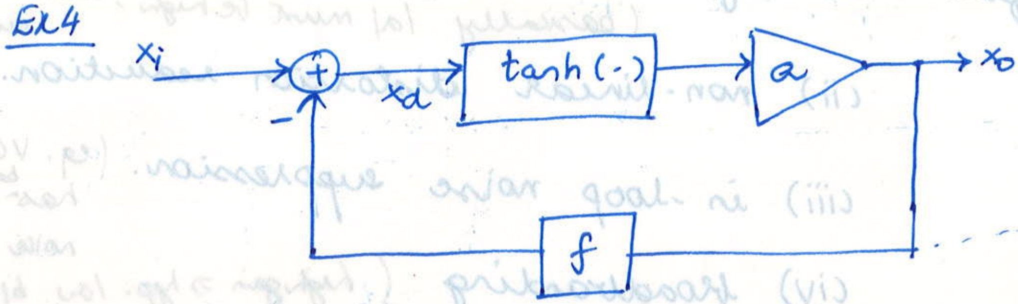
$$\therefore \quad \frac{\Delta A}{A} \cong -\underbrace{\left(\frac{T}{1+T}\right)}_{\cong 1 \text{ for large} |T|} \frac{\Delta f}{f}$$

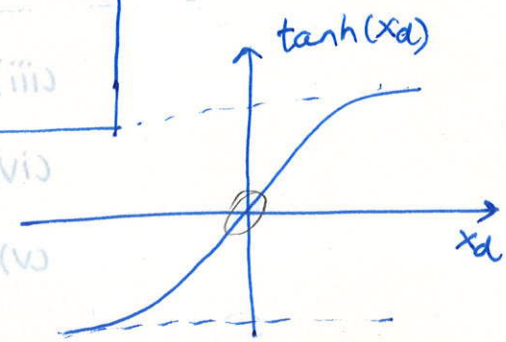
$\Rightarrow$ A is not "stable" (variable) w.r.t. variations in f.

$\Rightarrow$ Want high gain amp. (not necessarily well defined) and precise feedback factor.

(ii) <u>Non-Linear Distortion Reduction</u>: (Transfer fn. $\Rightarrow$ LTI system)

<u>Ex4</u>





$$x_o = a \tanh(x_d)$$
$$= a \tanh(x_i - x_o f)$$

$$\therefore \quad \tanh^{-1}\left(\frac{x_o}{a}\right) = x_i - f x_o$$

Intuitively, $x_d \to 0$ if $a \to \infty$ where tanh(x_d) is linear.

$$\therefore \quad \frac{x_o}{a} + \frac{x_o^3}{3a^3} + \frac{x_o^5}{5a^5} + \ldots = x_i - f x_o$$

$* a$

$$\Rightarrow \quad x_o + \underbrace{\frac{x_o^3}{3a^2} + \frac{x_o^5}{5a^4} + \ldots}_{\to 0 \text{ as } a \to \infty} = a(x_i - f x_o)$$

$\therefore \quad x_o \cong A x_i$ for large a & $x_o \to \underbrace{\dfrac{x_i}{f}}_{\text{no distortion.}}$ as $a \to \infty$

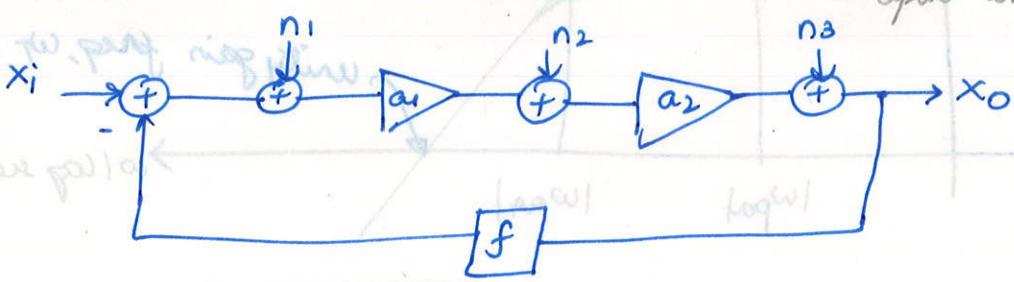
Heuristics:

What's intuitive is what we have seen before; therefore real inventions are w/ Math

1) Follows from gain densitivity (distortion prior to a $\Leftrightarrow$ i/p dependent gain variation)

2) Large $a \Rightarrow$ small $x_d \Rightarrow$ small portion of non linear curve is transversed $\Rightarrow \cong$ linear

(iii) $\exists$ In loop Noise Suppression :- (Whether noise gets suppressed depends upon where it is added)



$$\boxed{\text{Open loop gain} : a = a_1 a_2}$$

$[[(x_i + n_1) a_1 + n_2] a_2 + n_3] = x_o$

$$\text{MGF} \Rightarrow x_o = \frac{a}{1 + a_1 f}\left(x_i + n_1 + \frac{n_2}{a_1} + \frac{n_3}{a_1 a_2}\right)$$

$a_1 a_2 = a$

not suppressed — somewhat suppressed — most suppressed.

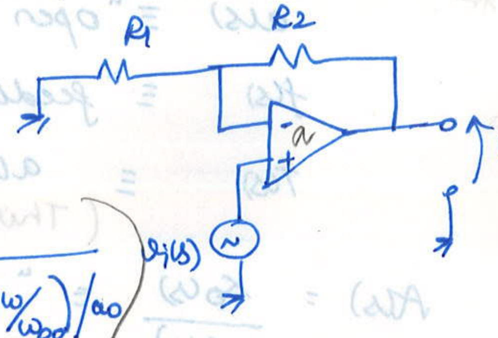
i.e. o.k. to have more noise @ o/p.

(iv) Broadbanding :-

Ex 5 Suppose $a \cong \dfrac{a_0}{1 - j\omega/\omega_{p0}}$ (dominant pole approx.)

$f \cong$ independent of ω.

eg:



Then ②

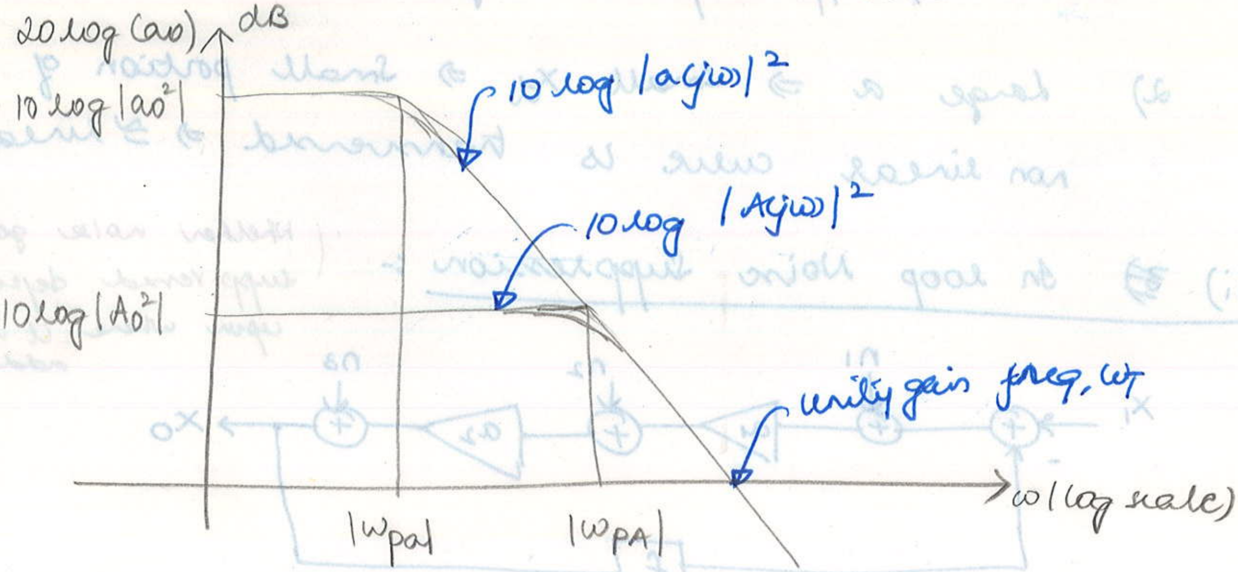
$$\Rightarrow A(j\omega) = \underbrace{\left(1 + \frac{R_2}{R_1}\right)}_{1/f} \frac{1}{1 + (1 + R_2/R_1)(1 - j\omega/\omega_{p0})/a_0}$$

$$f = \frac{R_1}{R_1 + R_2}$$

algebra $\Rightarrow$ $A(j\omega) = \dfrac{A_0}{1 - j\omega/\omega_{pA}}$

where $A_0 = \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + (1 + R_2/R_1)/a_0}$ (5)

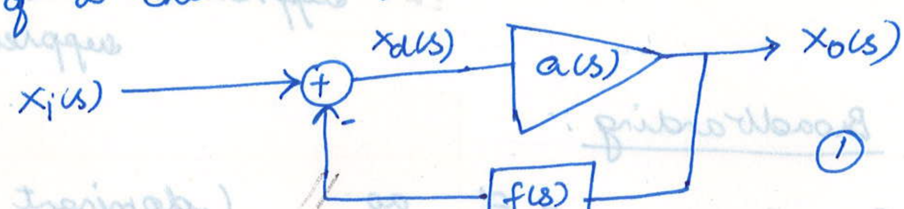
& $\omega_{PA} = \omega_{P0}\left(1 + a_0 \frac{R_1}{R_1 + R_2}\right)$ (6)



REC #10 ~~JAN~~ FEB 14 '08

Negative intentional active feedback (contd.)

Recall: Can always write the block diagram of a ckt. w/ feedback as:



(1)

Can model any system like this (1); but a(s) & f(s) are not unique then.

$a(s) \equiv$ "open loop gain"

$f(s) \equiv$ "feedback factor"

$T(s) \equiv a(s)f(s) \equiv$ "loop gain"

(This is diff. from M.G.F. where $T(s) = -a(s)f(s)$ defn. inconsistency)

$A(s) = \frac{x_o(s)}{x_i(s)} \equiv$ "closed loop gain"

MGF $\Rightarrow A(s) = \dfrac{a(s)}{1+a(s)f(s)}$ ②

Broadbanding (contd)

Suppose : $a(j\omega) = \underbrace{a_0}_{\text{DC gain}} \dfrac{1}{1-j\omega/\omega_{pa}}$ ③

→ dominant pole approximation.

What if we have C_c? ← $f(j\omega) = \text{const.}$

②, ③ $\Rightarrow A(j\omega) = A_0 \cdot \dfrac{1}{1-j\omega/\omega_{pA}}$ where

$A_0 = \dfrac{a_0}{1+a_0 f}$ (DC closed loop gain)

& $\omega_{pA} = (1+a_0f)\,\omega_{pa}$ ④

$|A_0\omega_{pA}| \equiv$ closed loop gain bandwidth product, $GBW_{\text{closedloop}}$

$|a_0\omega_{pa}| \equiv$ open loop gain bandwidth product, $GBW_{\text{open loop}}$

④ $\Rightarrow A_0\omega_{pA} = a_0\omega_{pa}$

$\therefore GBW_{\text{open loop}} = GBW_{\text{closed loop}}$ in this case.

$\therefore$ Increasing f (between 0 & 1) decreases a_0, but increases the 3dB B/W

$f>0$: we need -ve F.B.

→ Unity gain ($f=1$)

$f>1 \Rightarrow$ Attenuator.

↓ Using op-amps to build attenuators is not efficient

$f\uparrow \Rightarrow$ stability ↓

→ $f=1$ spec opamp spec

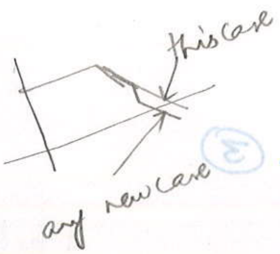
→ But this is used in switched caps.

"Unity Gain Freq"

$\equiv \omega_t$ such $|a(j\omega_t)| = 1$ $\omega_t \in \mathbb{R}$

$\therefore$ ③ $\Rightarrow \left| a_0 \cdot \dfrac{1}{1-j\omega_t/\omega_{pa}} \right| = 1$

$\Rightarrow (a_0^2-1)\,\omega_{pa}^2 = \omega_t^2$



$\Rightarrow \omega_t \cong a_0/\omega_{pa}|$ $(a_0 \gg 1)$

↳ This is true for just this case

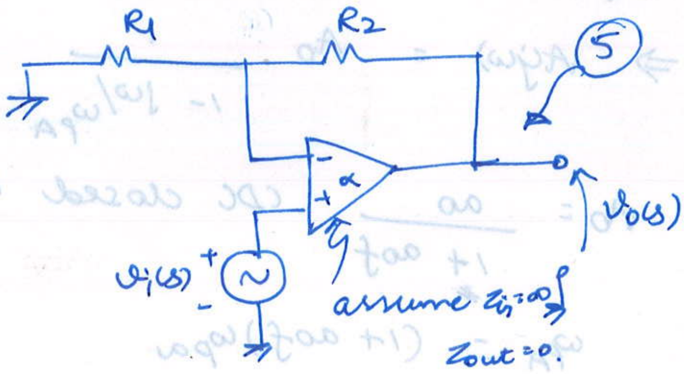
$\therefore$ GBW $= \omega_t$ in this case (not always in other cases)

↓ dominant pole + f indep. of freq. ✓

α dominant pole?

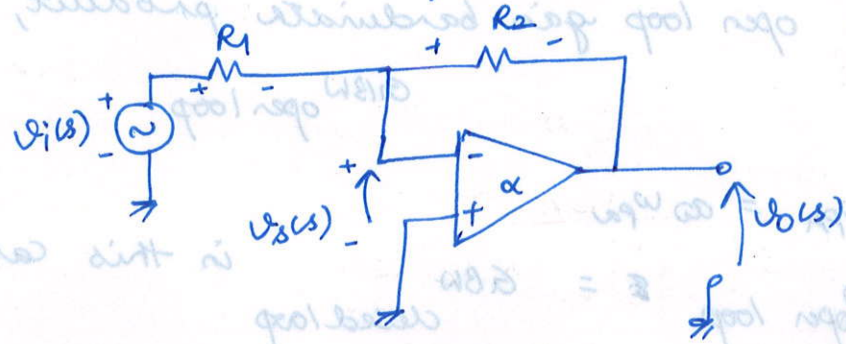
Ex1 (from last time)

Ex3.)



$\Rightarrow$ ① w/ $a=\alpha$; $f=\dfrac{R_1}{R_1+R_2}$

Ex2 Same except different i/p location:-



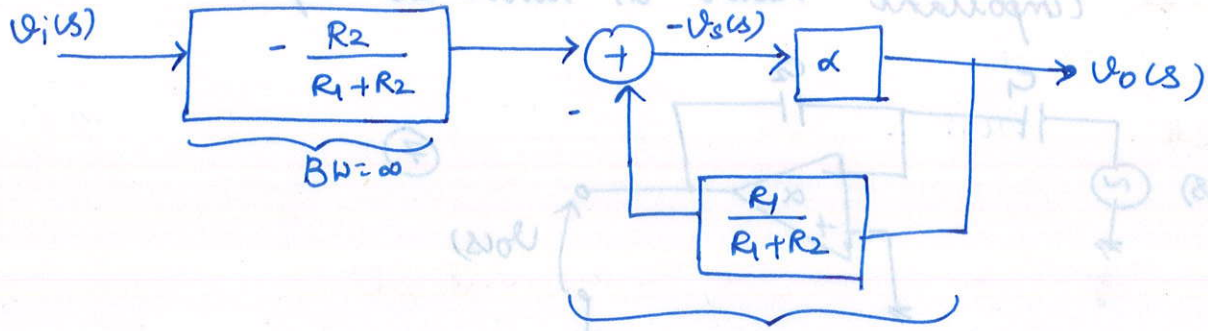
6

$$v_o(s) = \alpha(s)\left[-v_s(s)\right]$$

$$\therefore v_s(s) = v_i(s) - R_1\left[\frac{v_i(s)-v_o(s)}{R_1+R_2}\right]$$

$$\therefore -v_s(s) = -v_i(s)\underbrace{\left[1-\frac{R_1}{R_1+R_2}\right]}_{\frac{R_2}{R_1+R_2}} - v_o(s)\left[\frac{R_1}{R_1+R_2}\right]$$

∴ BD of ⑥ is:



$\equiv$ ① w/ $a = \alpha$ & $f = \frac{R_1}{R_1+R_2}$

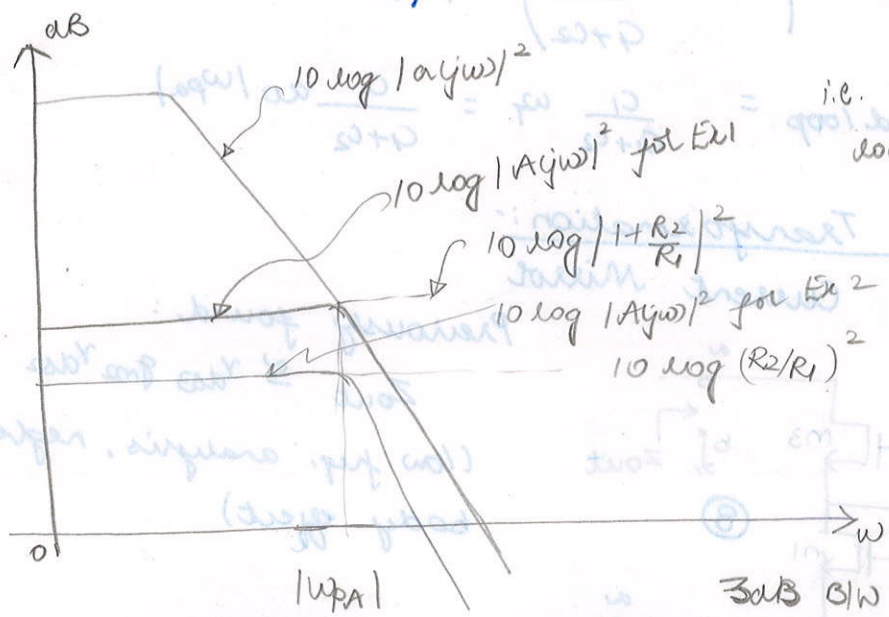
(same as Ex 1)

⇒ BW = same as Ex 1 (∵ α is same as before
$\frac{R_2}{R_1+R_2}$ in forward path doesn't limit b/w)

But GBW of ⑥ less than that of Ex 1 by $\frac{R_2}{R_1+R_2}$

Also, $GBW_{\text{closed-loop}} = \frac{R_2}{R_1+R_2}\cdot\omega_t = \frac{R_2}{R_1+R_2}\cdot \underbrace{a_0|\omega_{pa}|}_{\text{GBW open loop}}$

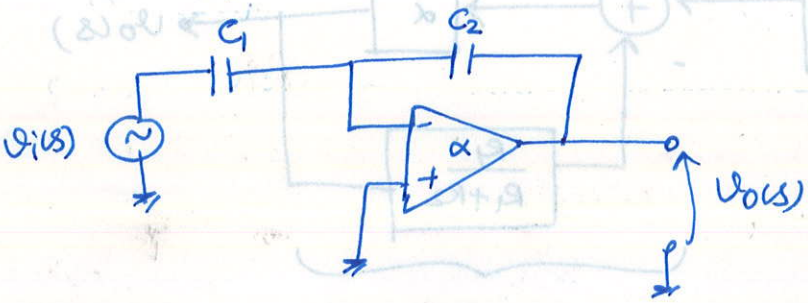
i.e. Open loop & closed loop GBWs are not equal.



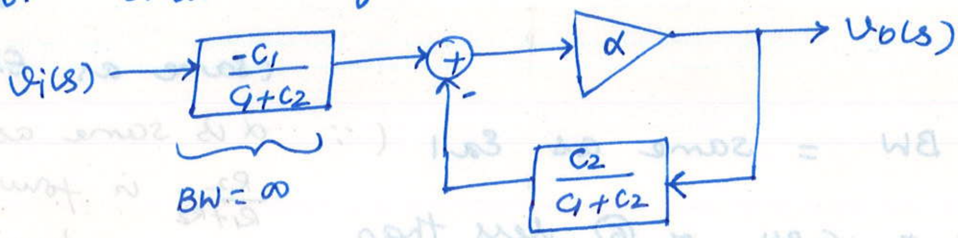
3dB B/W didn't change but closed loop GBW diff. from open loop GBW.

Ex 3   Same as Ex2 w/ R's replaced by C's.

(important later in switched-cap. circuits)



Exercise:- Show BD of ⑦ is:



$\equiv$ ① w/ $\begin{cases} a = \alpha \\ f = \dfrac{C_2}{C_1+C_2} \Rightarrow f = \text{const.} \ (\in \mathbb{R}) \end{cases}$

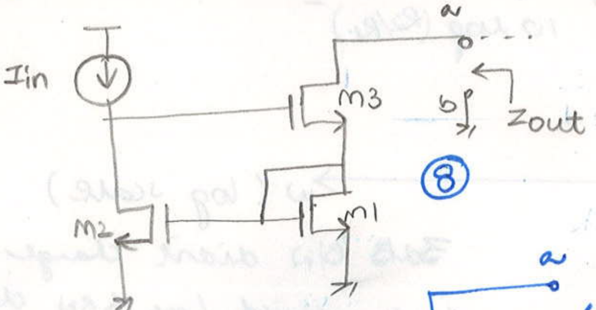
$\therefore$ ④ $\Rightarrow$ $$BW = \left(1 + a_0 \frac{C_2}{C_1+C_2}\right)|\omega_{pa}|$$

Also, $$GBW_{\text{closed loop}} = \frac{C_1}{C_1+C_2}\,\omega_t = \frac{C_1}{C_1+C_2}\,a_0|\omega_{pa}|$$

(V) Impedance Transformation:-

Ex4   Wilson Current Mirror

F.B. around C.S.A M2

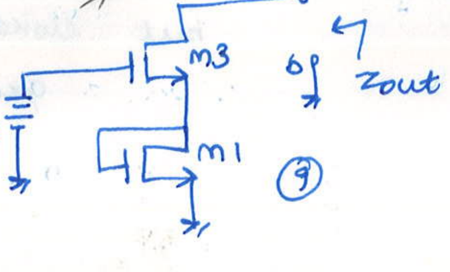


Previously found:

$Z_{out} \cong r_{ds3}\, g_{m2}\, r_{ds2}$

(low freq. analysis, neglecting body effect)

Now consider



where V_{G3} = DC component of V_{g3} in ⑧

Can show that: Z_{out} of 9 $\cong r_{ds3}\left(2 + \frac{1}{g_{m1} r_{ds3}}\right)$

$\cong 2 r_{ds3}$

Note: 9 is equivalent to 8 with feedback disabled

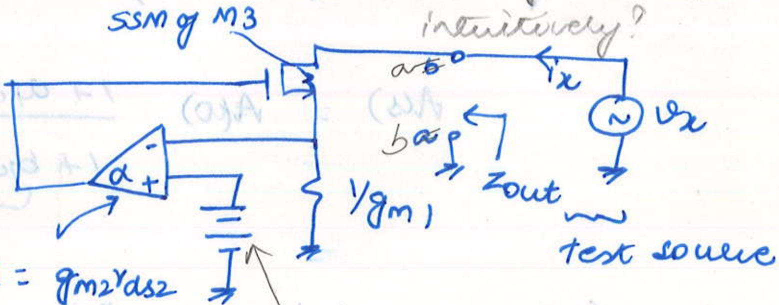
$\therefore$ Feedback increases Z_{out} by $\boxed{\frac{1}{2}} g_{m2} r_{ds2}$ factor

(This ckt. is typically used in high gain op amp loads; called as gain boosting)

$\hookrightarrow$ Why this factor intuitively?

Heuristics:

8 $\equiv$



If v_x increases, i_x increases

$\Rightarrow$ v_{g2} increases

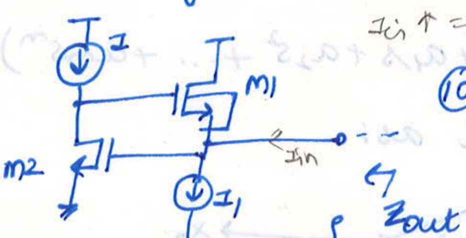
$\Rightarrow$ v_{g3} decreases

$\Rightarrow$ Feedback acts to reduce i_x ($\Rightarrow Z_{out}\uparrow$)

<u>Ex 5</u> <u>Voltage source</u> ($I_{ds1}\downarrow$) $I_{in} + I_{ds1} = I_1$

$I_{in}\uparrow \Rightarrow \uparrow V_{gs1}\uparrow \Rightarrow V_{s1}\uparrow \Rightarrow V_{g2}\uparrow \Rightarrow V_{g1}\downarrow$

M1: source follower, $\therefore V_{s1}$ need not change a lot to accomodate current changes.



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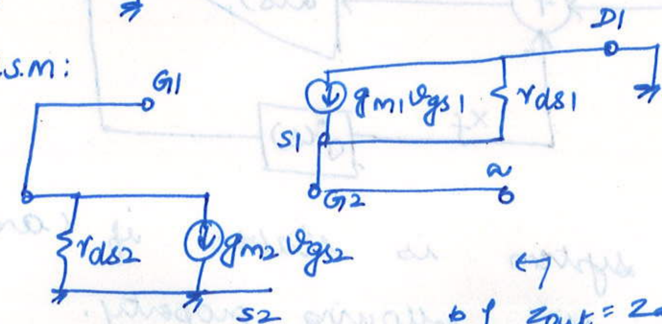
Low freq. S.S.M:



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$Z_{out} = Z_{ab}$

<u>BIR</u>: Let $g_{m1} v_{gs1}$ = ref source $\therefore v_{gs1} = x_i$, $\alpha = g_{m1}$,

$Z_{ab}^0 = r_{ds\phi}$, Exercise: show: $T_{sc} = 0$; $T_{oc} = + g_{m1} g_{m2} r_{ds1} r_{ds2}$

$\therefore Z_{ab} \cong \dfrac{1}{g_{m1} g_{m2} r_{ds2}}$ = small!! typ ~ few ohms.

Feedback reduced o/p impedance by factor of $g_{m2} r_{ds2}$

LEC #11 FEB 19'08

Stability:

Let $A(s) = \dfrac{a_0' + a_1's + a_2's^2 + \ldots + a_m's^m}{b_0' + b_1's + b_2's^2 + \ldots + b_n's^n}$; $m \le n$. (1)

If $A(0) \ne 0$ or ∞, can write (1) as,

$$A(s) = A(0)\,\frac{1 + a_1 s + a_2 s^2 + \ldots + a_m s^m}{\underbrace{1 + b_1 s + b_2 s^2 + \ldots + b_n s^n}_{= T(s)}} \quad (2)$$

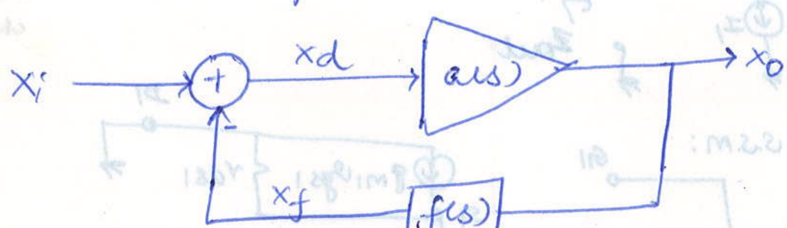
(Other choices of $T(s)$ are possible if we allow $a(s)$ to have poles).

$\therefore A(s) = \dfrac{a(s)}{1 + a(s)f(s)}$

where $a(s)f(s) = T(s)$

and $a(s) = A_0\,(1 + a_1 s + a_2 s^2 + \ldots + a_m s^m)$

$\therefore$ (2) can be implemented as:



Def: A system is stable if (and only if) it satisfies the following property.

For each bounded I/P, $X_i(t)$, the output signal, $x_o(t)$ must also be bounded.

($x(t)$ is bounded means $\exists$ (there exists) $B \in \mathbb{R}$ such that $|x(t)| < B \ \forall t$)

$\therefore$ "Stability" $\equiv$ $\underbrace{\text{"bounded i/p, bounded o/p"}}_{\text{"BIBO"}}$ stability

Claim 1: Let $h(t)$ be the impulse response of a linear time invariant system, and let $H(s)$ be its transfer function. (If not LTI, $H(s)$ is not defined).

Then the system is stable iff either of the following hold:-

(i) All poles of $H(s)$ are in the L.H.P. (not including imaginary axis) (i.e. if s_0 is a pole, then $Re\{s_0\}<0$)

(ii) $\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$

Proof: Exercise (Review Material) (Common Interview Q!!)

Claim 2: If an LTI has a transfer function given by ②, then it is stable iff

$$T(s_0) = -1 \Rightarrow Re\{s_0\}<0$$

Proof: Exercise.

Def "Nyquist Plot" $\equiv$ plot of $Im\{T(j\omega)\}$ vs. $Re\{T(j\omega)\}$ as ω increases from $-\infty$ to ∞.

Nyquist criterion:

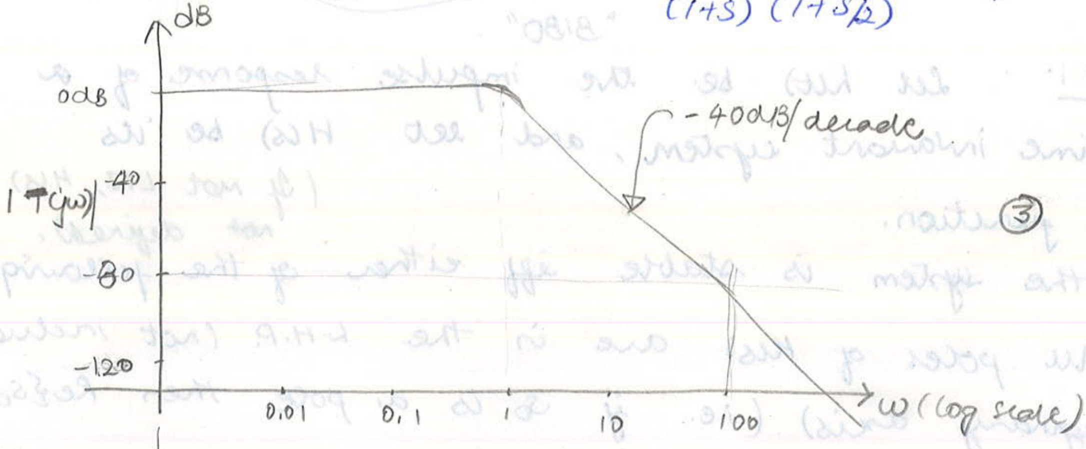
Provided there are no R.H.P. pole-zero cancellations in $T(s) = a(s)f(s)$, an LTI system is stable iff the net # of counter clockwise encirclements of the point $(-1,0)$ by the Nyquist plot equals the # of RHP poles in $T(s)$.

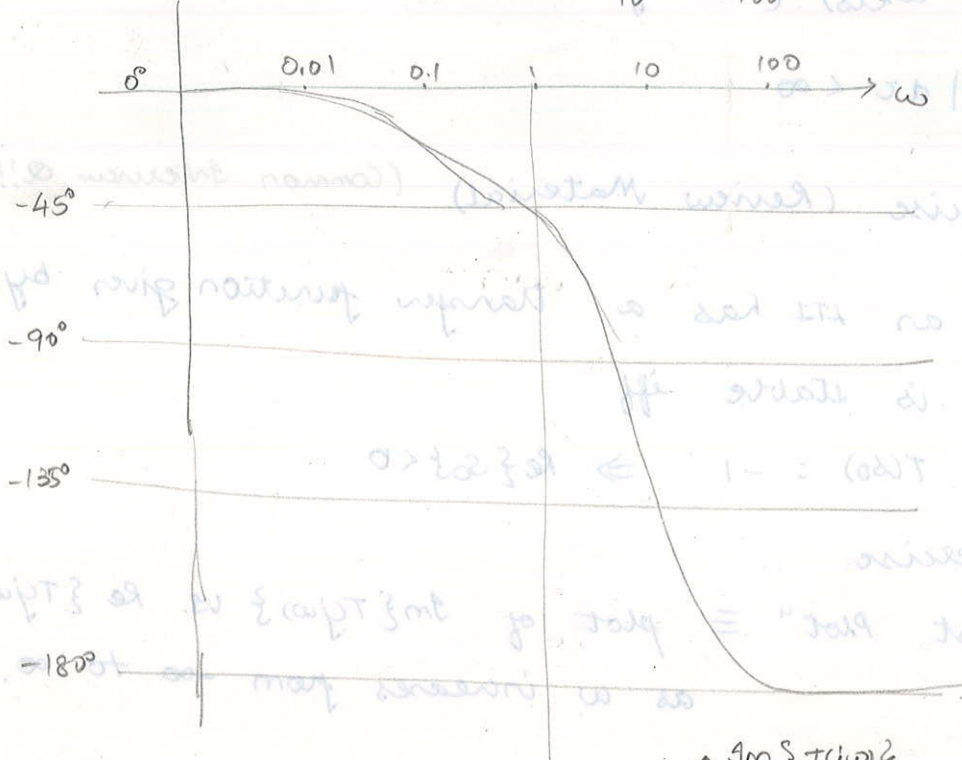
& does this mean that if O.L. is unstable C.L. is unstable?

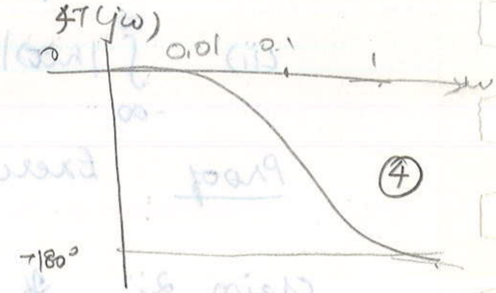
$\therefore$ Nyquist criterion $\Rightarrow$ stability test. } not so useful these days 'coz of computers.

Will soon see: Nyquist criterion $\Rightarrow$ insight $\Rightarrow$ concepts of phase & gain margins } Very useful.

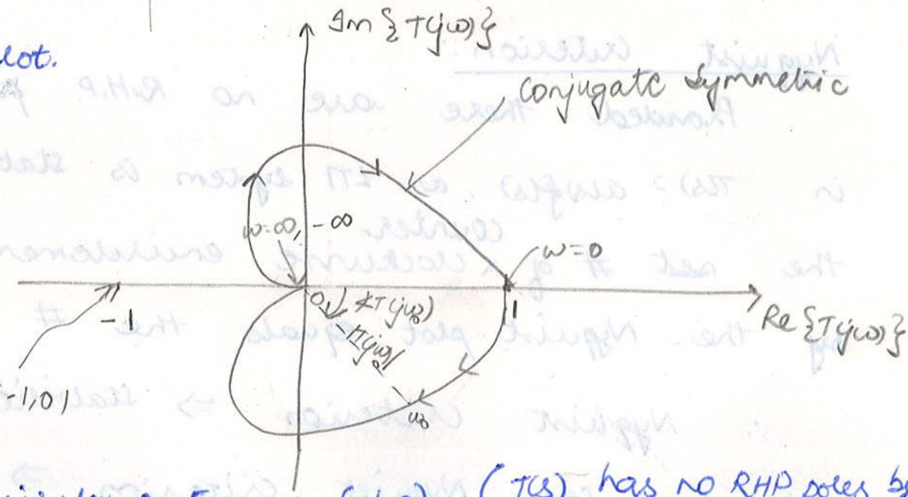
Ex 1:- Suppose $T(s) = \dfrac{1}{(1+s)(1+s/2)}$ poles : $-1, -2$







③, ④ ⇒ Nyquist plot.



⇒ No encirclements of $(-1,0)$ ($T(s)$ has no RHP poles by inspection)

Nyquist criterion ⇒ System w/ transfer function $A(s) = \dfrac{a(s)}{1+T(s)}$ is stable (provided there are no pole-zero cancellations in $a(s) f(s)$)

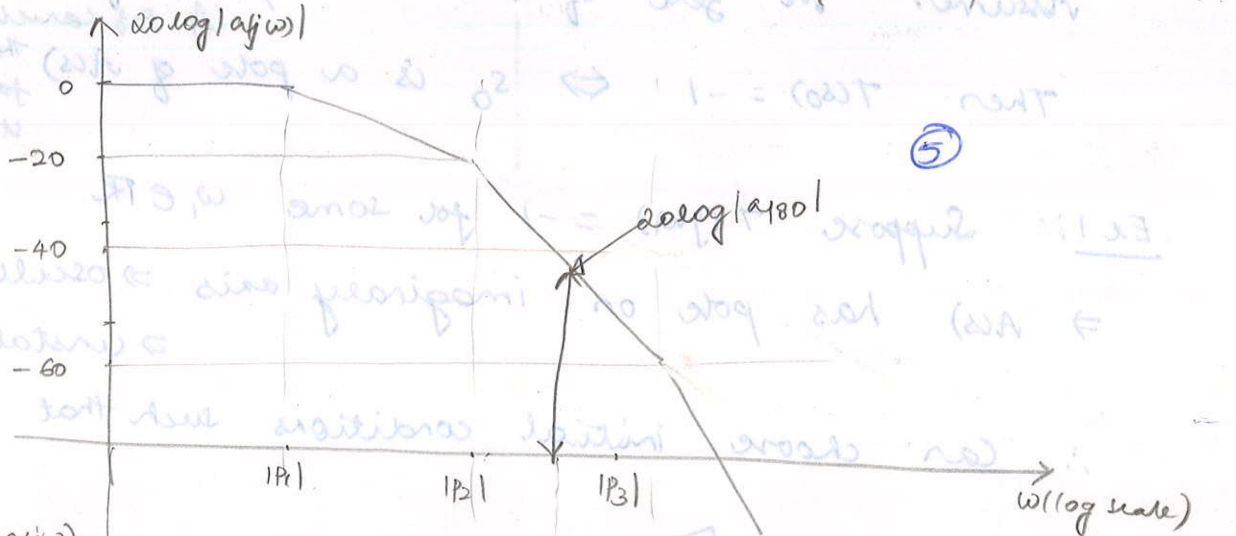
Why?

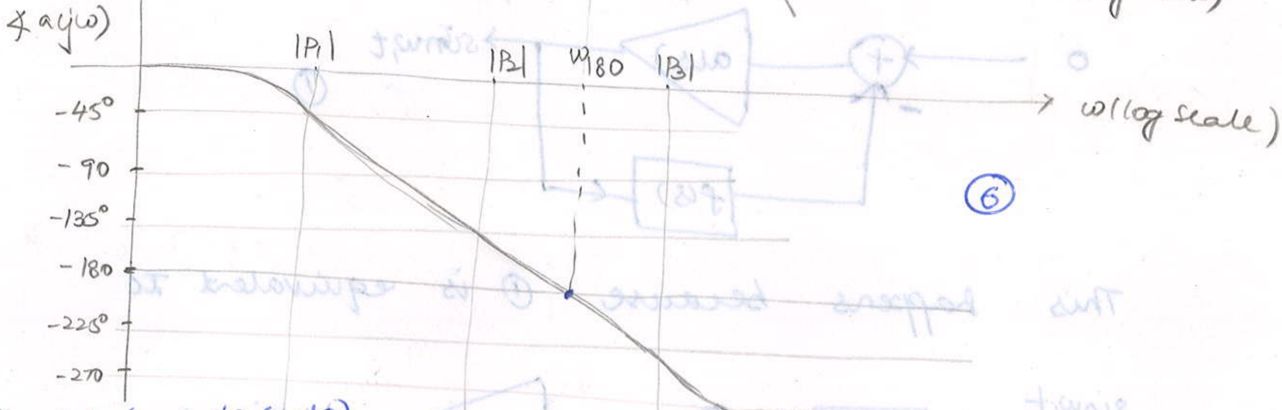
Ex2: Suppose $a(s) = \dfrac{a_0}{(1-s/p_1)\,(1-s/p_2)\,(1-s/p_3)}$ and

f = const.

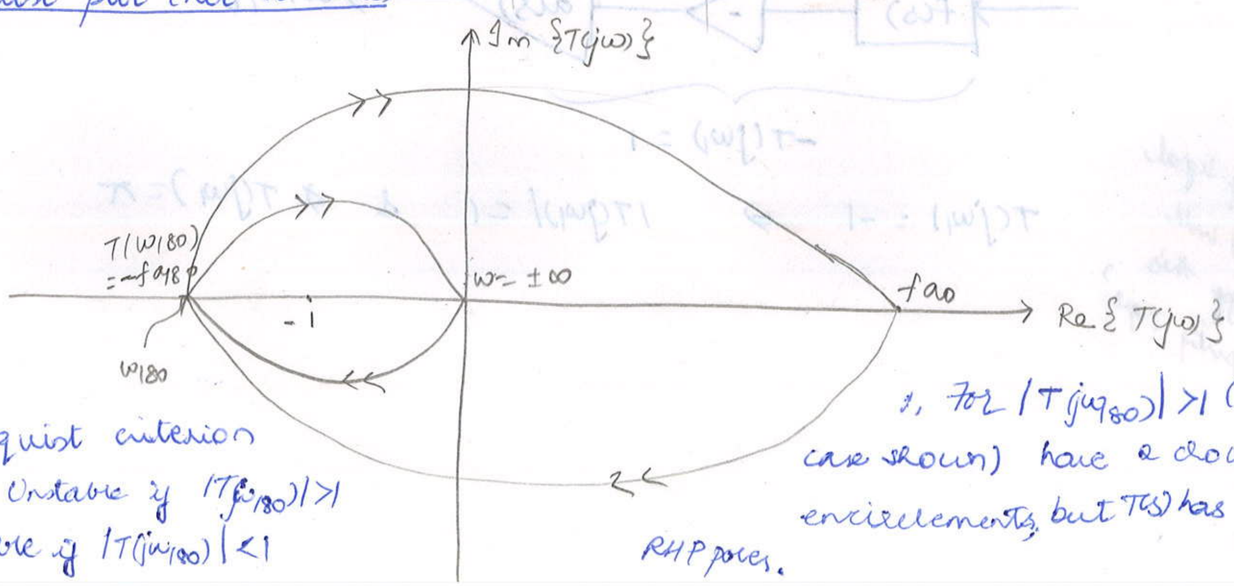
$\therefore \quad T(j\omega) = f \cdot a(j\omega)$

For $|p_1| \cong \frac{1}{10}|p_2| \cong \frac{1}{100}|p_3|$ & $Re\{p_i\} < 0 \quad i = 1,2,3$
(i.e. all poles in L.H.P.)





Nyquist plot (not to scale)



$\therefore$ Nyquist criterion
$\Rightarrow$ Unstable if $|T(j\omega_{180})| > 1$
$\Rightarrow$ stable if $|T(j\omega_{180})| < 1$

$\therefore$ For $|T(j\omega_{180})| > 1$ (the case shown) have a clockwise encirclements, but $T(s)$ has no RHP poles.

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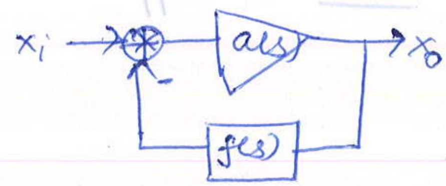
Nyquist Criterion (contd.)

Ret $A(s) = \dfrac{a(s)}{1+a(s)f(s)}$ $\Leftrightarrow$



$T(s) = a(s)f(s)$

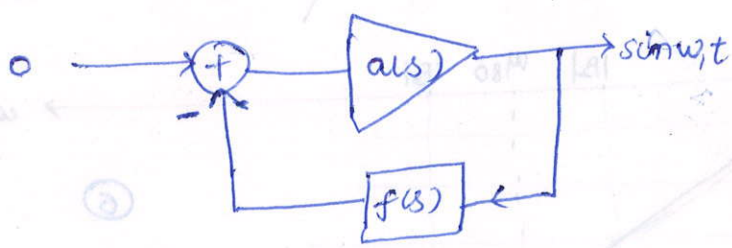
Assume: No zero of $f(s)$ is a pole of $a(s)$ (If p-z cancellation, then can falsely conclude stability)

Then $T(s_0) = -1 \Leftrightarrow s_0$ is a pole of $A(s)$

Ex 1: Suppose $T(j\omega_1) = -1$ for some $\omega_1 \in \mathbb{R}$

$\Rightarrow A(s)$ has pole on imaginary axis $\Rightarrow$ oscillation $\Rightarrow$ unstable.

$\therefore$ Can choose initial conditions such that



(1)

This happens because (1) is equivalent to

$-T(j\omega_1) = 1$

Q: Any signal $x(t)$ will satisfy this property right?

$T(j\omega_1) = -1 \Rightarrow |T(j\omega_1)| = 1$ & $\angle T(j\omega_1) = \pi$

Ex2 Suppose $T(s_0) = -1$ for $s_0 = \sigma_0 + j\omega_0$, $\sigma_0 \geq 0$

complex roots must occur in pairs right?

$\Rightarrow$ have ① w/ $\sin\omega_1 t$ replaced by $\underbrace{e^{\sigma_0 t}}_{\to\infty \text{ as } t\to\infty} e^{j\omega_0 t}$

$\Rightarrow$ 0 i/p $\Rightarrow$ Unbounded o/p. (Unstable)

Ex3 Suppose $A(s)$ is stable (i.e. the system whose T.F. / G.L.T.F. is $A(s)$ is stable)

but for some $\omega_1 \in \mathbb{R}$

$$|T(j\omega_1)| = 1 - |\epsilon| \quad \text{where } |\epsilon| = \text{small } \#$$

$$\measuredangle T(j\omega_1) = \pi$$

Then have ringing for any input step

(Basically underdamped system)
↓
but need complex conjugate poles?
↓
why is ringing good in regulators?

eg. $x_i =$  $x_o =$ 

In many systems, ringing is undesirable because it slows settling time.

$\Rightarrow$ Need to design w/ "gain margin" s.t. $|T(j\omega)| \neq 1$ when $\measuredangle T(j\omega) = \pi$ *(w/ sufficient "margin")* to minimise ringing.

Nyquist criterion = method of evaluating "marginal stability" using simulated or calculated freq. response plots.

Recall: Nyquist plot $\equiv$ Plot of $\text{Im}\{T(j\omega)\}$ vs $\text{Re}\{T(j\omega)\}$ as ω increases from $-\infty$ to ∞.

Nyquist criterion $\equiv$ An LTI system is stable iff the net # of counter clockwise encirclements of the point $(-1, 0)$ by the Nyquist plot equals the # of R.H.P. poles of $T(s)$.

i.e. We can have unstable O.L. systems which by f.b. become stable systems. (e.g. fighter planes)

The Nyquist criterion is based on "The Encirclement Property" (E.P.)

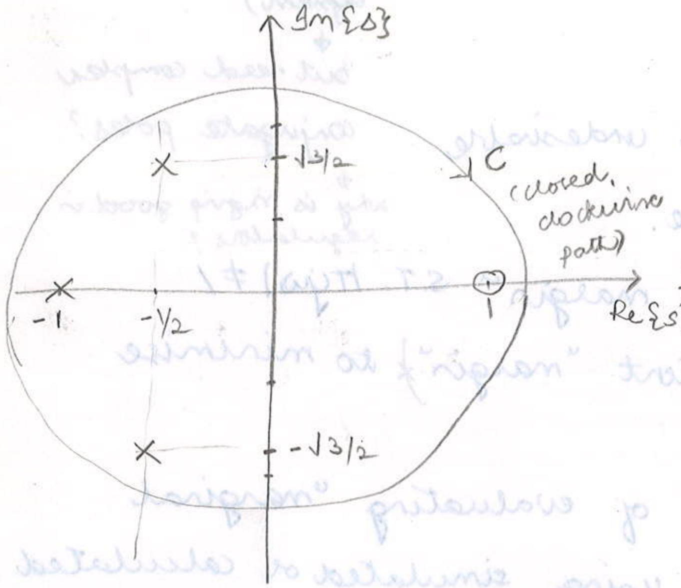
E.P. Let $R(s)$ be any rational function. The plot of $R(s)$ along a closed, clockwise path, C, in the complex s-plane encircles the point $s=0$ in a clockwise direction in a net number of times equal to the net number of zeros minus the number of poles within contour.

(Free to choose contour)

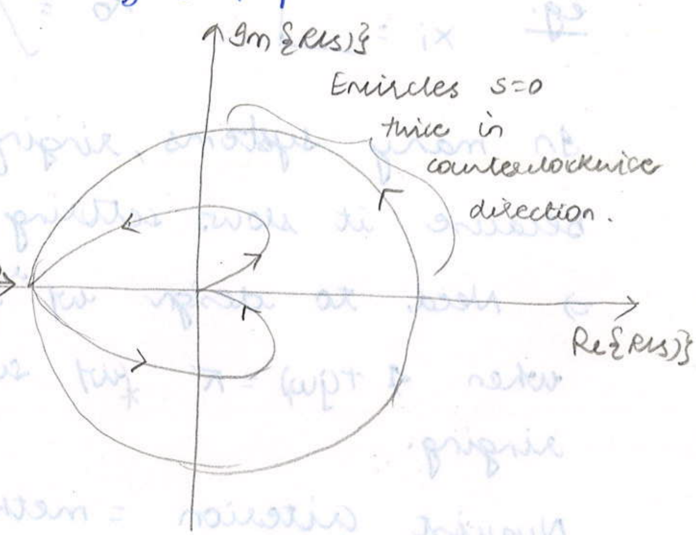
Rational
$\Rightarrow \#$ of $Z < \#$ of P.
net $\#$ of Z $-$ $\#$ of P $= -ve$.
$\Rightarrow$ counter-cw $= +ve$.

Ex 4: $R(s) = \dfrac{s-1}{(s+1)(s^2+s+1)}$ poles : -1, $-\frac{1}{2} \pm \frac{\sqrt{3}}{2} j$

zero : 1

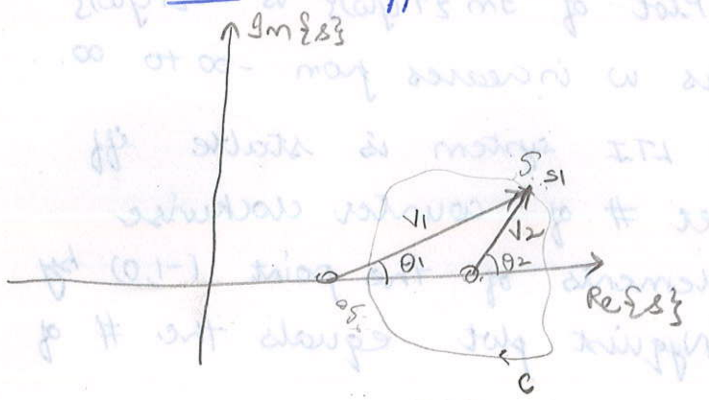


can show



Why does this work?

Ex 5 Suppose $R(s)$ has only 2 (RHP) zeros and no poles.



$(V_1, V_2 \in \mathbb{R}, >0)$

$\therefore R(s_1) = V_1 V_2 e^{j(\theta_1+\theta_2)}$ ②

Verify

$(s-z_1)(s-z_2)$

$\dfrac{V_1 e^{j\theta_1} \; V_2 e^{j\theta_2}}{V_3 e^{j\theta_3}}$

As C is traversed, net change in θ_1 is zero.
"net change in θ_2" is -360° (-2π)

② $\Rightarrow \angle R(s) = \theta_1 + \theta_2$

$\therefore$ As C is traversed, the net change in phase of $R(s)$ is -2π $\Rightarrow$ $R(s)$ evaluated

$\Rightarrow R(s)\big|_{s \in C}$ circles origin $(s=0)$ once (in clockwise direction)

contour

Proof of Nyquist criterion:

Restrictions of proof: 1) Rational $T(s)$

2) $|T(j\omega)| < \infty \quad \forall \, \omega \in \mathbb{R}$ (i.e. no poles on imaginary axis)

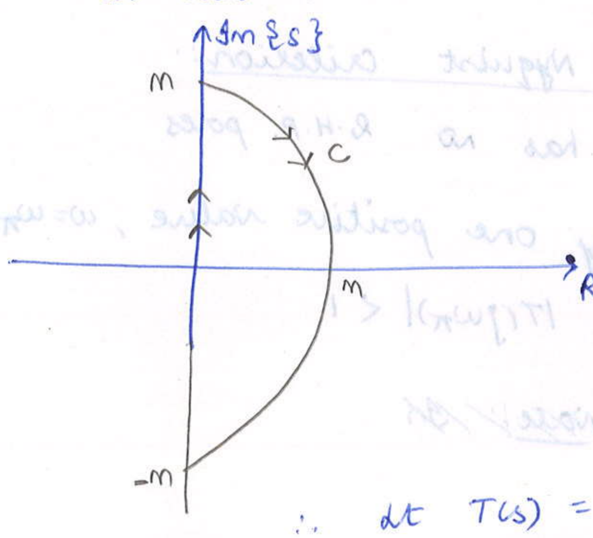
1) ≡ 3)?

3) $T(s)$ has # of poles $\geq$ # of zeros.

(But Nyquist criterion also applies w/o restrictions 1) & 3) and can be modified to handle poles on imaginary axis)

this is useful for integrators in F.B. loop (like in a PLL)

Consider contour C as



As $M \to \infty$, C contains all R.H.P. poles of $A(s)$

1), 3) $\Rightarrow$

$$T(s) = \frac{c_0 + c_1 s + \ldots + c_{N_1} s^{N_1}}{d_0 + d_1 s + \ldots + d_{N_2} s^{N_2}}$$

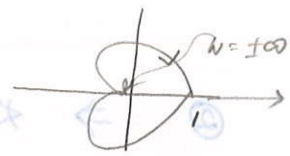
where $N_1 \leq N_2$.

$$\therefore \lim_{|s| \to \infty} T(s) = \begin{cases} \dfrac{c_{N_1}}{d_{N_1}} & \text{if } N_1 = N_2 \\ 0 & \text{if } N_1 < N_2 \end{cases} = \text{const.}$$

$\Rightarrow$ The plot of $T(s)$ along the semi-circular part of C approaches a single point on real axis as $M \to \infty$.

$\Rightarrow$ The Nyquist plot is equivalent to the plot of $T(s)$ along C as $M \to \infty$.

e.g. recall: Nyquist plot of $T(s) = \dfrac{1}{(1+s)(1+s/2)}$ is



$\Rightarrow$ Can apply encirclement property to Nyquist plot.

Let $R(s) = 1 + T(s)$. Then plot of $R(s)$ along C encircles origin exactly as many times as that of $T(s)$ encircles $(-1, 0)$.

$\therefore$ Encirclement property $\Rightarrow$ Net # of clockwise encirclements of $(-1,0)$ = # of zeros of $1+T(s)$ − # of poles of $1+T(s)$ inside C.

But
- zeros of $1+T(s)$ $\equiv$ poles of $A(s)$ (RHP)
- poles of $1+T(s)$ $\equiv$ poles of $T(s)$ (zeros of $A(s)$)
- inside C = all RHP as $M \to \infty$ $\quad A(s) \quad \dfrac{1}{1+T(s)}$ as $f(s)$

$\Rightarrow$ Nyquist criterion. $\quad N = Z - P$

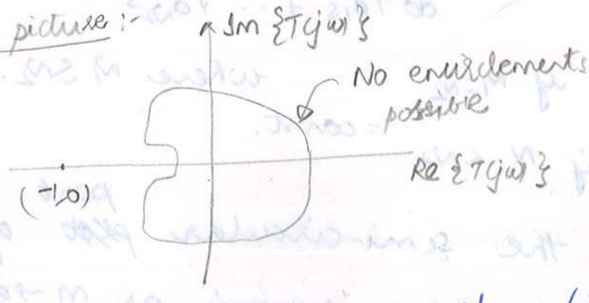
<u>Important Special Case of Nyquist Criterion</u>:

Suppose $\measuredangle T(0) = 0$ and $T(s)$ has no R.H.P. poles

If $|\measuredangle T(j\omega)| = \pi$ for only one positive value, $\omega = \omega_\pi$, then $A(s)$ is stable iff $|T(j\omega_\pi)| < 1$

<u>Note</u>: ~~If~~

<u>e.g. picture</u>:



<u>Note</u>: In general, $|T(j\omega_\pi)| > 1 \not\Rightarrow$ instability (!)